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# Module Overview

The Teaching Mathematics TEKS through Technology Professional development is designed to provide teachers an opportunity to increase their depth of understanding about the judicious use of technology in the mathematics classroom. Expected learning outcomes for participants include an understanding of how technology can:

- Provide access to a deeper understanding of mathematical content;
- Provide access to "real world" mathematical topics;
- Improve the economy and efficiency of teaching mathematics TEKS relative to time;
- Facilitate the use of various instructional tools in a mathematical setting.

The structure of the professional development will be designed around the inquiry based 5E instructional model. This model has a strong foundation in research and has been shown to be highly effective in instructional settings.

The components of the "5E" Instructional Model are:

**ENGAGE:**
The instructor initiates this phase by asking well-chosen questions, posing a problem to be solved, or showing something intriguing. The activity should be designed to interest participants in the problem and to make connections between past and present learning.

The goal of the Engage phase is to begin conversations about data. As participants see the value of data and the mathematics that can be explored and reinforced through the use of data, they will begin to seek data. Technology offers the tools to make sense of data efficiently. Technology also offers effective means for representing data so that analysis may take place. Participants work with data from the Internet, an almanac, data collection devices, and basic measuring tools. They compare the different methods and determine similarities and differences as well as the benefits of each method.

The presenter's role is to ask well-chosen questions to guide the activity but allow participants to proceed in a nonjudgmental fashion. These questions are provided in the leader notes of the training.

**EXPLORE/EXPLAIN:**
**Explore**
The exploration phase provides the opportunity for participants to become directly involved with the key concepts of the lesson through guided exploration that requires them to probe, inquire, and question. As we learn, the puzzle pieces (ideas and concepts necessary to solve the problem) begin to fit together or have to be broken down and reconstructed several times. In this phase, presenters observe and listen to participants as they interact with each other and the activity. Presenters ask probing questions to help participants clarify their understanding of major concepts and redirect the participants when necessary.

**Explain**
In the explanation phase, collaborative learning teams begin to logically sequence events and facts from the investigation and communicate these findings to each other and the presenter. The presenter, acting in a facilitation role, uses this phase to offer further explanation and provide additional meaning or information, such as formalizing correct terminology. Giving labels or correct terminology is far more meaningful and helpful in retention if it is done after the learner has had a direct experience. The explanation phase is used to record the learner's development and grasp of the key ideas and concepts of the lesson.

There are 3 Explore/Explain cycles in this module.

In the first Explore/Explain cycle, participants manipulate sketches created in dynamic geometric software. Problem-solving strategies of breaking a large problem into smaller components and working backwards are utilized to facilitate the constructions and development of geometry concepts.

In the second Explore/Explain cycle, participants use digital images to explore geometric properties such as parallel and perpendicular lines and planes, congruence, similarity, transformations, etc. Participants will then collect information to formulate and test conjectures about geometric properties. Participants will then compare and contrast traditional exploration methods with technological exploration.

In the third Explore/Explain cycle, participants will create a sketch using dynamic geometric software and collect and analyze data collected from their sketch using a variety of technologies. Problem-solving strategies of breaking a large problem into smaller components and working backwards are utilized to facilitate the constructions and development of geometry concepts.

The presenter's role in the Explore/Explain phases is to ask well-chosen questions to guide participants and clarify their understandings. These questions are provided in the leader notes of the training.

**ELABORATE:**
The elaboration phase allows for participants to extend and expand what they have learned in the first three phases and connect this knowledge with their prior learning to create understanding. It is critical that presenter verify participants' understanding during this phase.

In the Elaborate phase a problem is posed to the participants. Participants will utilize technology to plane, construct, and analyze a complex geometric figure. They will compare and contrast a pencil and paper approach to a technology based approach. Participants will then apply or extend their understandings acquired in the professional development by generating a list of attributes to guide judicious use of technology.

The presenter's role in the Elaborate phase is to ask well-chosen questions to guide participants' and extend their understandings. These questions are provided in the leader notes of the training.

**EVALUATE:**
Throughout the learning experience, the ongoing process of evaluation allows the instructor to determine whether or not the participant has reached the desired level of understanding of the key ideas and concepts. More formal evaluation can be conducted at this phase.

Participants will review the instructional phases of this professional development and the classroom-ready lessons according to the list of attributes generated in the elaborate phase of the professional development. Revisions to the list of attributes may occur. Participants will engage in discussion about how each lesson exhibits a judicious use of technology; i.e., participants will address the question, "How does the use of technology in this student lesson help me teach the concepts and skills more effectively and efficiently?"

The presenter's role in the Evaluate phase is to ask well-chosen questions to assess participants' understandings as they evaluate student lessons for judicious use of technology. These questions are provided in the leader notes of the training.

**STUDENT LESSONS**
This training is specifically designed for adult learners. Student lessons with detailed teacher notes and resources are provided to facilitate the implementation of the knowledge acquired by teachers in the professional development.

# TABLE OF CONTENTS

## Technology Tutorials

**Geometry**

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

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| **Basic Elements** | (G.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system. | The student is expected to:<br>(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;<br>(B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and<br>(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries. |
| **Making Conjectures** | (G.2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures. | The student is expected to:<br>(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and<br>(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic. |

The provisions of this subchapter were adopted by the State Board of Education in February 2005 to be implemented beginning with the 2006-2007 school year. This implementation date supersedes any other implementation date found in this subchapter.

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| **Axiomatic Systems** | (G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements. | The student is expected to:<br>(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;<br>(B) construct and justify statements about geometric figures and their properties;<br>(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;<br>(D) use inductive reasoning to formulate a conjecture; and<br>(E) use deductive reasoning to prove a statement. |
| **Representations** | (G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems. | The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems. |
| **Patterns and Transformations** | (G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems. | The student is expected to:<br>(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;<br>(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;<br>(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and<br>(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples. |
| **Solids: Representations** | (G.6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems. | The student is expected to:<br>(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;<br>(B) use nets to represent and construct three-dimensional geometric figures; and<br>(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems. |
| **Coordinate Geometry** | (G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. | The student is expected to:<br>(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;<br>(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and<br>(C) derive and use formulas involving length, slope, and midpoint. |

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| **Area, Surface Area, Volume** | (G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. | The student is expected to:<br>(A) find areas of regular polygons, circles, and composite figures;<br>(B) find areas of sectors and arc lengths of circles using proportional reasoning;<br>(C) derive, extend, and use the Pythagorean Theorem; and<br>(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations. |
| **Properties of Planar and Solid Figures** | (G.9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures. | The student is expected to:<br>(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;<br>(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;<br>(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and<br>(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models. |
| **Congruence** | (G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems. | The student is expected to:<br>(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and<br>(B) justify and apply triangle congruence relationships. |
| **Proportion and Similarity** | (G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems. | The student is expected to:<br>(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;<br>(B) use ratios to solve problems involving similar figures;<br>(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and<br>(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems. |

| **Algebra 1** |
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(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

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| **Foundations of Functions: Functional Relationships** | (A.1) **Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. | The student is expected to:<br>(A) describe independent and dependent quantities in functional relationships;<br>(B) gather and record data and use data sets to determine functional relationships between quantities;<br>(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;<br>(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and<br>(E) interpret and make decisions, predictions, and critical judgments from functional relationships. |

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| **Foundations of Functions: Graphical Representations** | (A.2) **Foundations for functions.** The student uses the properties and attributes of functions. | The student is expected to:<br>(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;<br>(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;<br>(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and<br>(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations. |
| **Foundations: Using Variables** | (A.3) **Foundations for functions.** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. | The student is expected to:<br>(A) use symbols to represent unknowns and variables; and<br>(B) look for patterns and represent generalizations algebraically. |
| **Foundations: Symbolic Manipulation** | (A.4) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. | The student is expected to:<br>(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;<br>(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and<br>(C) connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$. |
| **Linear Functions: Representations** | (A.5) **Linear functions.** The student understands that linear functions can be represented in different ways and translates among their various representations. | The student is expected to:<br>(A) determine whether or not given situations can be represented by linear functions;<br>(B) determine the domain and range for linear functions in given situations; and<br>(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions. |

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| **Linear Functions: Meanings of Slope and Intercepts** | (A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. | The student is expected to:<br>(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;<br>(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;<br>(C) investigate, describe, and predict the effects of changes in $m$ and $b$ on the graph of $y = mx + b$;<br>(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and $y$-intercept;<br>(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;<br>(F) interpret and predict the effects of changing slope and $y$-intercept in applied situations; and<br>(G) relate direct variation to linear functions and solve problems involving proportional change. |
| **Linear Functions: Solving Problems** | (A.7) **Linear functions.** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. | The student is expected to:<br>(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;<br>(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and<br>(C) interpret and determine the reasonableness of solutions to linear equations and inequalities. |
| **Systems of Linear Equations** | (A.8) **Linear functions.** The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. | The student is expected to:<br>(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;<br>(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and<br>(C) interpret and determine the reasonableness of solutions to systems of linear equations. |
| **Quadratic Functions: Graphical Representation** | (A.9) **Quadratic and other nonlinear functions.** The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions. | The student is expected to:<br>(A) determine the domain and range for quadratic functions in given situations;<br>(B) investigate, describe, and predict the effects of changes in $a$ on the graph of $y = ax^2 + c$;<br>(C) investigate, describe, and predict the effects of changes in $c$ on the graph of $y = ax^2 + c$; and<br>(D) analyze graphs of quadratic functions and draw conclusions. |

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| **Solving Quadratic Equations** | (A.10) **Quadratic and other nonlinear functions.** The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. | The student is expected to:<br>(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and<br>(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (*x*-intercepts) of the graph of the function. |
| **Other Nonlinear Functions** | (A.11) **Quadratic and other nonlinear functions.** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. | The student is expected to:<br>(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;<br>(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and<br>(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods. |

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# Presenter Preparation Checklist

**Table set (per group of 4):**

- □ Post-it notes
- □ Rulers
- □ Tape measures (metric)
- □ Highlighters
- □ Post-it flags
- □ Tape
- □ Masking tape
- □ Flip chart markers
- □ Sticky dots
- □ Pencils
- □ **Technology Tutorial T$^2$** binder (one per computer)
- □ Chart paper

**Manipulatives/Materials**

Engage:

- □ Rulers-1 per participant
- □ Polygons cut out from Activity Master

Explore/Explain 2:

- □ Rulers-1 per participant
- □ Protractors-1 per participant

Elaborate:

- □ Rulers-1 per participant
- □ Protractors-1 per participant
- □ Compass-1 per participant
- □ Patty paper-several sheets available per participant

Evaluate:

- □ Sentence strips – blue and yellow, one of each per participant

**Prepare**

Engage:

- □ Copy on cardstock and cut out polygons from the Activity Master—one set per Polygons Rule station.
- □ Copy on cardstock and cut out Data Station Tents—Polygons Rule, Techno Polly
- □ Chart Paper
  - Statements about technology with Likert scale—one per statement.
  - Reflections on Data Venn Diagram (1 per 12 participants)

Elaborate:

- □ Copy on cardstock and cut out Rose Hint Cards from the Activity Master—one set per group of 2 participants.

**Technology**

- □ Presentation computer loaded with most recent update of:
  - PowerPoint (optional)
  - Geometer's Sketchpad
  - TI-Interactive
  - TI-Connect
  - Excel
  - Word
  - Internet access
  - Hyperlink document (optional)
  - Sketches found on the CD Resource Disk—labeled on the desktop as "Geometry Jump Drive"
- □ Data projector
- □ Overhead projector
- □ One computer per two participants loaded with most recent update of:
  - Geometer's Sketchpad
  - TI-Interactive
  - TI-Connect
  - Excel
  - Word
  - Internet access
  - Hyperlink document (optional)
  - Sketches found on the CD Resource Disk—labeled on the desktop as "Geometry Jump Drive"
- □ TI-83/84 calculator – one per participant
  - Graph link (optional)
- □ Jumpdrives (optional)

**Transparencies or Power Point Slides**

Engage:

- Reflections on Data
- Debriefing the Exploration of Data

Explore/Explain 1:

- Putting It All Together

Explore/Explain 2:

- Explore the World with Geometric Properties

Explore/Explain 3:

- Dome Floor Dilemma “Posing the Problem”
- Explain

Elaborate:

- Rose Window
- Transparency 1:  Looks Like—Sounds Like
- Transparency 2: Looks Like, Sounds Like
- Teaching Strategies
- Student Research

Evaluate:

- Encouraging Judicious Use of Technology

**Handouts**

Prepare one folder for each participant to use through out the training.  The handout for Planning for Intentional Use of Data in the Classroom from the Engage phase and the Explore/Explain phases should all be copied on the same particular color (i.e. green). The other handouts should be copied on different colors for each phase (i.e.  light pink for the Engage and light blue for Explore/Explain 1, etc.).  It also might be helpful to staple these colored pages together.

Engage:

- Polygons Rule:  Data Collection
- Polygons Rule:  Questions About Data
- Techno Polly:  Data Collection
- Techno Polly:  Questions About Data
- Reflections on Data
- Debriefing the Exploration of Data
- Polly Polly In Come Free Intentional Use of Data in the Classroom (copy on green paper)

Explore/Explain 1:

- □ Polygarden Landcaping Company
- □ Putting It All Together
- □ Polygarden Landscaping Company Intentional Use of Data (copy on green paper)

Explore/Explain 2:

- □ Copy of a magazine cover
- □ Sketchpad Skills Investigation
- □ Explore the World with Geometric Properties
- □ Geometric Properties and Sketchpad Skills Intentional Use of Data in the Classroom (copy on green paper)

Explore/Explain 3:

- □ Dome Floor Dilemma
- □ Analyze the Data
- □ Explain
- □ Dome Floor Dilemma Intentional Use of Data in the Classroom (copy on green paper)

Elaborate:

- □ Ring Around the Rose Window
- □ Understanding the Problem and Planning the Solution
- □ Constructing the Rose

Evaluate:

- □ Gallery Walk Observations
  - Polygarden Landscaping Company
  - Sketchpad Skills Investigation and Exploring the World
  - Dome Floor Dilemma
  - Ring Around the Rose Window

# Polly Polly In Come Free

## Engage

### Purpose:

Provide participants the opportunity to investigate a variety of data derived from the measurement of a variety of polygons. Assess participants' experience and comfort level with various avenues and tools for collecting data. Compare and contrast the use of technology-based exploration and technology-free traditional methods.

### Descriptor:

Participants will rotate between two stations to gather and explore data:

- Polygons Rule: technology-free traditional method
- Techno Polly: technology-based method

Upon completion of both activities, participants will compare and contrast their experiences. Participants will then be introduced to the formulation of questions that will spark data collection and investigation.

### Duration:

2 hours

### TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, powerful and accessible hand-held calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.2B Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

G.5A Use numeric and geometric patterns to develop algebraic expressions representing geometric properties and to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

## TAKS Objectives:

- Objective 3: Linear Functions
- Objective 4: Formulate and Use Linear Equations and Inequalities
- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of geometric relationships and shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

## Technology:

- Dynamic geometry software (Geometer's Sketchpad)

## Materials:

**Advance Preparation:**

- Participant access to computers with Geometer's Sketchpad(latest version update available from http://www.keypress.com/sketchpad) and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool.
- TechnoPolly.gsp sketch loaded in a folder on the desktop entitled Geometry Jump Drive.
- Cut out sets of polygons (Activity Master) on cardstock.
- Chart paper statements about technology, one statement per page.
- Chart paper Venn diagram Reflections on Data

**For each group of four:**

- Rulers
- 1 set of card stock polygons (cut out).

**For each participant:**

- Transparency Pen

Handouts

- Polygons Rule: Data Collection
- Polygons Rule: Questions About Data
- Techno Polly: Data Collection
- Techno Polly: Questions About Data
- Reflections on Data
- Debriefing the Exploration of Data
- Polly Polly In Come Free Intentional Use of Data (printed on green paper)

## Polly Polly Income Fee—Leader Notes:

*The goal of the Engage phase is to begin conversations about data. As teachers see the value of data and the mathematics that can be explored and reinforced through the use of data, they will begin to seek out data. Technology offers the tools to make sense of data in an efficient way. Technology also offers effective means for representing data so that analysis may take place. Participants should interact with each other. The presenter(s) should be moving around the room facilitating the activity. Use the* ***Facilitation Questions*** *to guide and redirect participants, as needed.*

1. *Record the following statements on chart paper. Post these statements around the room.*

Technology offers the opportunity to strengthen mathematical learning in my classroom.

| | |
|---|---|
| Strongly Disagree | Strongly Agree |

Students should learn first with paper-and-pencil methods and then with technology.

| | |
|---|---|
| Strongly Disagree | Strongly Agree |

My students know how to discern which of these methods best serves the purposes of a given problem: mental strategies, paper-and-pencil techniques, and technology applications.

| | |
|---|---|
| Strongly Disagree | Strongly Agree |

The best technology tool for the geometry classroom is the dynamic geometric software.

| | |
|---|---|
| Strongly Disagree | Strongly Agree |

2. *As participants enter the session, direct them to respond to the posted statements by placing a marker, such as a sticky dot, in the location that best corresponds to their response. Use only one color of sticky dot for this activity.*

3. *As you provide a welcome and introduction to this professional development session, direct the participants' attention to the posted statements, sharing that continued reflection about these statements will be explored in greater detail during the course of this professional development.*

4. *Separate the participants into two groups. Explain that they will have 15 minutes for each activity. Half of them will do the* ***Polygons Rule*** *activity at their tables while the other half will do the* ***Techno Polly*** *activity at their computer station. Both groups will have the appropriate* ***Questions About Data*** *activity sheet and will be answering questions as they collect data. Float among the groups and use the* ***Questions About Data*** *to encourage discussion among the group members. Distribute handouts as appropriate. Switch the groups after 15 minutes. A count-down timer is a beneficial tool for keeping participants on task.*

5. *After both groups have completed the activities at each station distribute the* ***Reflection on Data*** *activity sheet (see Reflection on Data—Leader Notes).*

6. *After debriefing the Reflection on Data Venn diagram activity distribute the* ***Debriefing the Exploration of Data*** *activity sheet. Prompt the participants to reflect upon the discussion summarized by the Venn diagrams and record their responses to each of the questions posed on the activity sheet. After a few minutes of recording time, prompt the participants to share their responses with another participant. Debrief the responses in whole-group setting, keeping in mind that the goal of this phase of the professional development is to consider data.*

# Polygons Rule: Data Collection—Leader Notes

1. *Distribute a set of card stock polygons (cut out) to each group.*

2. *Distribute the* **Polygons Rule: Data Collection** and **Polygons Rule: Questions About Data** *activity sheet to each participant.*

3. *Prompt participants to measure all attributes of the polygons possible, recording their data on the hand out. If they need to draw lines on the polygons they can use transparency pens.*

4. *Prompt participants to answer the questions about data on the handout.*

## Polygons Rule: Questions About Data

*(possible participant answers)*

| **Data Source** | **Rulers** |
|---|---|
| **How would you describe this set of data? Why?** | *Numerical, because the data is actual measurements of different sized polygons.* |
| **What relationships occur within this set of data? Why?** | *Linear relationships such as side length to perimeter, because the length and perimeter are both linear measurements and as one changes, the other changes proportionally.*<br>*Quadratic relationships such as area to apothem, because the apothem length is one dimensional and as it changes it affects two dimensions in the area.*<br>*The relationship between the vertex angle and central angle is supplementary, because their sum is always $180^o$.* |
| **How would you represent this data? Why?** | *Graph it, to have a visual picture of the relationships.*<br>*Develop an equation, to emphasize the algebraic relationships.* |
| **What question(s) can we pose to students that this set of data helps to answer?** | *What is the relationship between the side length of a polygon and it perimeter? Justify your answer.*<br>*What is the relationship between the vertex angle and the central angle in any polygon? Justify your answer.*<br>*What is the relationship between the apothem length and the area of a polygon? Why?* |
| **How might this data extend what students already understand about our course content?** | *This would tie the geometry and algebra concepts together.* |

# Techno Polly—Leader Notes

1. *Have participants move to a computer. Two people per computer would be ideal, larger groups of 3 or 4 can work as well.*

2. *Distribute the* **Techno Polly: Data Collection** and **Techno Polly: Questions about Data** *activity sheets to each participant.*

3. *Participants will need to open the sketch* ***Techno Polly*** *in the Geometer's Sketchpad program. Directions about where this program is on their particular computer will need to be given at this time.*

## Techno Polly—Questions About Data

*(possible participant answers)*

| **Data Source** | **Geometer's Sketchpad** |
|---|---|
| **How would you describe this set of data? Why?** | *Numerical, because the data are actual measurements of different sized polygons.* |
| **What relationships occur within this set of data? Why?** | *Linear relationships such as side length to perimeter, because the length and perimeter are both linear measurements and as one changes, the other changes proportionally.*<br>*Quadratic relationships such as area to apothem, because the apothem length is one dimensional and as it changes it affects two dimension in the area.*<br>*The relationship between the vertex angle and central angle is supplementary because their sum is always 180°.* |
| **How would you represent this data? Why?** | *Graph it, to have a visual picture of the relationships.*<br>*Develop an equation, to emphasize the algebraic relationships.* |
| **What question(s) can we pose to students that this set of data helps to answer?** | *What is the relationship between the side length of a polygon and it perimeter? Justify your answer.*<br>*What is the relationship between the vertex angle and the central angle in any polygon? Justify your answer.*<br>*What is the relationship between the apothem length and the area of a polygon? Why?* |
| **How might this data extend what students already understand about our course content?** | *This would tie the geometry and algebra concepts together.* |

# Reflection on Data—Leader Notes

1. *Upon completing rotation through each station, reorganize participants into groups of 4. Prompt the participants to complete the* ***Reflections on Data*** *activity sheet individually. Allow approximately 5 minutes for the completion of these activity sheets.*

2. *While the participants are completing their individual* ***Reflections on Data*** *activity sheets, post 1 set of Venn Diagrams for every 12 participants.*

3. *Prompt participants to move to the chart paper Venn diagrams in groups of 12 by combining 3 existing groups of 4 participants. Share with participant that they will work silently in these groups of 12 to create summary Venn diagrams of the three groups' discussions.*

4. *Prompt the group to identify the person with the longest hair. This person will be the first recorder. Prompt this person to record one statement on the large chart paper Venn diagrams. The statement may be a personal observation or an observation from the group's Venn Diagrams.*

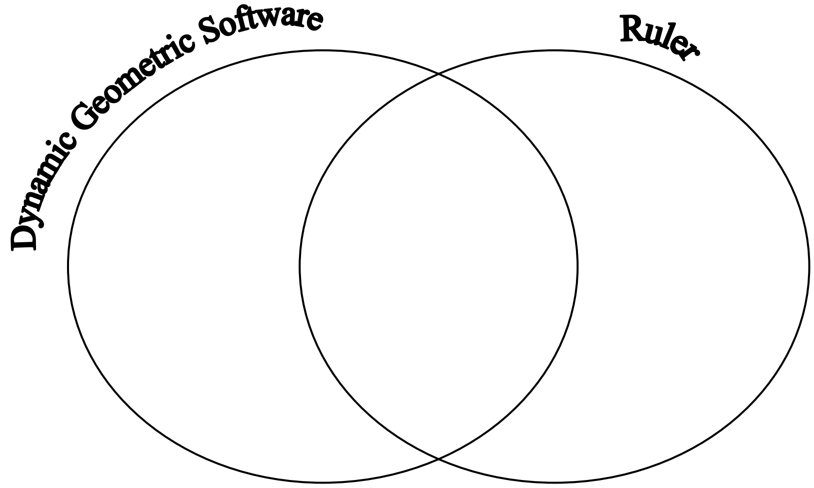
5. *Prompt the participant to pass the marker to a new recorder, preferably a person who was not a member of his or her discussion group. This person will record a new statement on the Venn diagram. Prompt participants to continue this process until each participant has had an opportunity to record a statement. Participants may record new observations or statements that occur as a result of seeing the reflections of others. Participants may record one statement, one at a time on the Venn diagram. This should be done silently with the whole group looking on and reading as one participant adds one statement to the Venn diagrams. Allow approximately 5 minutes for this process. Debrief using these facilitation questions:*

**Facilitation Questions**

- Which similarities did each group note?
- Which similarities were new to you?
- Which differences did each group note?
- Which differences were new to you?
- What are the benefits of a computer-based tool over a measurement tool?
  *Answers might include: The computer-based tool measures more accurately. The computer based-tool is quicker.*
- What are the benefits of a ruler over a computer-based tool?
  *Answers might include: The rule helps students develop a kinesthetic sense of measurement. Rulers are readily available.*

## Reflections on Data

**Complete the following Venn diagram to compare and contrast the uses of the dynamic geometric software and a ruler as data sources.**




**What are the benefits of using data derived from the dynamic geometric software?**
*Possible answers might include: The dynamic geometric software allowed for the same collection of data in a greatly reduced amount of time. The dynamic geometric software made it easy to focus on the mathematical concepts.*

**What are the benefits of using data derived from actual measurement?**
*Possible answers might include: Actual measurement helps students gain a tactile experience of measurement. Actual measurement helps students develop measurement conceptually.*

**How might these data sources function in a geometry classroom?**
*Possible answers might include: The dynamic geometric software can provide efficient exploration of data and quick analysis. The actual measurement might work better in situations where long distances or objects might need to be measured.*

## Debriefing the Exploration of Data—Leader Notes

1. *Distribute the **Debriefing the Exploration of Data** activity sheet. Prompt participants to reflect upon the discussions summarized by the Venn diagrams and record their responses to each of the questions posed on the activity sheet. After a few minutes of recording time, prompt the participants to share their responses with another participant. Debrief the responses in whole-group setting, keeping in mind that the goal of this phase of the professional development is to consider data.*

2. *Pose the questions listed below to the whole group. Explain to the participants that these questions serve as "filtering questions" when seeking to incorporate the use of data into classroom instruction.*

*a.* *What TEKS in Geometry does the use of data enhance?*

*b.* *What data are essential to enhance the study of these TEKS?*

*c.* *What question(s) does using data answer?*

*d.* *How does using data allow one to increase the rigor of the learning experience? How might using data move the learner from remembering, understanding, and applying to analyzing and evaluating?*

*e.* *What type of data would be most useful for the stated TEKS?*

*f.* *What setting will be available during instruction related to these mathematical goals?*

*g.* *What actual data source(s) may prove helpful in enhancing mathematical learning related to these TEKS?*

## Debriefing the Exploration of Data

**1. What questions can we ask as reflective practitioners to determine the appropriateness of a data source for promoting mathematical learning?**

*Participant answers might include: Will this help my students develop a conceptual understanding of the TEKS? Will this data source be interesting to my students?*

**2. How does the technology-based data offer an opportunity to strengthen mathematical learning?**

*Participant answers might include: Technology is quicker allowing students to focus on the analysis portion rather then collection portion of data. Technology is more accurate in measure allowing relationships easier to see.*

**3. How might hands-on activities complement the judicious use of technology?**

*Participant answers might include: This might allow for good compare and contrast situations. Technology can analyze data collected by hand.*

**4. What paper-and-pencil methods do students need to know to make sense of the data we explored?**

*Participant answers might include: They need to have some experience with creating a table, developing a function rule, plotting points, etc.*

## Polly Polly In Come Free Intentional Use of Data—Leader Notes

1. *Distribute the* ***Polly Polly Income Free Intentional Use of Data*** *activity sheet to each participant. Share with the participants that these reflective questions form the basis for the* ***Planning for Intentional Use of Data in the Classroom*** *activity done during the Evaluate phase of the professional development. Share with the participants that these filtering questions helped develop each of the activities contained within this professional development. This template will serve as a reflection tool to summarize each phase of the professional development in order to identify elements that support the judicious use of technology.*

2. *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.*

Facilitation Questions

- Which TEKS formed the primary focus of this activity?
- What additional TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. *As a whole group, share responses for two to three minutes.*

4. A*s a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Engage cycle. Allow three minutes for discussion.*

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

5. *As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.*

Facilitation Questions

- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- Could this activity be done using graphing calculators instead of computer-based applications? If so how?
- How might we have made additional use of available technologies during this activity?
- Why was technology withheld during the ruler measurements part of this activity?
- How does technology enhance learning?

6. *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the elaborate phase as prompts for generating attributes of judicious users of technology.*

# Polly Polly In Come Free Intentional Use of Data

*(possible participant answers)*

| | | | |
|---|---|---|---|
| **TEKS** | | *a(5), a(6), G.2B, G.5A* | |
| **Question(s) to Pose to Students** | **Math** | *What type of relationships could be found among the measurements you gathered?* | |
| | **Tech** | *How did technology help you with the gathering of data?* | |
| **Cognitive Rigor** | | **Knowledge** | √ |
| | | **Understanding** | √ |
| | | **Application** | √ |
| | | **Analysis** | √ |
| | | **Evaluation** | √ |
| | | **Creation** | √ |
| **Data Source(s)** | | **Real-Time** | *When using the computer sketch.* |
| | | **Archival** | *none* |
| | | **Categorical** | *none* |
| | | **Numerical** | *When using the rulers.* |
| **Setting** | | **Computer Lab** | *Each student uses the computer.* |
| | | **Mini-Lab** | *In groups students take turns or groups switch out.* |
| | | **One Computer** | *A student operates the control as other students read directions, entire class records data.* |
| | | **Graphing Calculator** | *Could be used to enter data and find relationships.* |
| | | **Measurement Based Data** | *Could be done at stations or individually.* |
| **Bridge to the Classroom** | | *This activity transfers directly to the classroom with the only modifications being the settings addressed above.* | |

## Data Station Tents
## Activity Master

(fold)

# Polygons Rule

Use the following tools to gather data and answer questions.

- Ruler—measuring with centimeters
- Handout—Polygons Rule:  Data Collection
- Handout—Polygons Rule:  Questions About Data

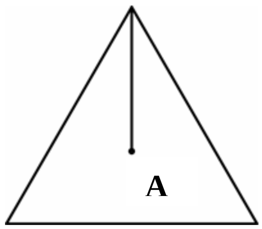
(fold)

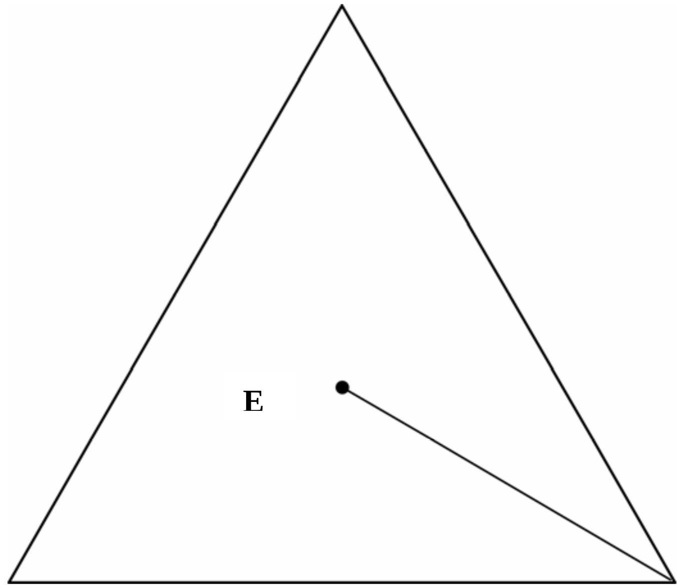
# Techno Polly

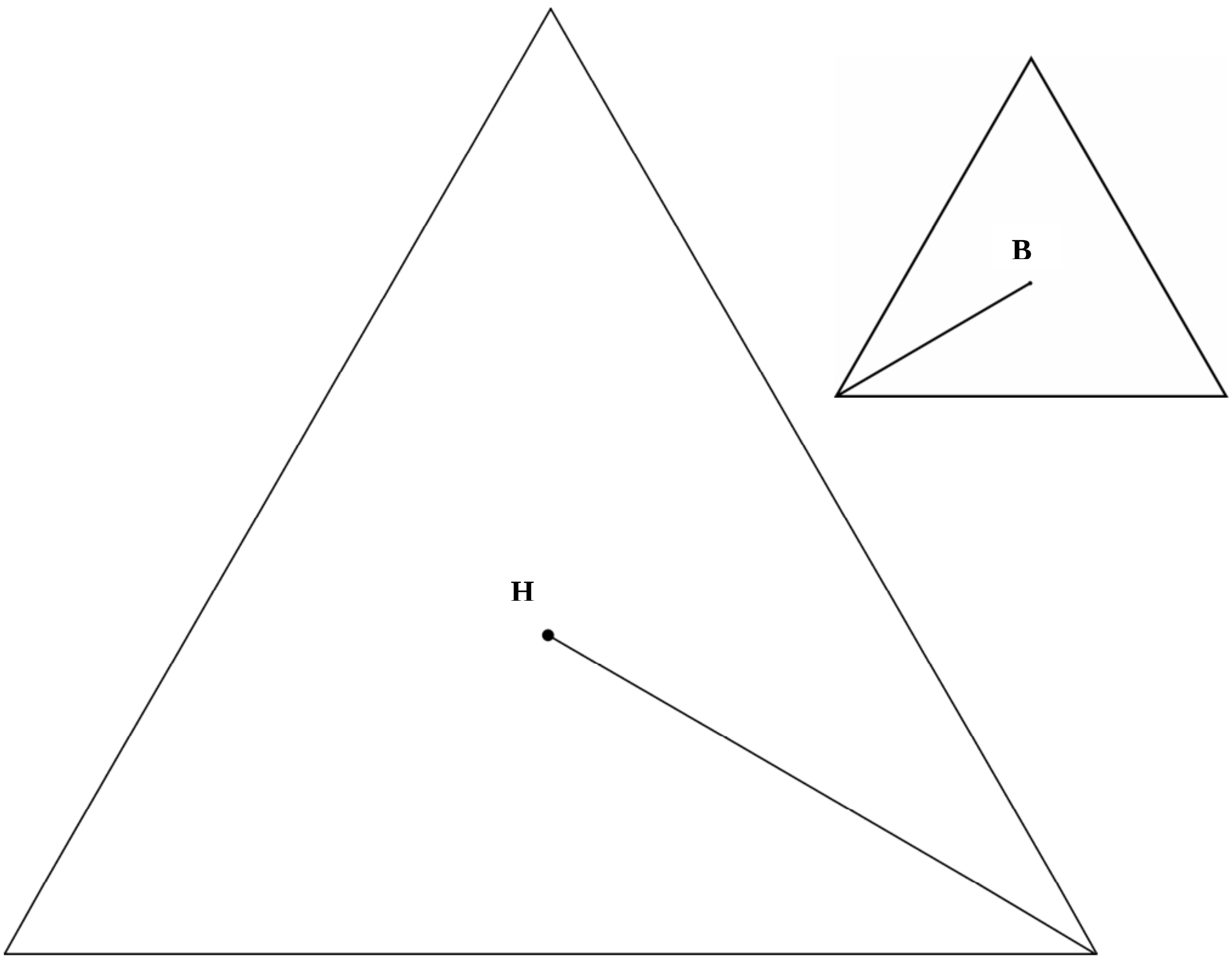
Use the following tools to gather data and answer questions.

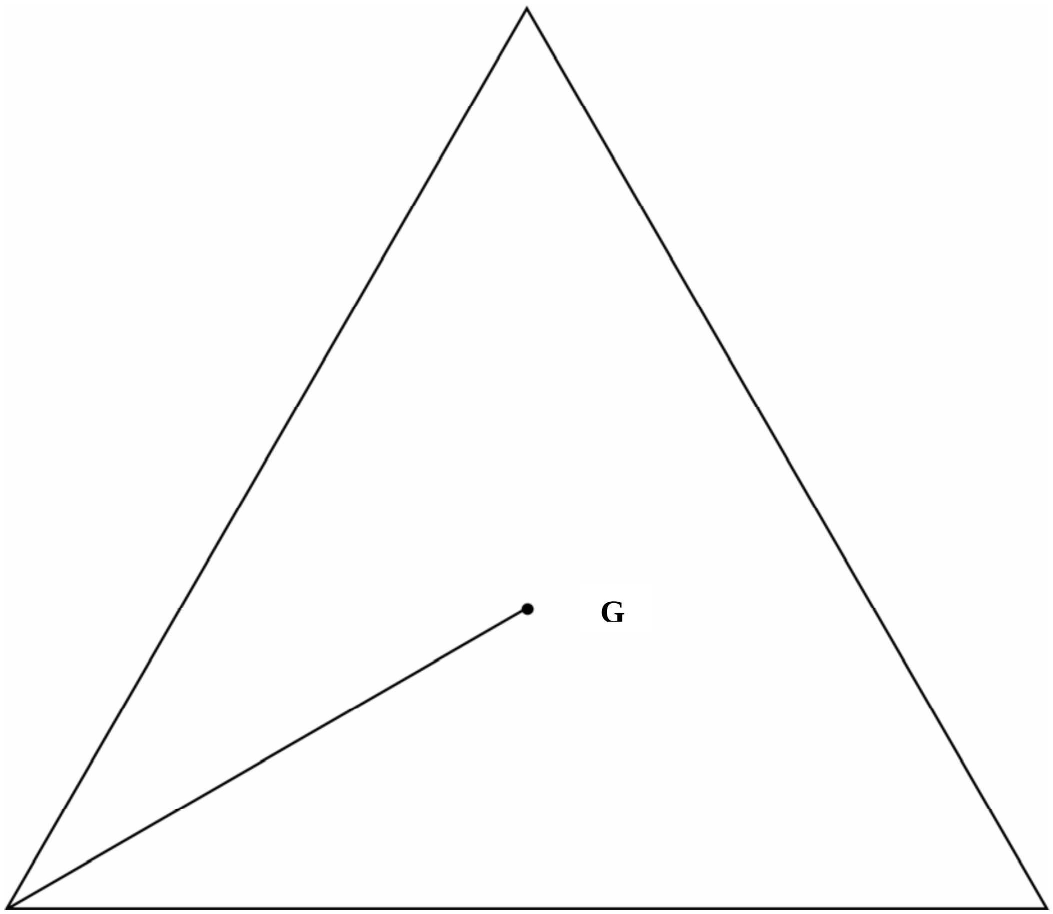
- **Techno Polly.gsp** sketch (found in the folder on the desktop "Geometry Jump Drive" in the "Geometry-Engage-Sketches)
- Handout—Techno Polly: Data Collection
- Handout—Techno Polly: Questions About Data

# Polygons Rule
# Activity Master

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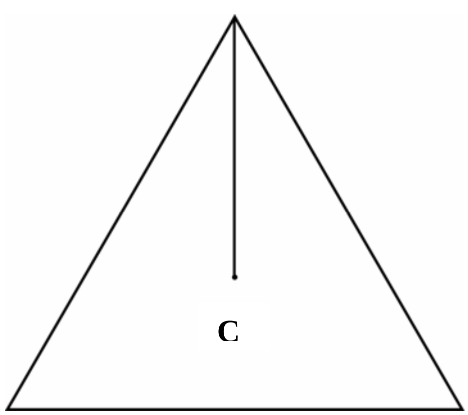
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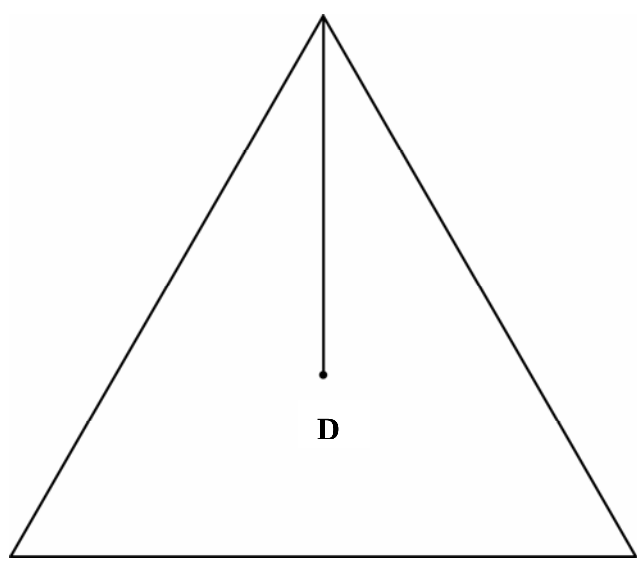
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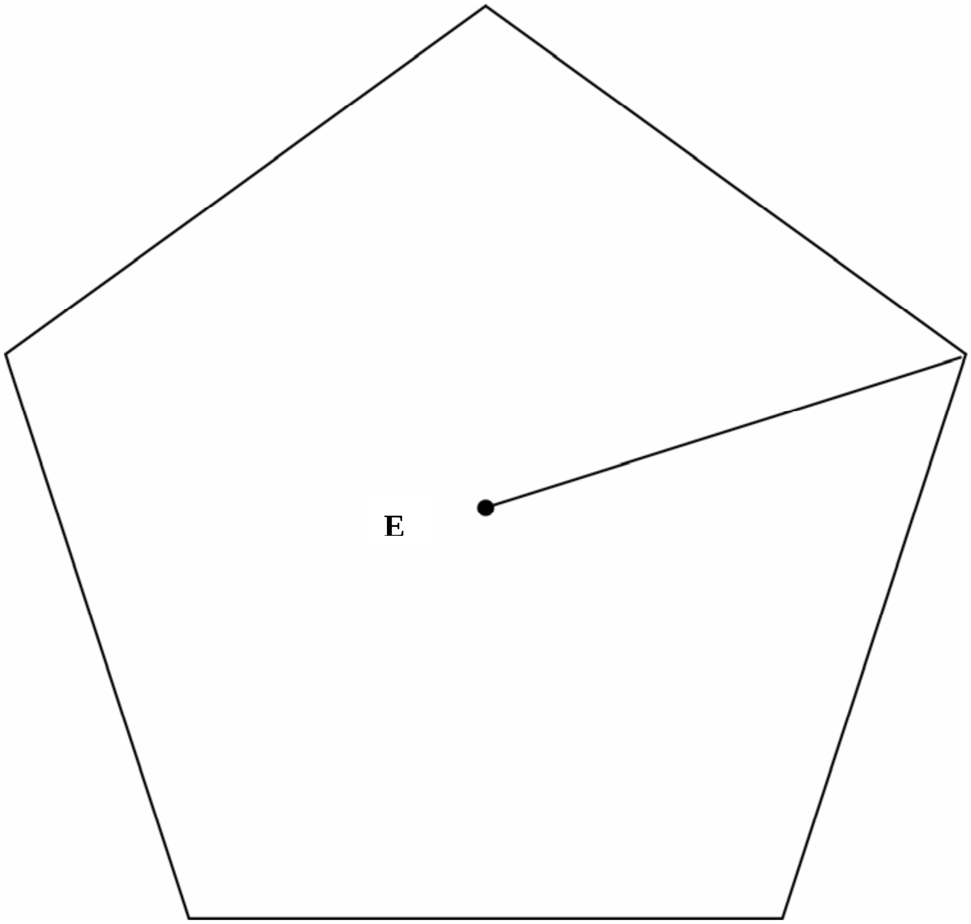
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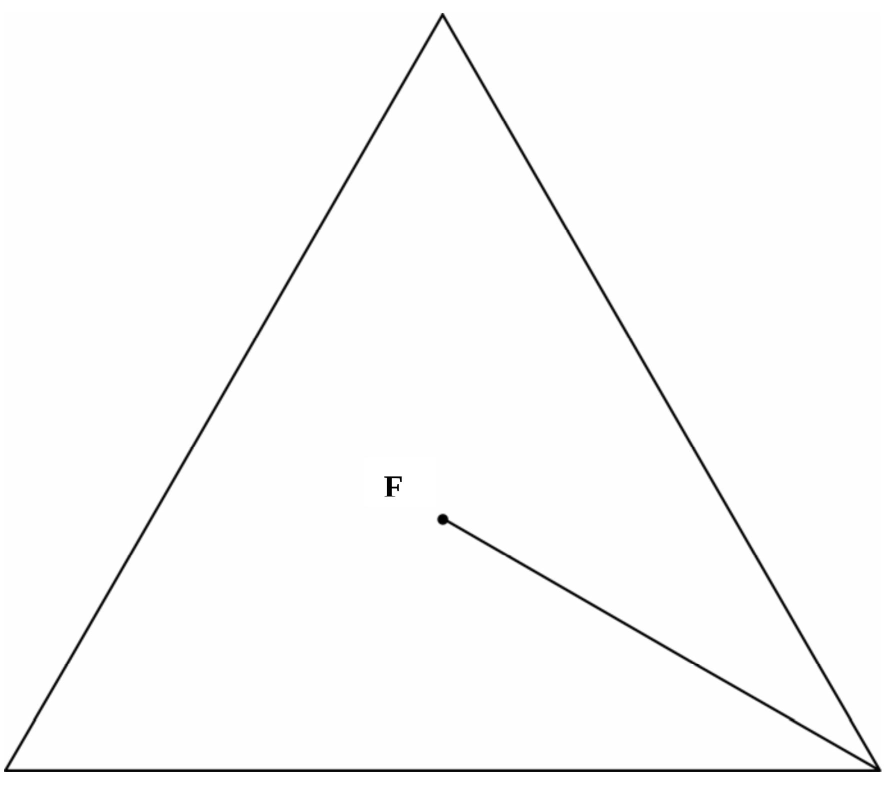
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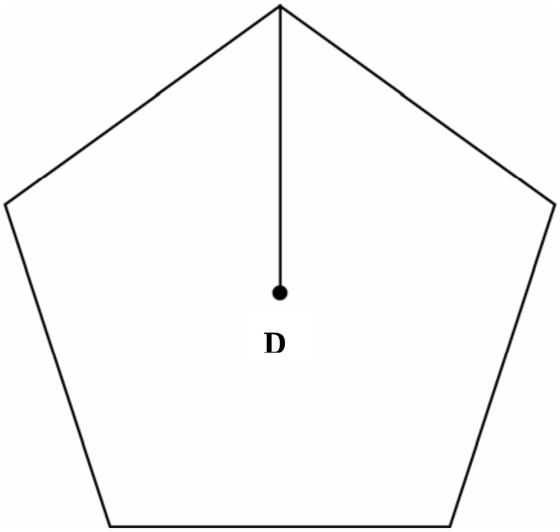
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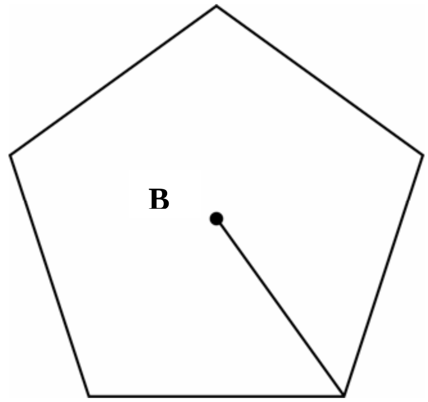
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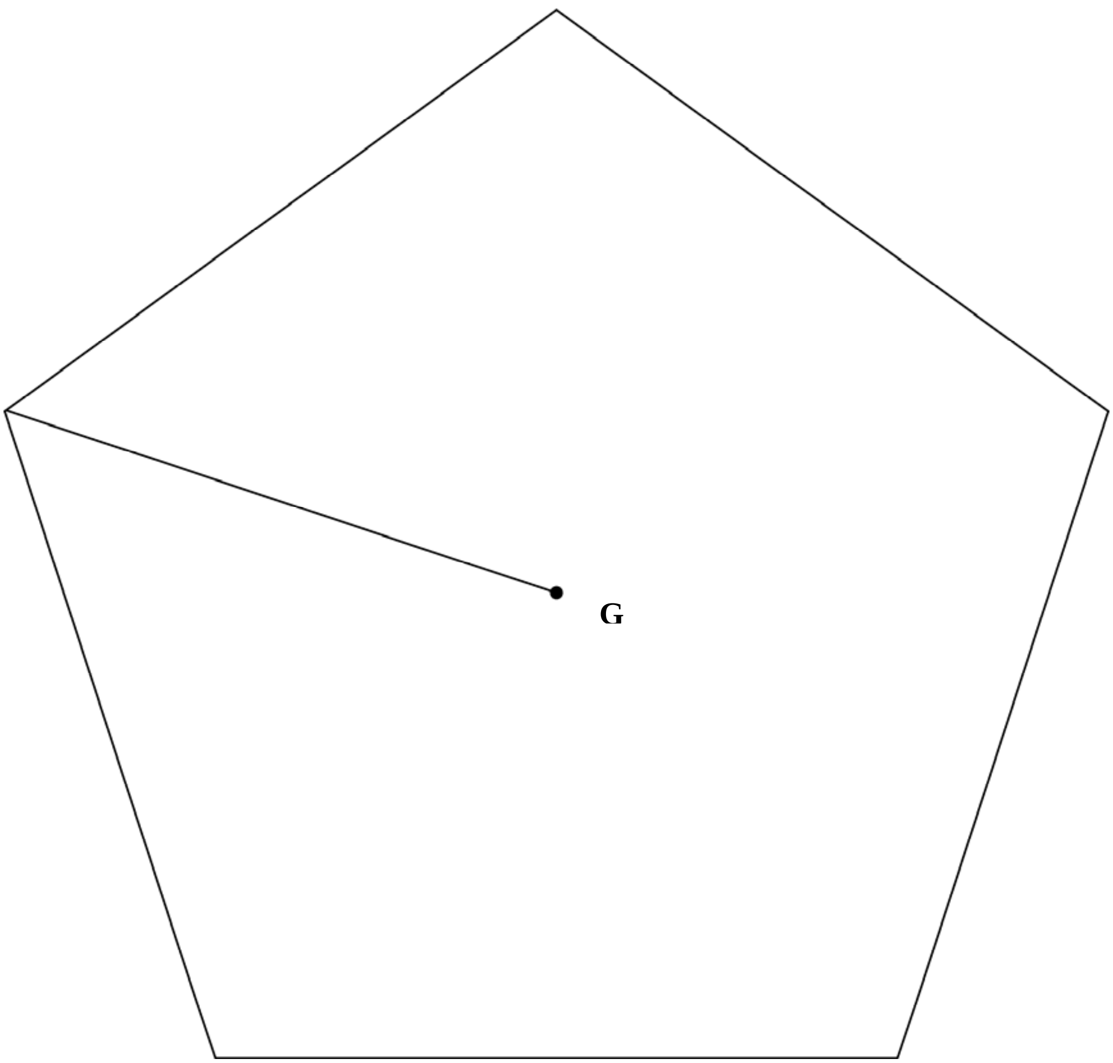
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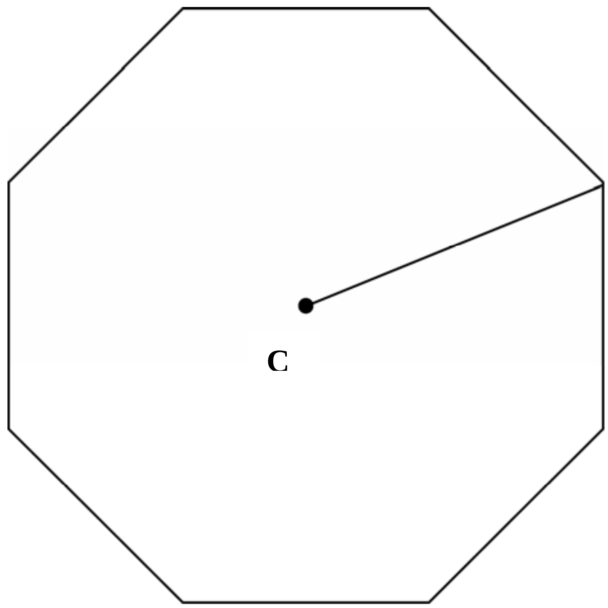
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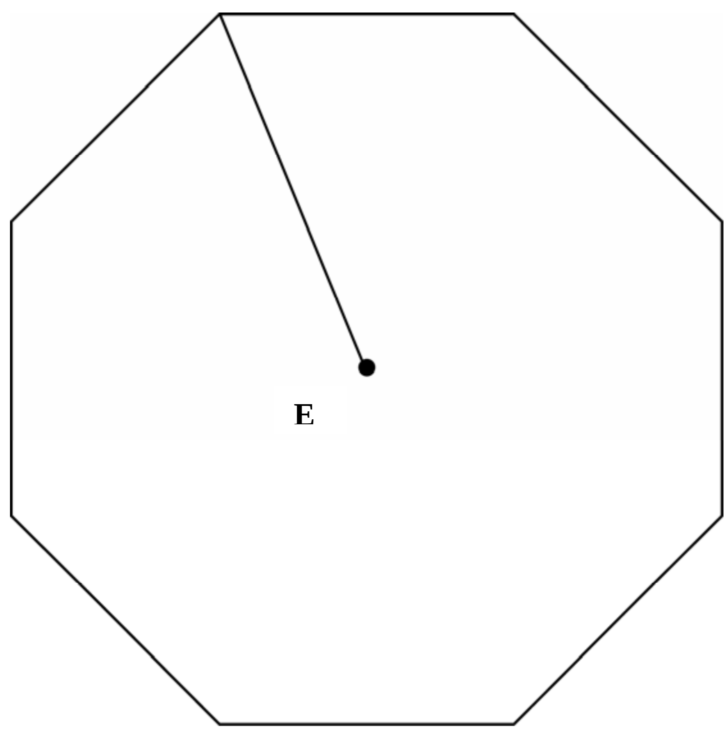
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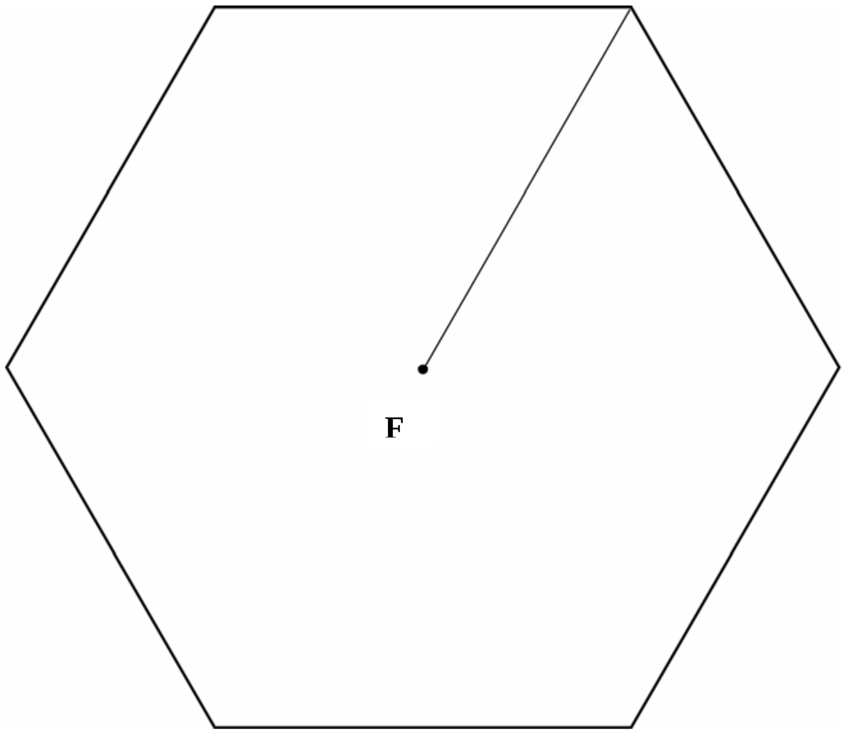
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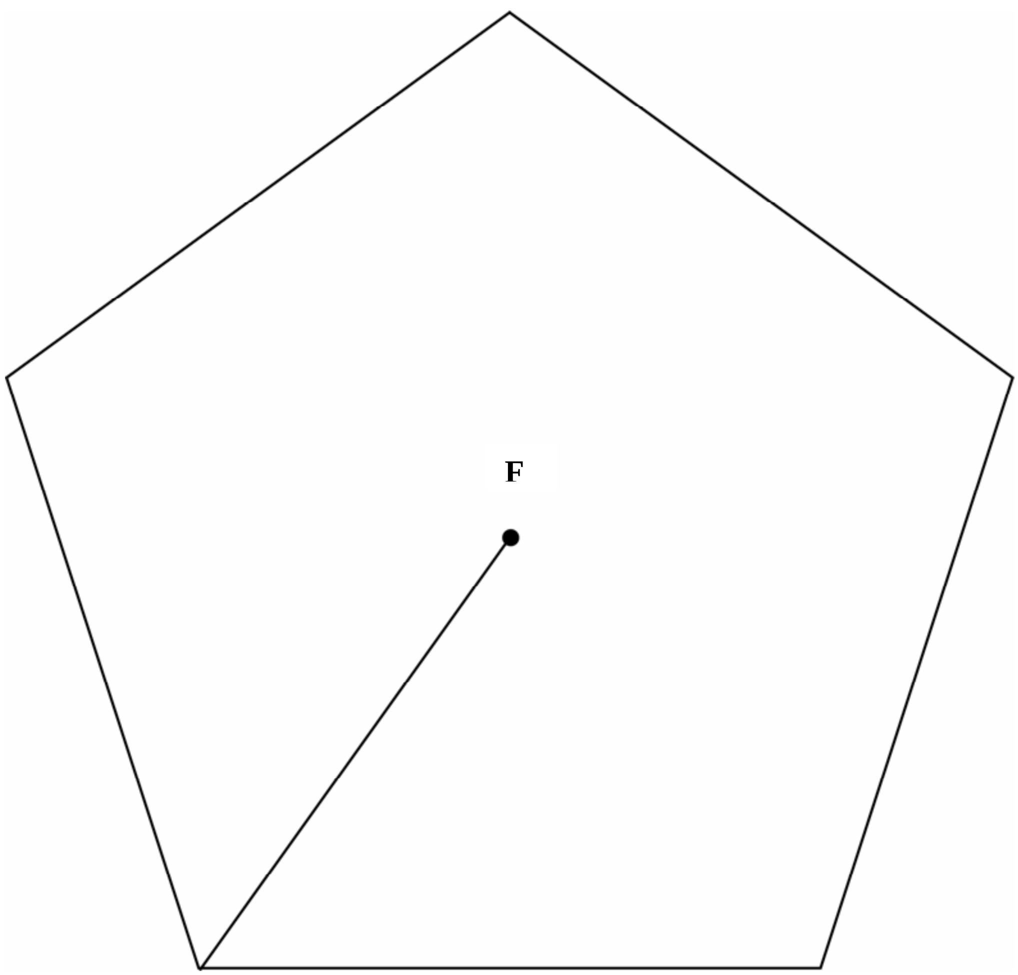
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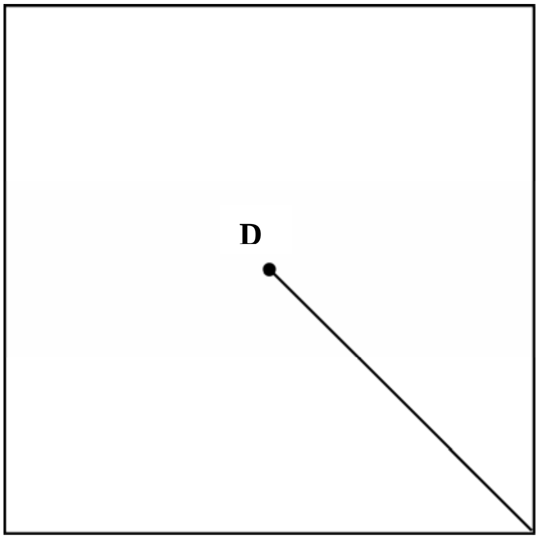
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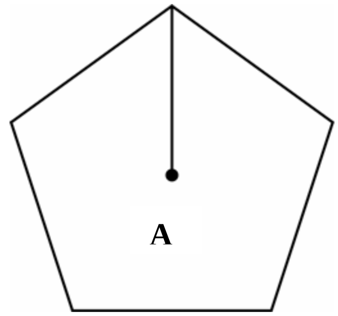
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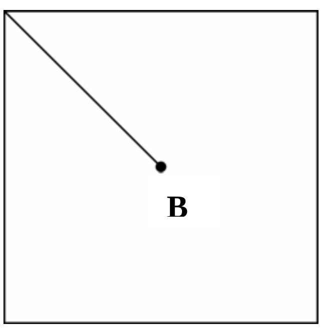
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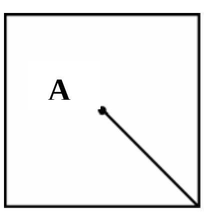
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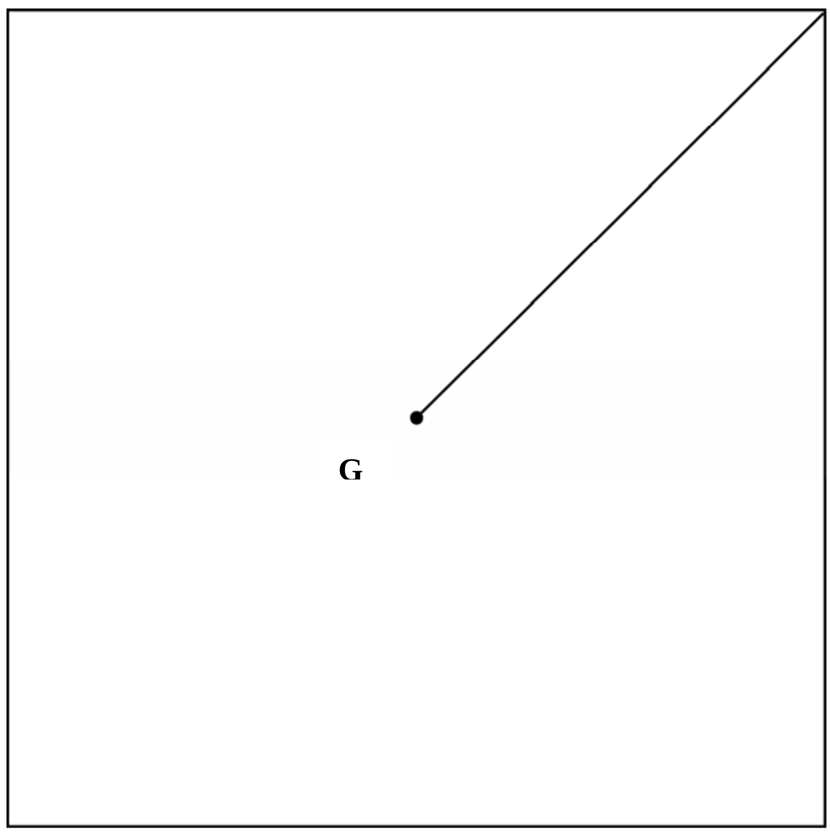
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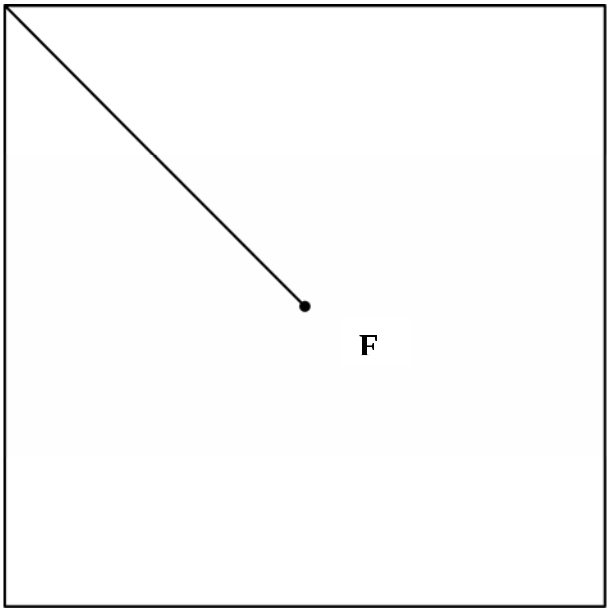
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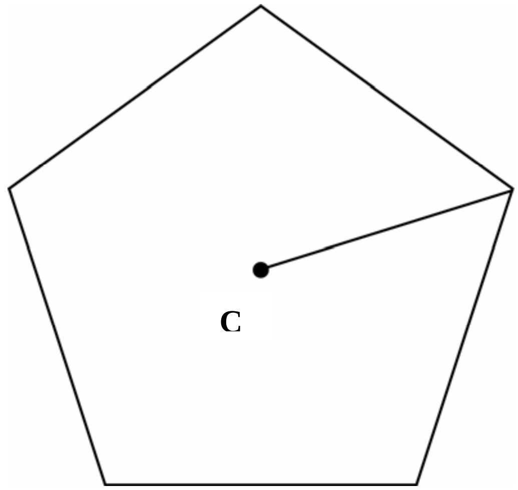
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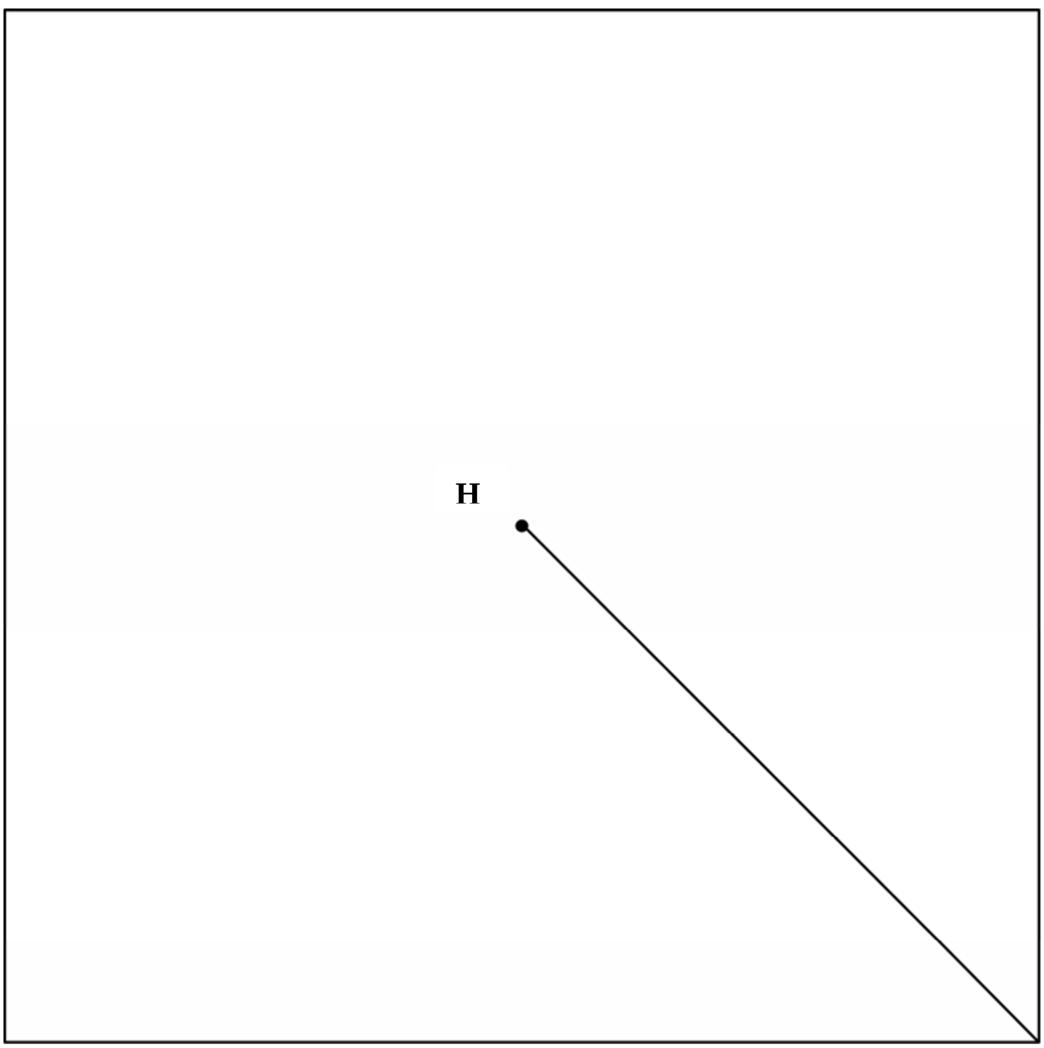
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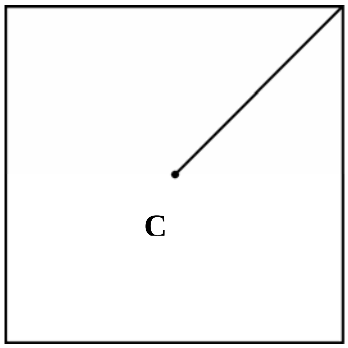
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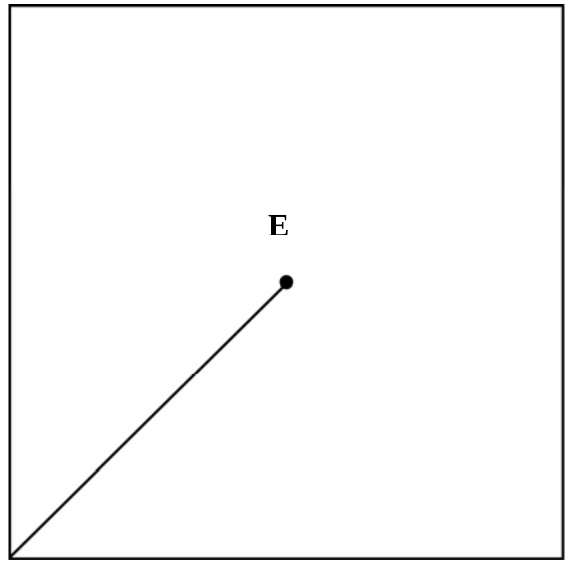
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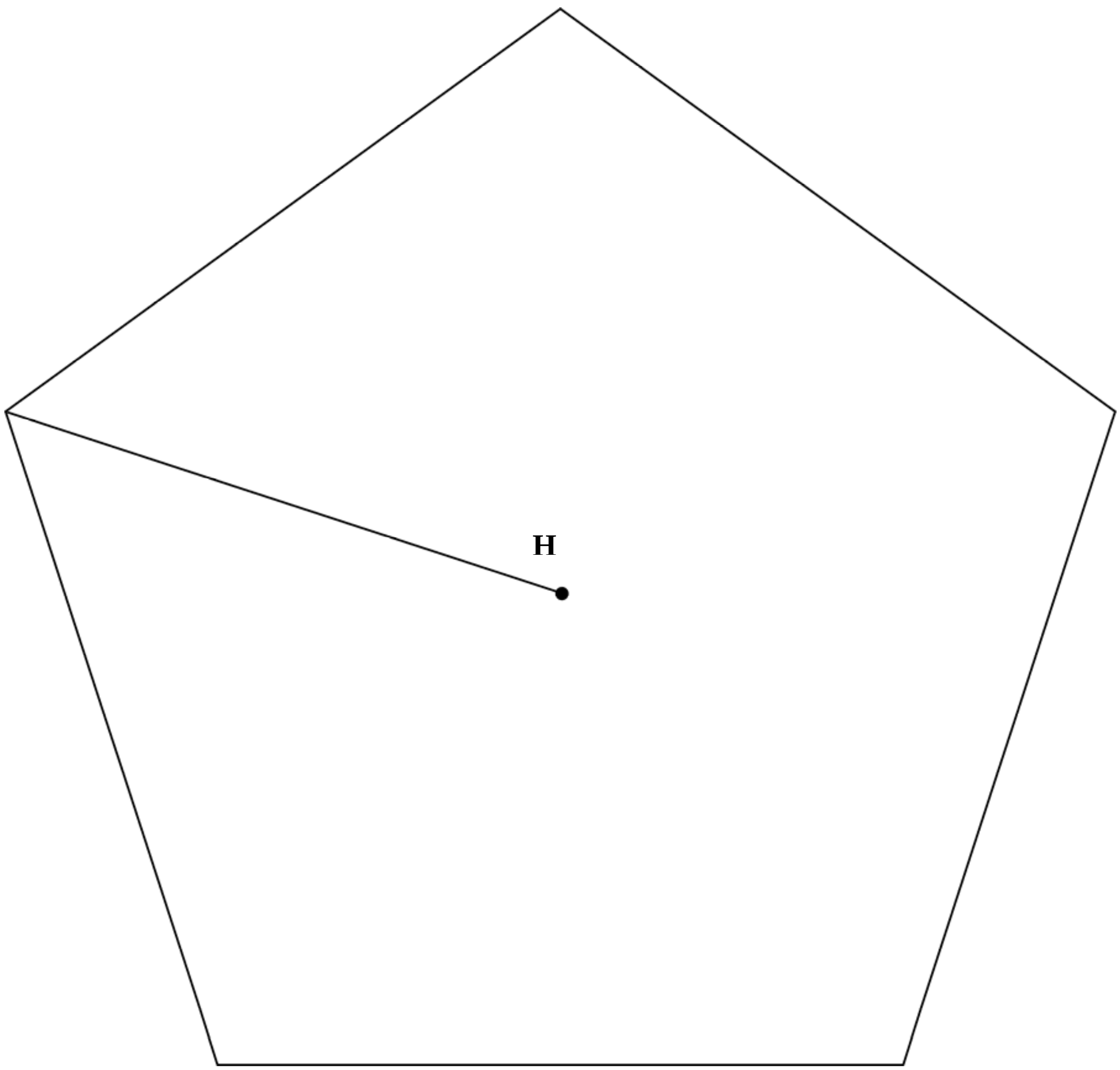
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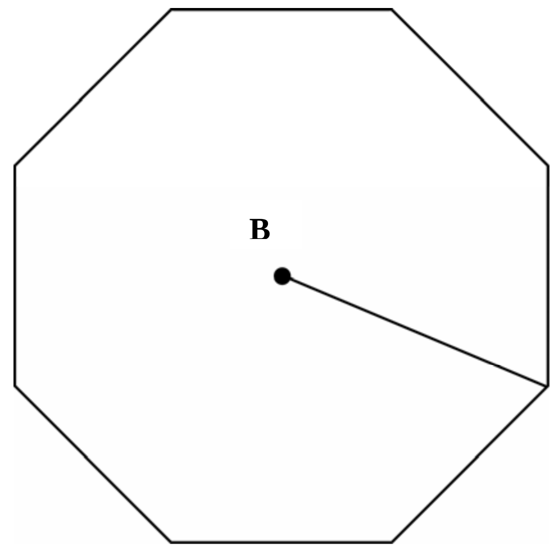
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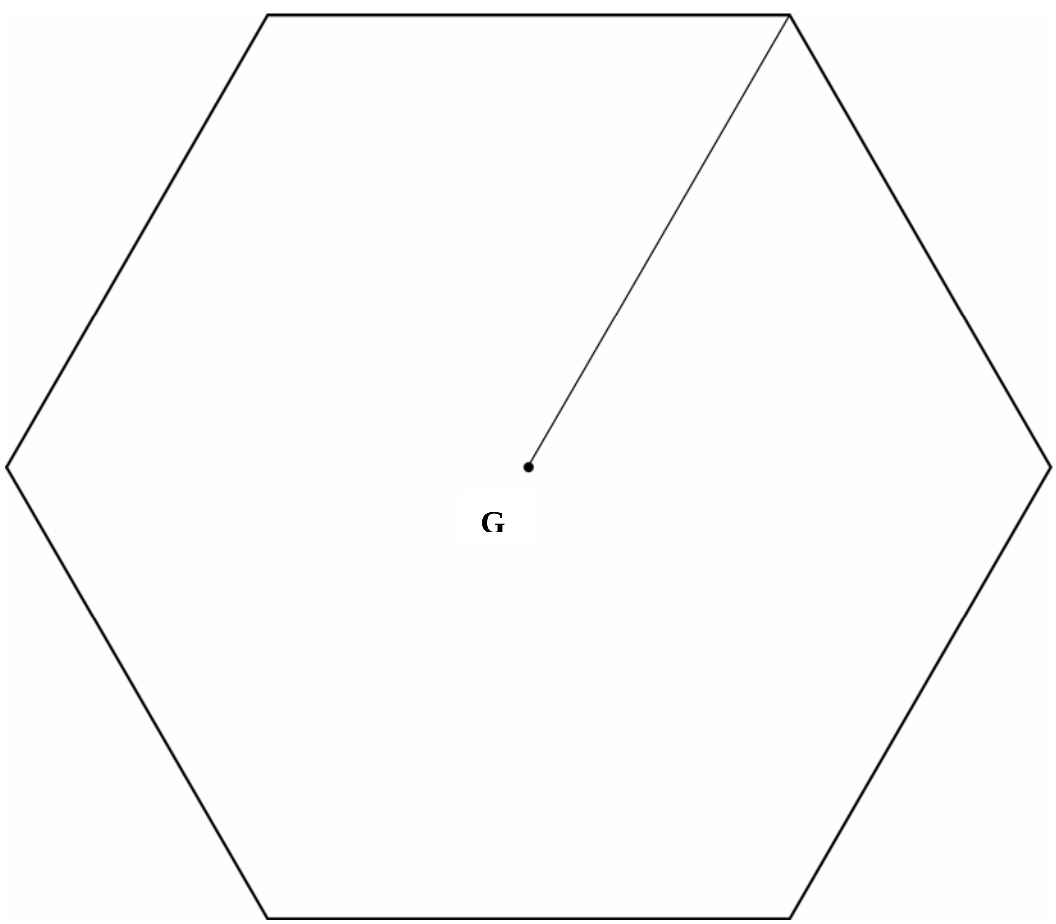
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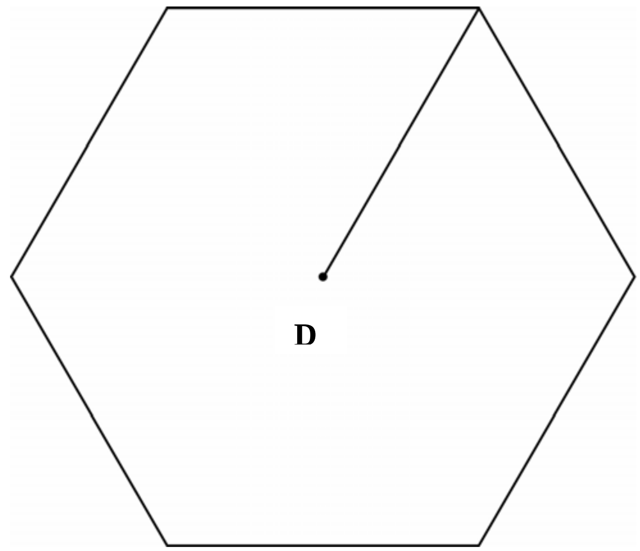
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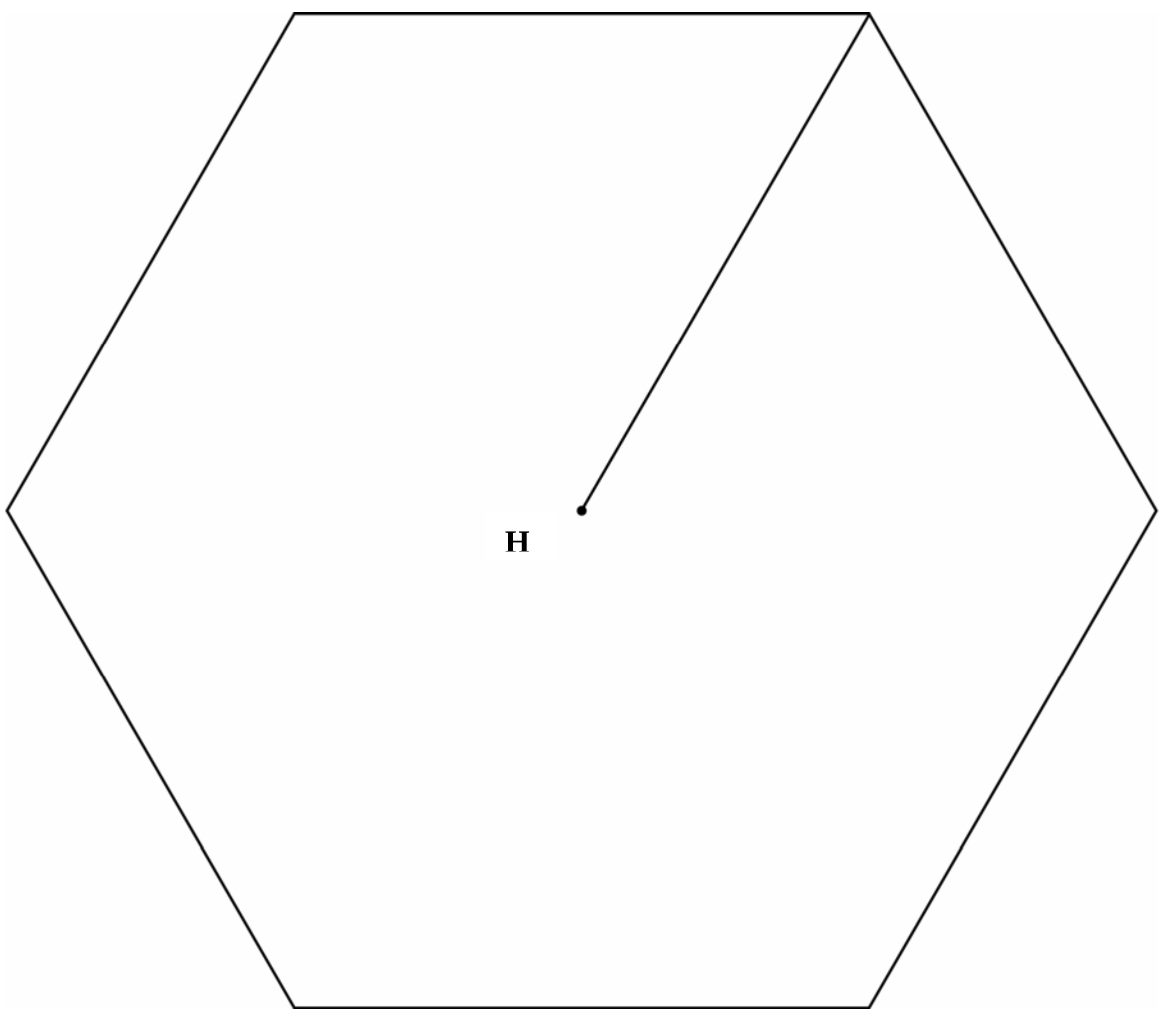
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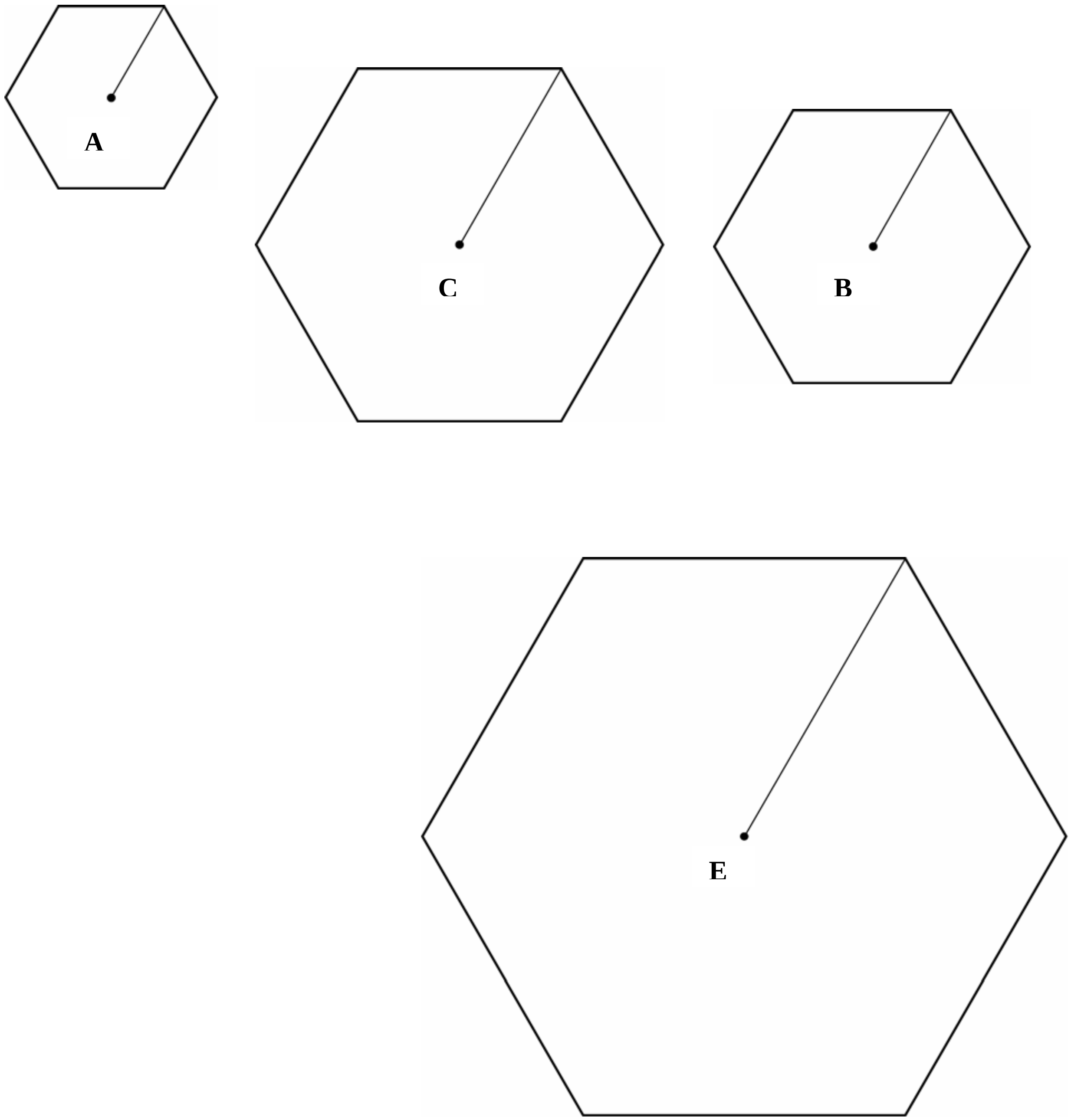
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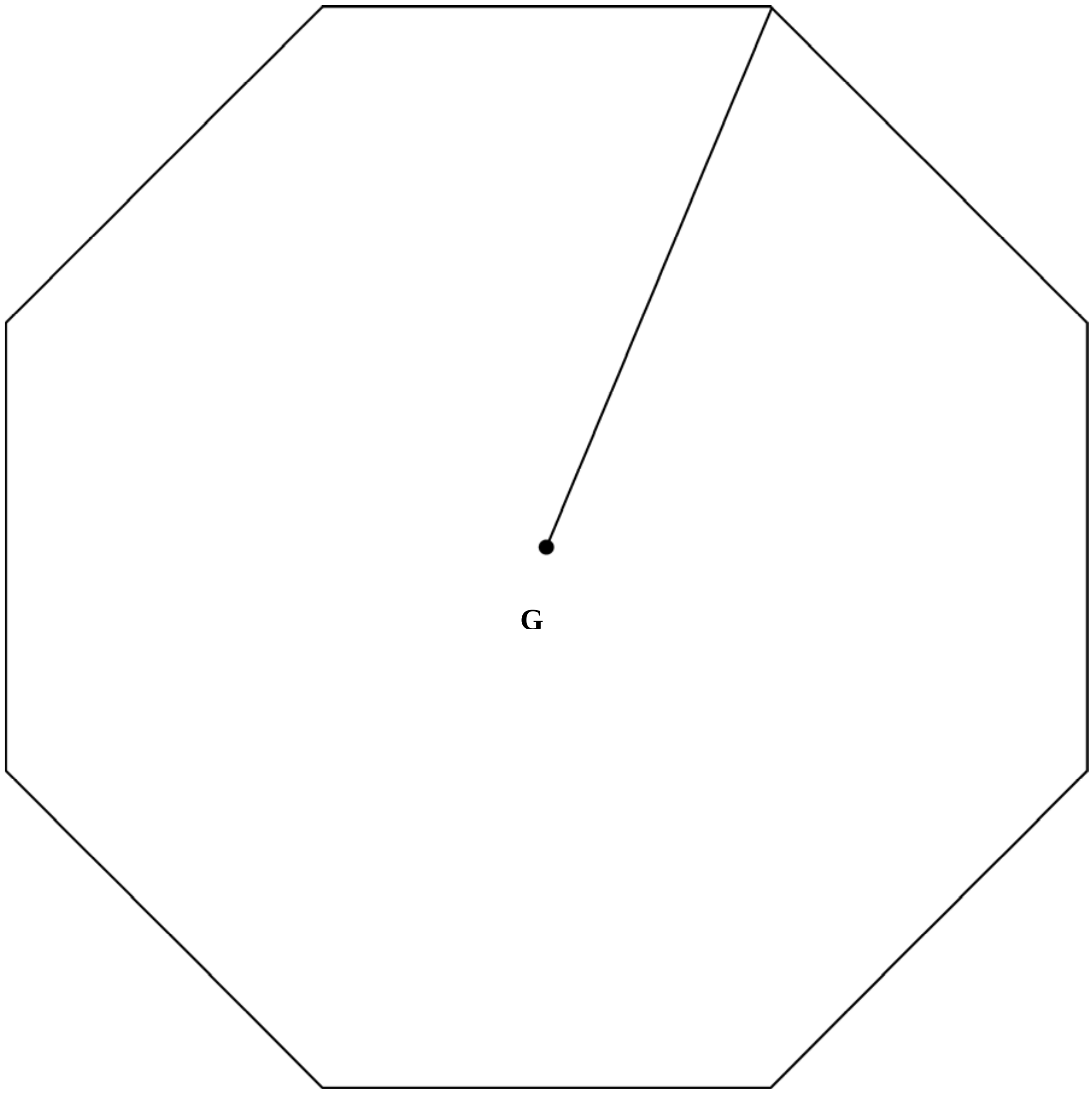
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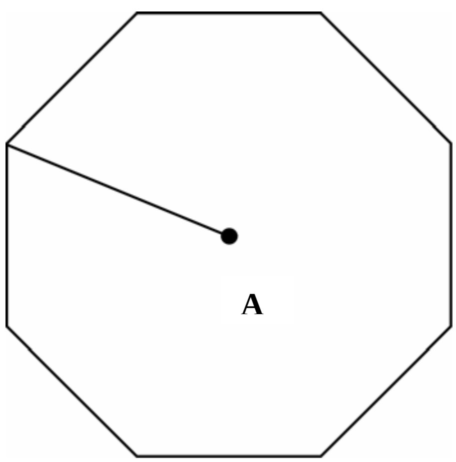
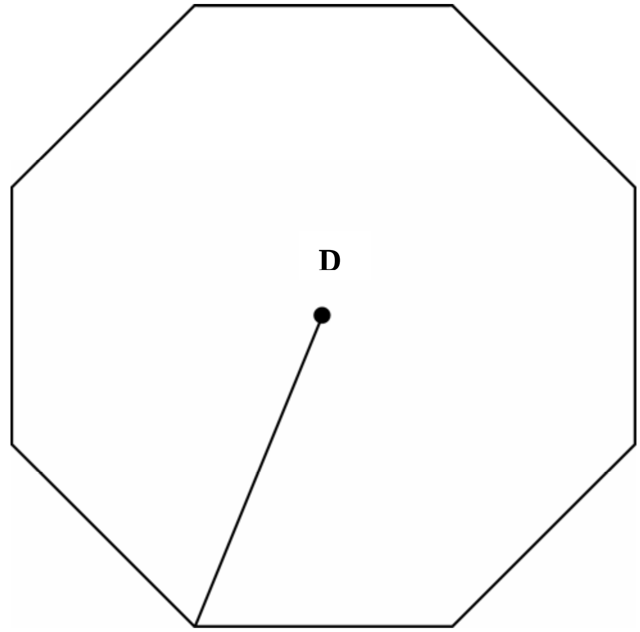

A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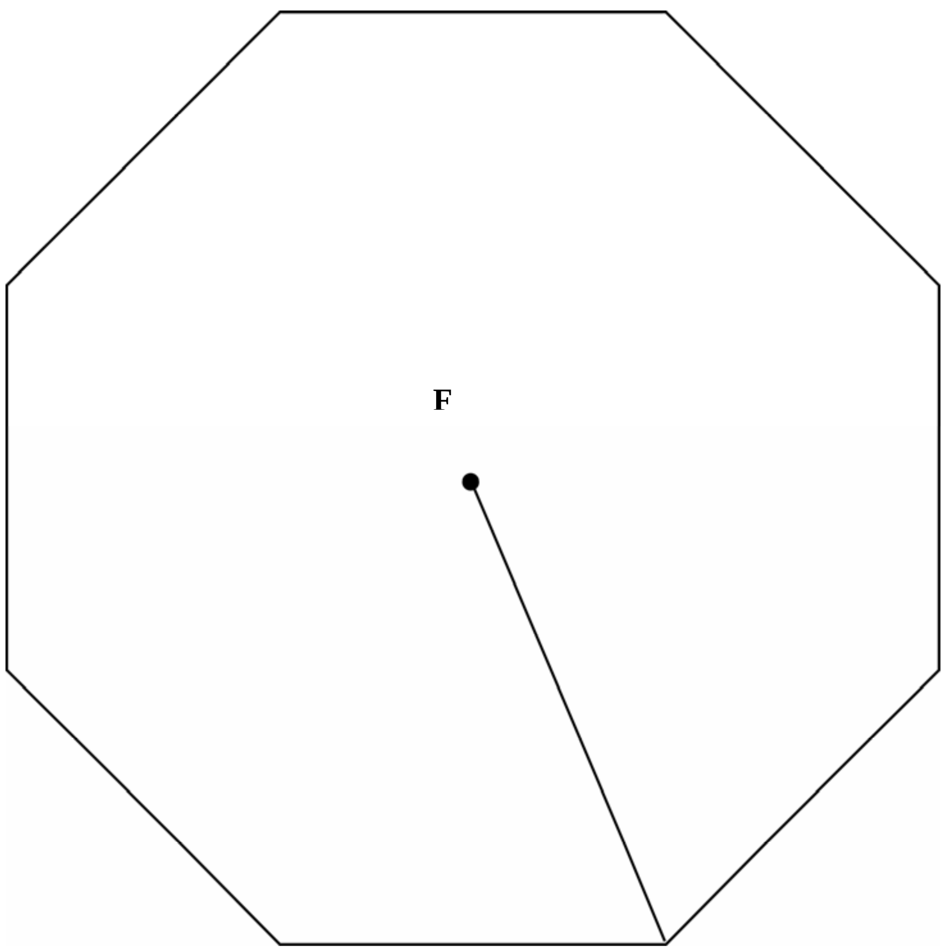
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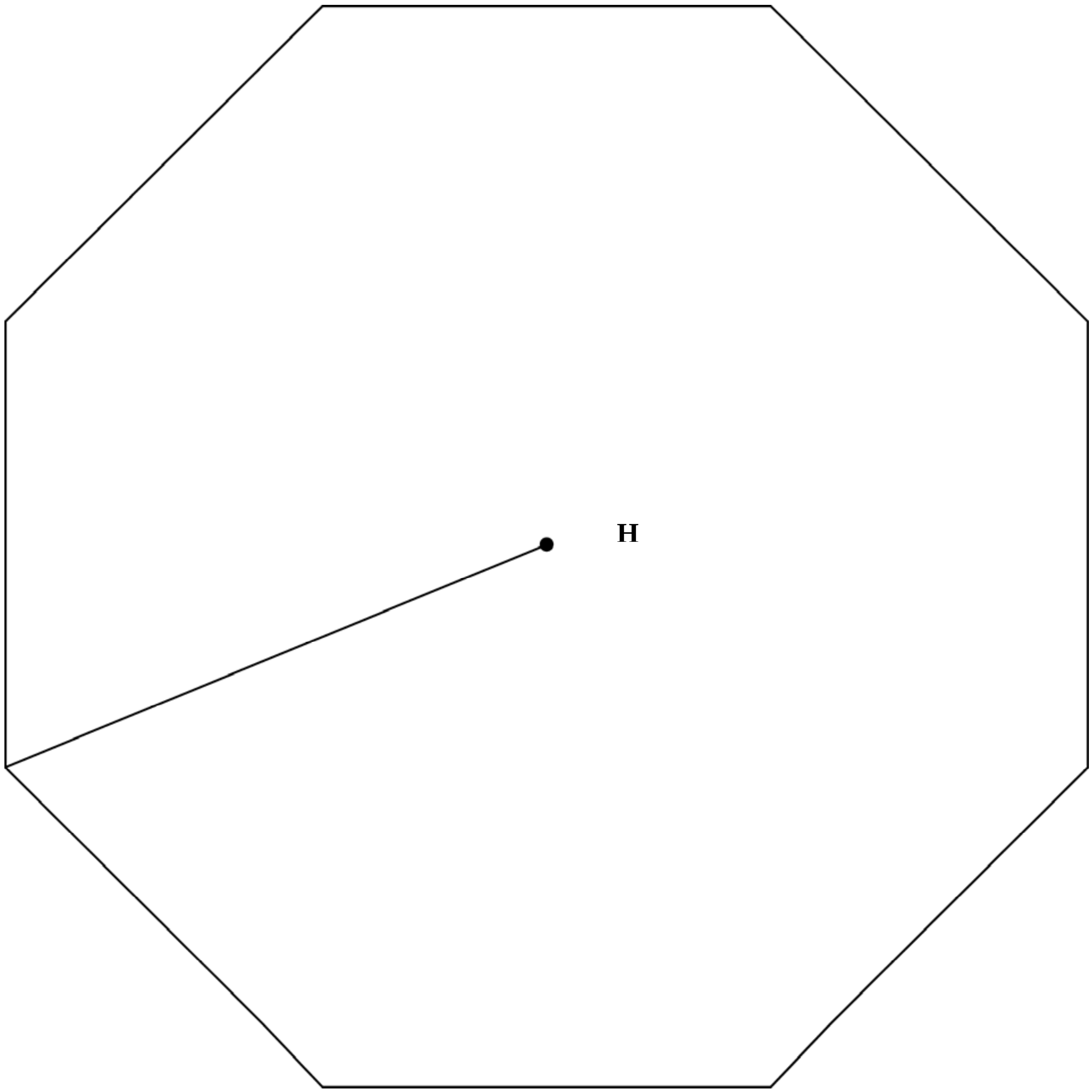
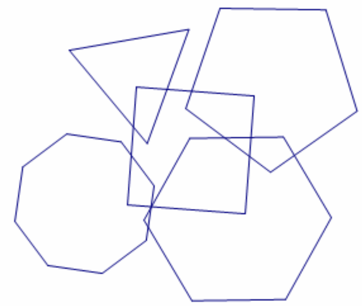

H

# Polygons Rule: Data Collection



Using the polygons provided, measure in ***centimeters*** attributes and fill in the data on the appropriate table.

Triangles Rule

| | Side Length | Radius Length | Apothem Length | Perimeter | Area | Vertex Angle | Central Angle |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |
| H | | | | | | | |

Squares Rule

| | Side Length | Radius Length | Apothem Length | Perimeter | Area | Vertex Angle | Central Angle |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |
| H | | | | | | | |

Pentagons Rule

| | Side Length | Radius Length | Apothem Length | Perimeter | Area | Vertex Angle | Central Angle |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |
| H | | | | | | | |

Hexagons Rule

| | Side Length | Radius Length | Apothem Length | Perimeter | Area | Vertex Angle | Central Angle |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |
| H | | | | | | | |

Octagons Rule

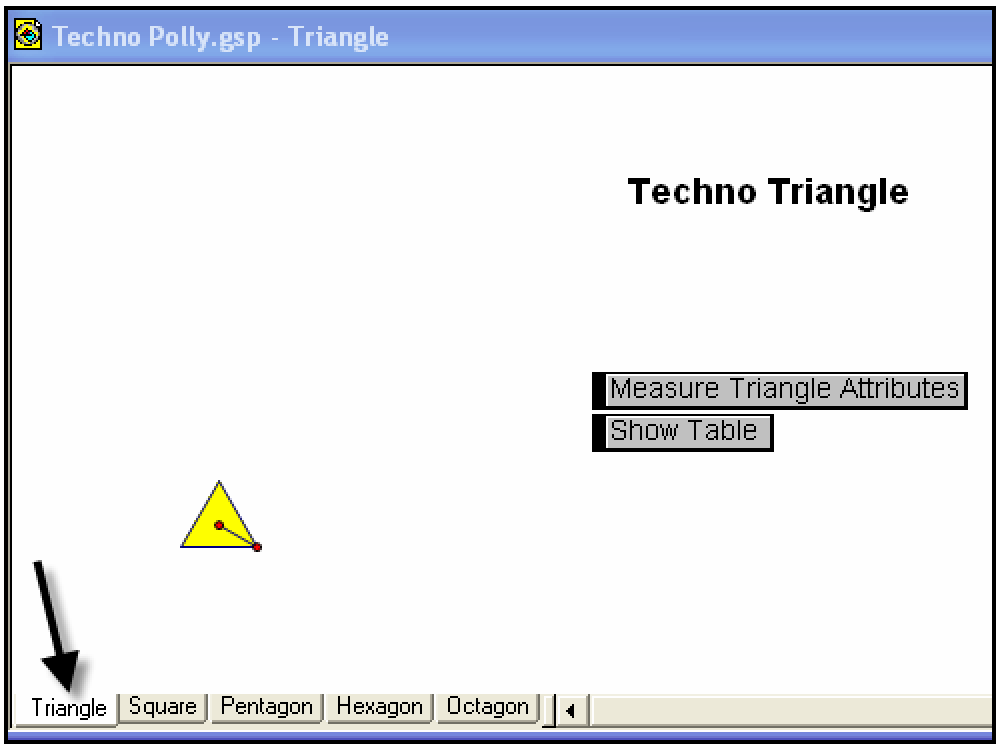
| | Side Length | Radius Length | Apothem Length | Perimeter | Area | Vertex Angle | Central Angle |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |
| H | | | | | | | |

# Polygons Rule: Questions About Data

| | |
|---|---|
| **Data Source** | Rulers |
| How would you describe this set of data? Why? | |
| What relationships occur within this set of data? Why? | |
| How would you represent this data? Why? | |
| What question(s) can we pose to students that this set of data helps to answer? | |
| How might this data extend what students already understand about our course content? | |

# Techno Polly: Data Collection

Open the sketch, **Techno Polly.** Notice the tabs at the bottom of the sketch that say **Triangle**, **Square**, **Pentagon**, **Hexagon** and **Octagon** respectively. Use the same set of direction for each tab, working through them sequentially.




1. Click on the Measure Attributes button. What happens?

2. Click on the Show Table button.

3. Double click on the table to add another row, and then drag the vertex of the polygon increasing the length of the side to approximately 2 cm. What do you observe?

4. Double click on the table again to add another row, and then drag the vertex of the polygon, increasing the length of the side to approximately 3 cm. What do you observe?

5. Repeat this process until you have 8 rows in your table, increasing the side length of the polygon by approximately 1 cm each time.

6. Repeat the above steps for each polygon.

7. To view your data use the tabs at the bottom to transfer from data set to data set.

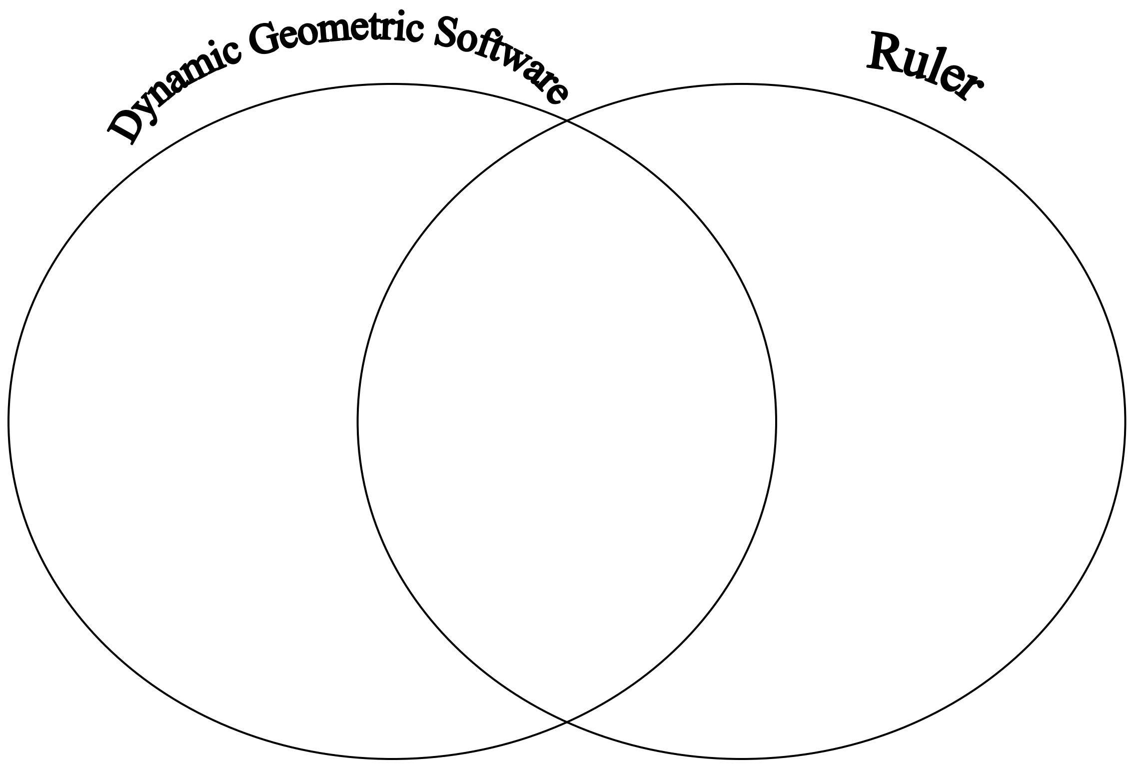
8. Upon exiting the Geometer's Sketchpad, the program will ask if you wish to save…select NO.

# Techno Polly—Questions About Data

| | |
|---|---|
| **Data Source** | Geometer's Sketchpad |
| How would you describe this set of data? Why? | |
| What relationships occur within this set of data? Why? | |
| How would you represent this data? Why? | |
| What question(s) can we pose to students that this set of data helps to answer? | |
| How might this data extend what students already understand about our course content? | |

# Reflections on Data

Complete the following Venn diagram to compare and contrast the uses of the dynamic geometric software and a ruler as data sources.




What are the benefits of using data derived from the dynamic geometric software?

What are the benefits of using data derived from actual measurement?

How might these data sources function in a geometry classroom?

# Debriefing the Exploration of Data

1. What questions can we ask as reflective practitioners to determine the effectiveness of a data source for promoting mathematical learning?

2. How does the technology-based data offer an opportunity to strengthen mathematical learning?

3. What paper-and-pencil methods do students need to know to make sense of the data we explored?

4. How do you define the use of technology in your classroom?

# Polly Polly In Come Free Intentional Use of Data

| | | |
|---|---|---|
| TEKS | | |
| | | |
| Question(s) to Pose to Students | Math | |
| | Tech | |
| Cognitive Rigor | Knowledge | |
| | Understanding | |
| | Application | |
| | Analysis | |
| | Evaluation | |
| | Creation | |
| Data Source(s) | Real-Time | |
| | Archival | |
| | Categorical | |
| | Numerical | |
| Setting | Computer Lab | |
| | Mini-Lab | |
| | One Computer | |
| | Graphing Calculator | |
| | Measurement Based Data | |
| Bridge to the Classroom | | |

# Polygarden Landscaping Company

## Explore/Explain Cycle I

### Purpose:
Provide participants the opportunity to use technology to explore relationships in geometric figures that yield linear data, such as proportional change of one dimension in a two-dimensional figure. Participants will make connections between algebraic and geometric concepts that enhance their student's conceptual understanding of the Geometry TEKS.

### Descriptor:
In a guided exploration, participants will manipulate sketches created in Geometer's Sketchpad. Problem-solving strategies of breaking a large problem into smaller components and working backwards will be utilized to facilitate the constructions and the development of geometry concepts.

### Duration:
2 hours

### TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including but not limited to calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.5A Use numeric and geometric patterns to develop algebraic expressions representing geometric properties.

G.7A Use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures

G.7B Use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons.

G.8A Find areas of regular polygons, circles, and composite figures.

G.11D Describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems

## TAKS Objectives:

- Objective 3: Linear Functions
- Objective 4: Formulate and Use Linear Equations and Inequalities
- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of geometric relationships and shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

## Technology:

- Spreadsheet technology
- Hand-held graphing calculator
- Dynamic geometry software (Geometer's Sketchpad)
- Graph link technology

## Materials:

**Advanced Preparation*:***

- Participant access to computers with Geometer's Sketchpad (latest version update available from http://www.keypress.com/sketchpad) and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool
- Sketches **Growing Pollys.gsp** and **Inscribed Circles.gsp** found on the CD.

**For each participant:**

- Graphing calculator
- Graph link (optional)
- **Polygarden Landscaping Company** activity sheets
- **Putting It All Together** activity sheet
- **Polygarden Landscaping Company Intentional Use of Data** activity sheet printed on green paper

**For each group of 2 participants:**

- Computers with Geometer's Sketchpad and Microsoft Excel
- Copy of the Technology Tutorial $T^2$

## Polygarden Landscaping Company—Leader Notes

*In this exploration the presenter will ask the participants to use Geometer's Sketchpad to collect and analyze data to discover the relationship between the length of the apothem of a regular polygon and its perimeter.*

*The relationship is a linear relationship in the form* ***y = kx*** *, where* ***k*** *is the constant of proportionality or constant of variation. Participants will gather the data and analyze it on their own. During the Explain phase, participants will discuss several methods of analyzing the data and identify comparative advantages and disadvantages of each method.*

# Polygarden Landscaping Company

# Explore

## Posing the Problem:

**Polygarden Landscaping Company builds brick borders for flowerbeds that are always in the shape of regular polygons. To calculate the number of bricks necessary for a flowerbed, Brad, a bricklayer, needs to know the perimeter of the garden. On his last job Brad was not able to measure the perimeter of the flowerbed. He could only measure the distance from the center of the polygon to one side of the polygon. This distance is called the apothem. Is it possible for Brad to calculate the perimeter of the flowerbed if the only information he has is the length of the apothem and the number of sides of the garden?**

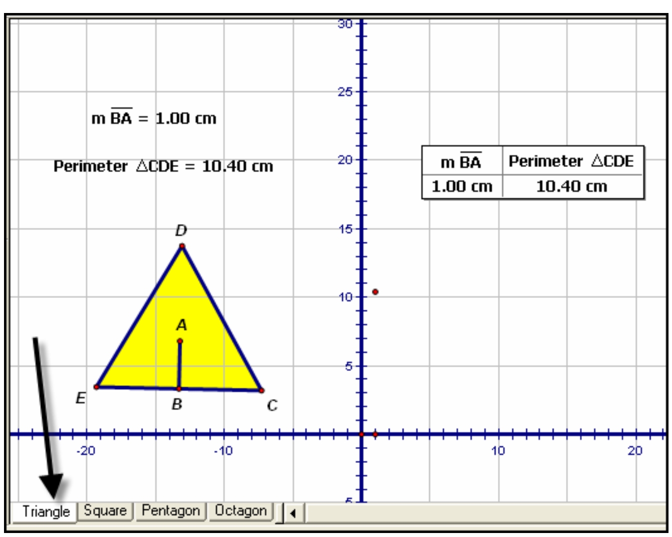
## Obtaining and Analyzing the Data:

**To solve this problem, use the problem-solving strategy of "solving a simpler problem." To do so, use geometric sketches to collect and analyze data.**

**Open the sketch Growing Polly's.**

*Participants might need instruction at this point about how to open Geometer's Sketchpad and find the sketch on the particular computers that are being used in the professional development.*

**Select the Triangle tab.**



1. **Double click on the table to add another row; then click and drag point *C* away from point *B*. What do you observe?**

*The measures change. The points are plotted and traced to create a graph.*

2. **Double click on the table again, and then move point *C* farther away from point *B*. Repeat this process until you have 10 rows in your table.**

3. **What patterns do you observe in the table?**

*Answers may vary. Participants may observe that this is a proportional relationship.*

4. **What observations can you make about your graph?**

*Participants may observe that the graph appears to be linear and passes through the origin.*

5. **Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.**

*$y= 10.39x$*

6. **Verify that your function rule models your data. Explain your verification.**

*Participants may have graphed the function rule over the scatterplot.*

7. **Write a verbal description of the relationship between the length of the apothem of an equilateral triangle and its perimeter.**

*The perimeter of an equilateral triangle can be calculated by multiplying the length of the apothem by 10.39.*

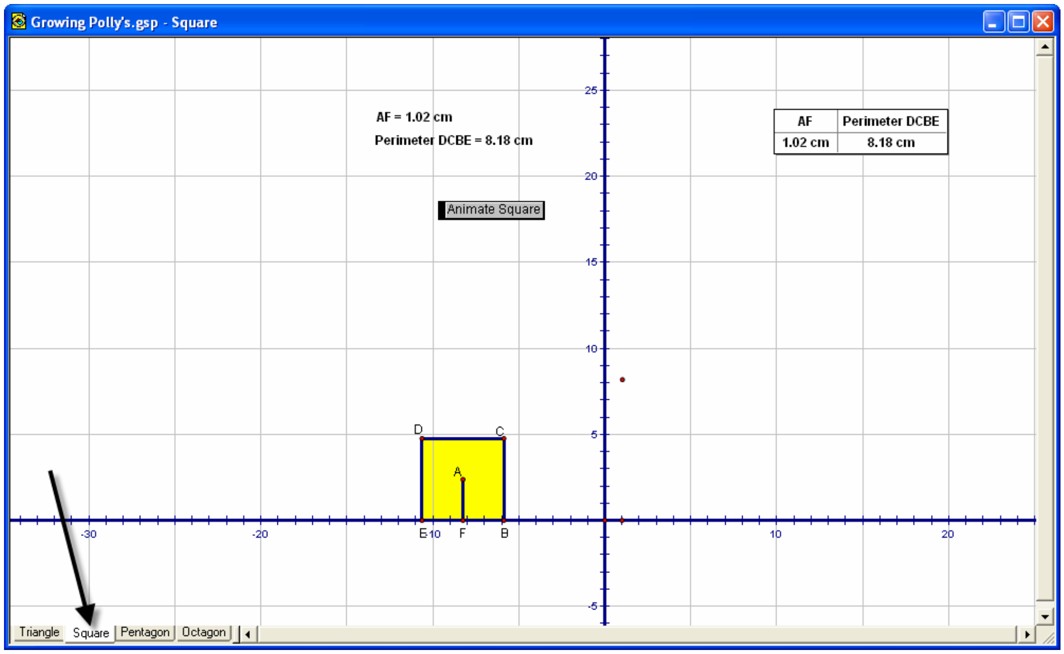
8. **What is the approximate perimeter of a flowerbed that is in the shape of an equilateral triangle with an apothem of 7.23 centimeters?**

*75.12 centimeters*

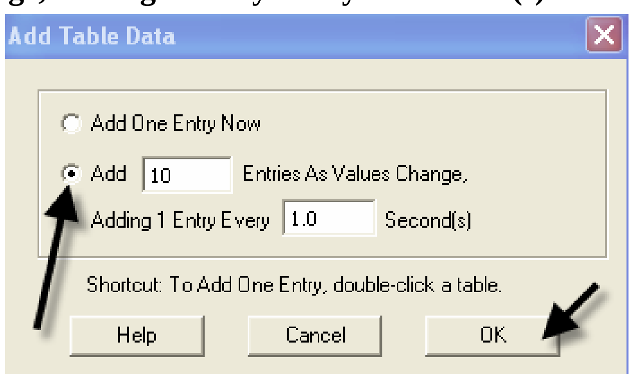
9. **What is the approximate length of the apothem of an equilateral triangle whose perimeter is 68.5 centimeters?**

*6.6 centimeters*

Select the **Square** tab.




1. ***Right* click in the table and select the Add Table Data option. Select the Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s) and click OK.**




2. **Start the data collection process by clicking on the Animate Square button. After your table fills with data, stop the animation by clicking on the Animate Square button again. What happened?**

   *The plotted points graphed creating a line. The table filled up as the square changed sizes.*

3. **What patterns do you observe in the table?**

   *Participants may observe that this is a proportional relationship.*

**4. What observations can you make about your graph?**

*Participants may observe that the graph appears to be linear and passes through the origin.*

**5. Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.**

$y = 8x$

**6. Verify that your function rule models your data. Explain your verification.**

*Participants may have graphed the function rule over the scatterplot.*

**7. Write a verbal description of the relationship between the length of the apothem of square and its perimeter.**

*The perimeter of a square can be calculated by multiplying the length of the apothem by 8.*

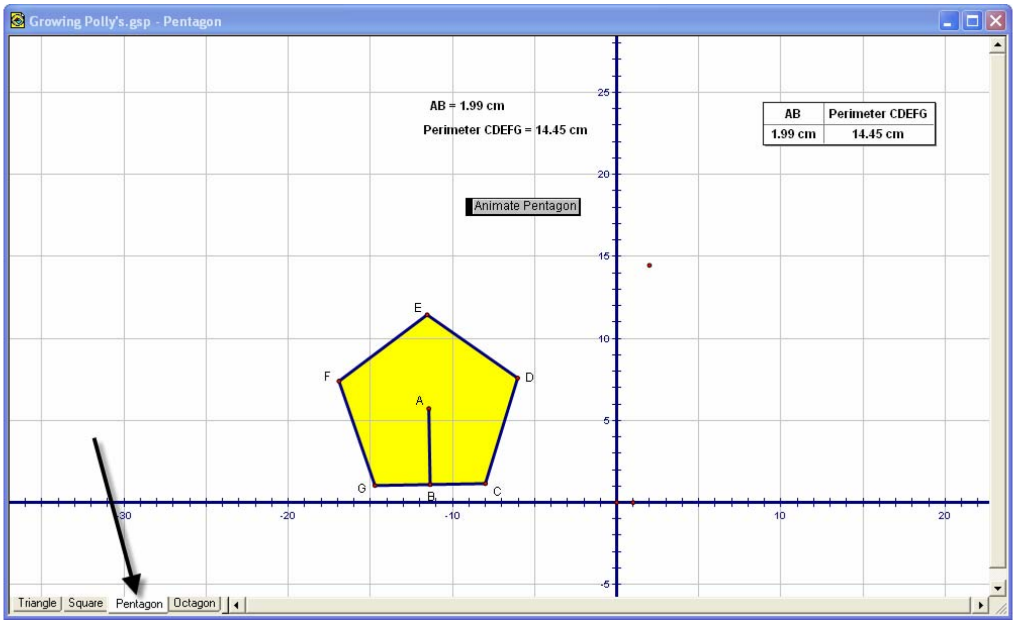
**8. What is the approximate perimeter of a flowerbed that is in the shape of a square with an apothem of 7.23 centimeters?**

*57.84 centimeters*

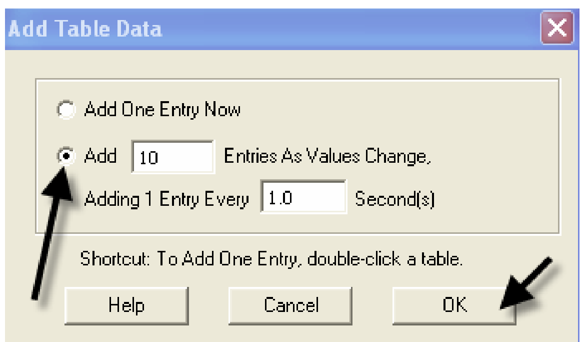
**9. What is the approximate length of the apothem of a square whose perimeter is 68.5 centimeters?**

*8.6 centimeters*

**Select the Pentagon tab.**




1. ***Right*** **click in the table and select the Add Table Data option. Select the Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s) and click OK.**




2. **Start the data collection process by clicking on the Animate Pentagon button. After your table fills with data, stop the animation by clicking on the Animate Pentagon button again. What happened?**

   *The plotted points graphed creating a line. The table filled up as the square changed sizes.*

3. **What patterns do you observe in the table?**

   *Participants may observe that this is a proportional relationship.*

**4. What observations can you make about your graph?**

*Participants may observe that the graph appears to be linear and passes through the origin.*

**5. Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.**

*y= 7.27x*

**6. Verify that your function rule models your data. Explain your verification.**

*Participants may have graph the function rule over the scatterplot.*

**7. Write a verbal description of the relationship between the length of the apothem of a regular pentagon and its perimeter.**

*The perimeter of a pentagon can be calculated by multiplying the length of the apothem by 7.27.*

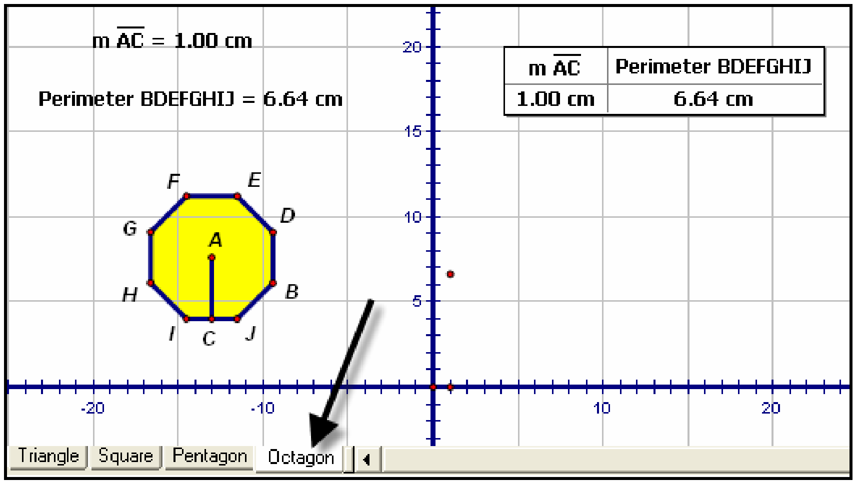
**8. What is the approximate perimeter of a flowerbed that is in the shape of a regular pentagon with an apothem of 7.23 centimeters?**

*52.56 centimeters*

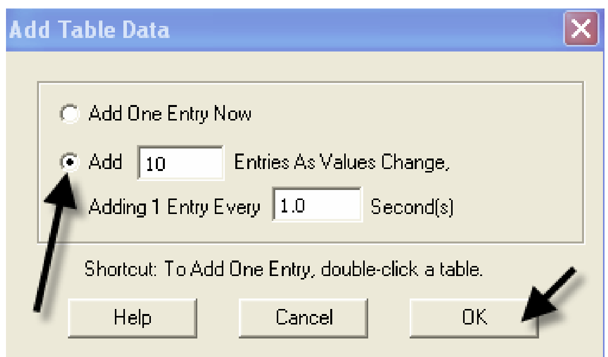
**9. What is the approximate length of the apothem of a regular pentagon whose perimeter is 68.5 centimeters?**

*9.4 centimeters*

**Select the Octagon tab.**




1. ***Right*** **click in the table and select the Add Table Data option. Select the Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s) and click OK.**




2. **Start the data collection process by clicking on the Animate Octagon button. After your table fills with data, stop the animation by clicking on the Animate Octagon button again. What happened?**

   *The plotted points graphed creating a line. The table filled up as the square changed sizes.*

3. **What patterns do you observe in the table?**

   *Participants may have observed that this is a proportional relationship.*

4. **What observations can you make about your graph?**

   *Participants may observe that the graph appears to be linear and passes through the origin.*

5. **Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.**

   *$y = 6.63x$*

6. **Verify that your function rule models your data. Explain your verification.**

   *Participants may have graphed the function rule over the scatterplot.*

7. **Write a verbal description of the relationship between the length of the apothem of regular octagon and its perimeter.**

   *The perimeter of an octagon may be calculated by multiplying the length of the apothem by 6.63.*

8. **What is the approximate perimeter of a flowerbed that is in the shape of a regular octagon with an apothem of 7.23 centimeters?**

   47.93 centimeters

9. **What is the approximate length of the apothem of a regular octagon whose perimeter is 68.5 centimeters?**

   *10.33 centimeters*

## Putting It All Together

**1. Complete the table.**

**Perimeter *versus* Apothem**

| Regular Polygon | Function Rule |
|---|---|
| **Triangle** | *y = 10.39x* |
| **Square** | *y = 8x* |
| **Pentagon** | *y = 7.27x* |
| **Octagon** | *y = 6.63x* |

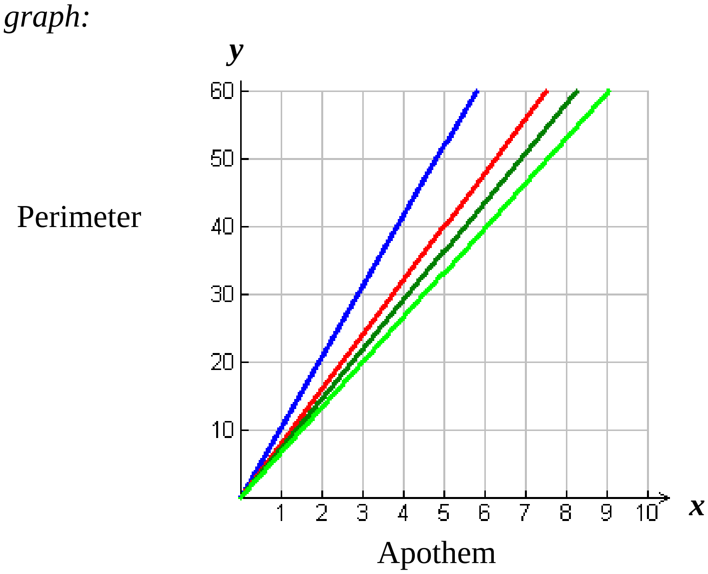
**2. In what ways are the function rules the same?**

*They are all in the form y = kx.*

**3. In what ways are the function rules different?**

*The constant of proportionality is different.*

**4. Graph all four-function rules on the same set of axes. Sketch your graph. Label each line with the name of the polygon.**
*Sample graph:*



**5. What observations can you make about your graph? Connect your observations to geometric properties observed in this exploration.**

*Participants should explain why the slopes of the lines decrease as the number of sides of the polygon increase.*

**6. Look back at Brad's problem. Is it possible for Brad to calculate the perimeter of the flowerbed if the only information he has is the length of the apothem and the number of sides of the garden? Why or why not?**

*Yes, because that is what we did in the activity: each time we increased the number of sides we were able to find a function rule to find the perimeter given the length of the apothem.*

**7. Is there a general rule or trend you can develop using the information gathered? If so what is it?**

*Yes, the higher the number of sides of the polygon the closer the measure of perimeter comes to its inscribed circle.*

**8. If the length of the apothem remains constant, what is the effect on perimeter as the number of sides of the polygon increases?**

*The perimeter decreases.*

**9. If you continue to increase the number of sides of the polygon while keeping the length of the apothem constant, what value will the perimeter approach?**

*The perimeter of the polygon approaches the circumference of the inscribed circle.*

## Explain

## Debrief the Polygarden Landscaping Company

*In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. Participants may have used graphing calculators as a tool for their data analysis. Have them discuss how they used or could have used the calculators to help them analyze their data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.*

1. **What knowledge of geometric properties is necessary to complete each of the constructions?**
   *Participants should discuss the properties of the polygons. For example the central angle of a regular hexagon equals 60° and the apothem is perpendicular to a side of the polygon.*

*After participants have answered, demonstrate the construction of the "Triangle Sketch" using Geometer's Sketchpad. Ask questions as you demonstrate. Point out to participants that this demonstration is not intended to train them in the use of Geometer's Sketchpad; they will get the opportunity to become familiar with it throughout the workshop with detailed steps available in the Technology Tutorial $T^2$. This demo is intended to provide an understanding of how the construction depends on the properties of geometric figures. Demonstrate only the "Triangle Sketch," however, be sure to connect the construction techniques and geometric properties to the sketch of the square, regular pentagon, regular hexagon and regular octagon. For detailed steps on the construction see the* ***Technology Tutorial $T^2$***.

2. **Construct a circle and its radius.**

**Facilitator Questions**
- How many degrees are in the central angle of an equilateral triangle?
  *120 degrees*
- What about a square, regular pentagon or octagon?
  *90, 72 and 45 degrees respectively*

3. **Demonstrate a rotation of the radius 120°.**

4. **Demonstrate a second rotation of the radius 120°.**

**Facilitator Question**
- Ask participants to predict the next step in the construction.
  *Connect the points on the circle with line segments.*

**5. Construct segments joining the end points of the radii.**

**Facilitator Questions**

- How do we know this is an equilateral triangle?
  *Because the inscribed angles are each 60 degrees (the measure of an inscribed angle is equal to one half of the measure of the intercepted arc, which in this case was the same as the central angle that we rotated 120 degrees).*
- What is the relationship between the apothem of a regular polygon and a side of the polygon?
  *They are perpendicular.*
- Do you have an idea of what we need to do in order to construct a perpendicular to the side of the polygon?
  *We need the perpendicular from one side through its opposite vertex.*

**6. Construct the line through the center of the circle that is perpendicular to one side of the triangle.**

**Facilitator Questions**

- Do we want this entire line?
- If not, what parts of it do we want?
  *We only want the apothem (from the center of the triangle to the point on the side).*

**7. Construct the point of intersection of the perpendicular line and the side of the triangle.**

**8. Construct the segment joining the center of the circle to the point of intersection.**

**9. Hide the circle and all unnecessary lines and segments.**

**Facilitator Question**

- What are the things we want to measure?
  *The apothem and the perimeter.*

**10. Measure the length of the apothem.**

**11. Highlight the three vertices and show how the Measure /Perimeter option is unavailable or "grayed" out on the selection menu.**

**12. Construct the triangle interior.**

**13. Show how the program automatically labeled the points and rename if desired.**

**Facilitator Questions**
- What relationship are we interested in?
  *How the apothem is related to the perimeter.*
- What are the independent and dependent variables?
  *The perimeter is the dependent and the apothem length is the independent.*
- How can we explore that relationship?
  *Build a table, plot the points.*

**14. Create the table.**

**15. Plot points to create the graph.**

**16. Trace the point.**

**17. Manipulate the triangle.**

**Facilitator Questions**
- What type of function does this appear to be?
  *Linear*
- What other kinds of parent functions are there in this family?
- How can you determine the value of the constant of proportionality?

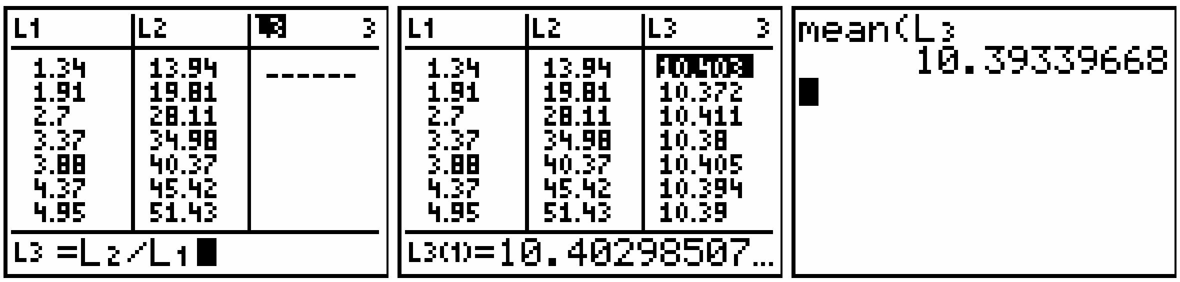
## Debrief Putting It All Together

*The explanations that follow come from the data collected for the triangle. The other three polygons use the same kind of analysis. Use facilitation questions to connect the explanations for the triangle to the other polygons.*

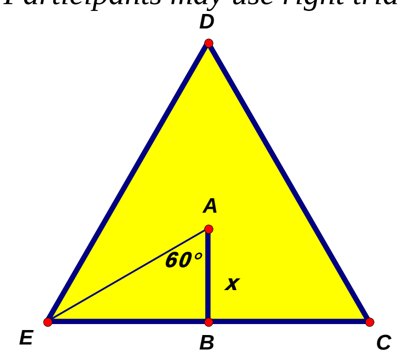
**1. What process did you use to develop your algebraic rules?**

*Participants should share their methods. Sample methods are shown below. If participants do not discuss each of these methods, the leader will bring them into the discussion.*

*Participants may use the list feature of a graphing calculator to find a constant of proportionality, the write the rule in the form y = kx. In this case y = 10.39x.*




*Participants may use right triangle trigonometry to develop the rule,* $P = 6x(\tan(60°))$.

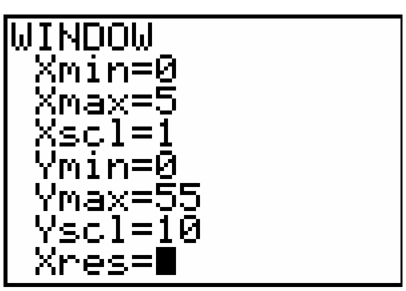


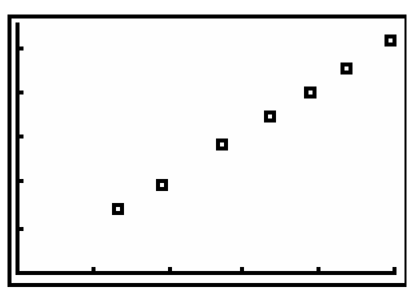

**EB = xtan(60°)**, **EC = 2EB** so **EC = 2xtan(60°)**.

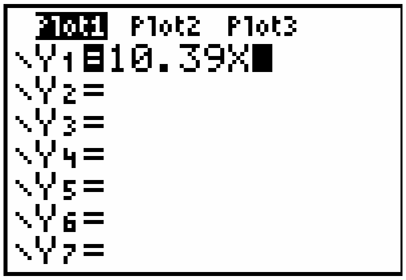
*The perimeter equals* **3EC** *so* **P = 6xtan(60°)**.

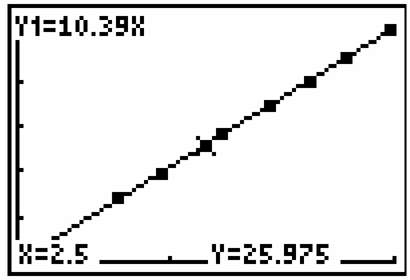
**2. How did you verify your function rule?**

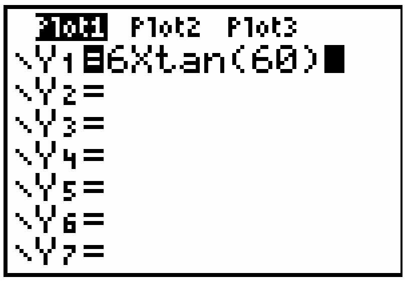
*Participants may have created a scatterplot using a graphing calculator then graphed the rule over the scatter plot.*

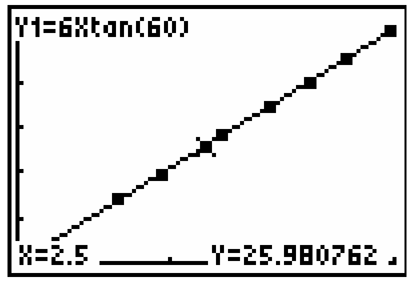





*Explain how to verify their function rule using Geometer's Sketchpad. For detailed steps on the verification of the function rule see the* ***Technology Tutorial T²***.

**3. How did you determine the approximate perimeter of an equilateral triangle with an apothem of 7.23 centimeters?**

*Participants may have used the table feature of the calculator.*

| X | Y1 | |
|---|---|---|
| 7.2 | 74.825 | |
| 7.21 | 74.929 | |
| 7.22 | 75.032 | |
| 7.23 | 75.136 | |
| 7.24 | 75.24 | |
| 7.25 | 75.344 | |
| 7.26 | 75.448 | |

X=7.23

**4. How did you determine the approximate length of the apothem of an equilateral triangle with a perimeter of 68.5 centimeters?**

*Participants may have used the table feature of the calculator.*

| X | Y1 | |
|---|---|---|
| 6.68 | 69.421 | |
| 6.69 | 69.525 | |
| 6.7 | 69.628 | |
| 6.71 | 69.732 | |
| 6.72 | 69.836 | |
| 6.73 | 69.94 | |
| 6.74 | 70.044 | |

X=6.69

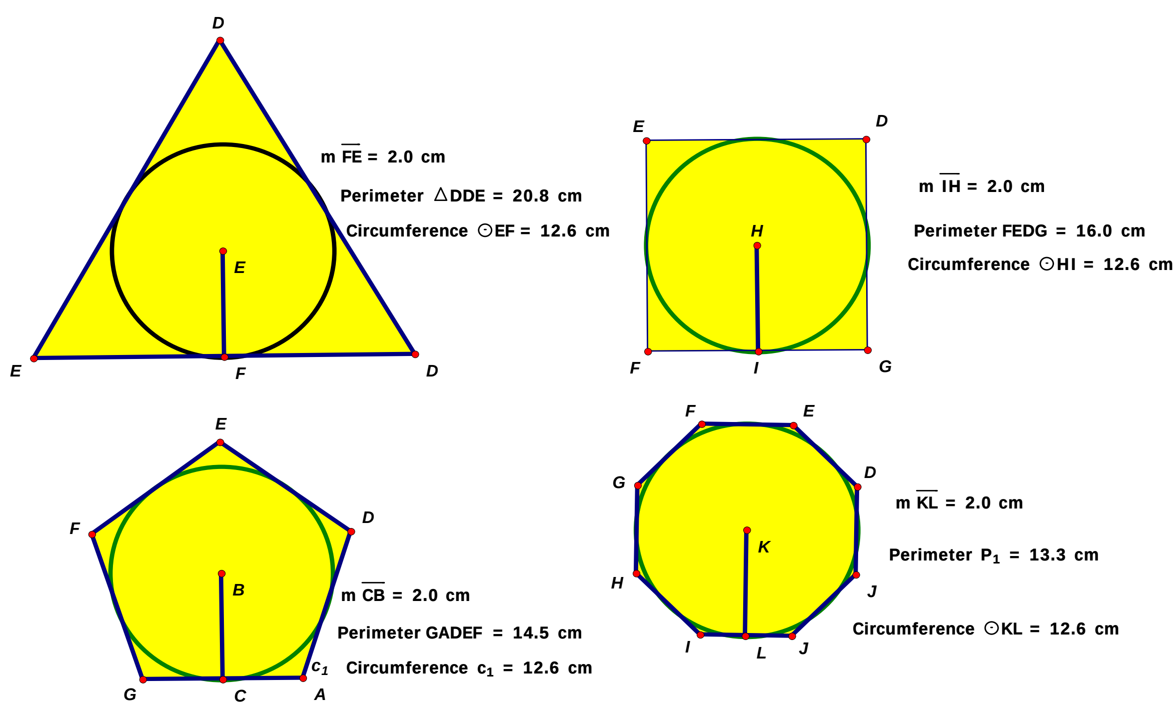
5. **How did you explain your graph of all four functions in a geometric context?**
*The slopes of the lines decrease as the number of sides of the polygon increase because the apothem is the radius of the inscribed circle. So as the number of sides increases the ratio of the perimeter to circumference decreases.*

6. **If the length of the apothem remains constant, what is the effect on perimeter as the number of sides of the polygon increases?**
*The perimeter decreases.*

7. **If you continue to increase the number of sides of the polygon while keeping the length of the apothem constant, what value will the perimeter approach?**
*The perimeter of the polygon approaches the circumference of the inscribed circle. This is illustrated in the sketch below.*

*To view these sketches electronically, open the sketch* ***Inscribed Circles****.*

*Participants might be confused about the concept of the inscribed circle, especially since the equilateral triangle's construction used a circle circumscribed about triangle. This sketch can help them see the relationship between the apothem and the radius of the inscribed circle.*



**8. How will the use of these technologies promote a better understanding of the targeted mathematical concepts?**
*Participant answers might include:*

*- Students can easily see that the apothem is related to the inscribed circle.*

*- The tie of algebra to geometry becomes obvious, thus opening up the idea of exploring relationships in other areas.*

*- Technology allows students to see several different cases, enabling them to make and test conjectures quickly.*

## Polygarden Landscaping Company
## Intentional Use of Data—Leader Notes

1. *At the close of the* ***Putting it All Together,*** *distribute the* ***Polygarden Landscaping Company Intentional Use of Data*** *activity sheet to each participant.*

2. *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.*

Facilitation Questions

- Which TEKS formed the primary focus of this activity?
- What additional TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. *As a whole group, share responses for two to three minutes.*

4. *As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.*

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

5. *As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.*

Facilitation Questions

- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

**6.** *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the elaborate phase as prompts for generating attributes of judicious users of technology.*

## Polygarden Landscaping Company
## Intentional Use of Data *(possible participant answers)*

| | | | |
|---|---|---|---|
| TEKS | | *a(5), a(6), G.5A, G.7A, G.7B, G.8A, G.11D* | |
| Question(s) to Pose to Students | Math | *What type of relationships could be found among the measurements you gathered?* | |
| | Tech | *How did technology help you with the gathering of data?* | |
| Cognitive Rigor | | Knowledge | √ |
| | | Understanding | √ |
| | | Application | √ |
| | | Analysis | √ |
| | | Evaluation | √ |
| | | Creation | √ |
| Data Source(s) | | Real-Time | *When using the computer sketch.* |
| | | Archival | *none* |
| | | Categorical | *none* |
| | | Numerical | *none* |
| Setting | | Computer Lab | *Each student uses the computer.* |
| | | Mini-Lab | *In groups students take turns or groups switch out.* |
| | | One Computer | *A student operates the control as other students read directions, entire class records data.* |
| | | Graphing Calculator | *Could be used to enter data and find relationships.* |
| | | Measurement Based Data | *Could be done at stations or individually.* |
| Bridge to the Classroom | | *This activity transfers directly to the classroom with the only modifications being the settings addressed above.* | |

# Polygarden Landscaping Company

## Explore

### Posing the Problem:

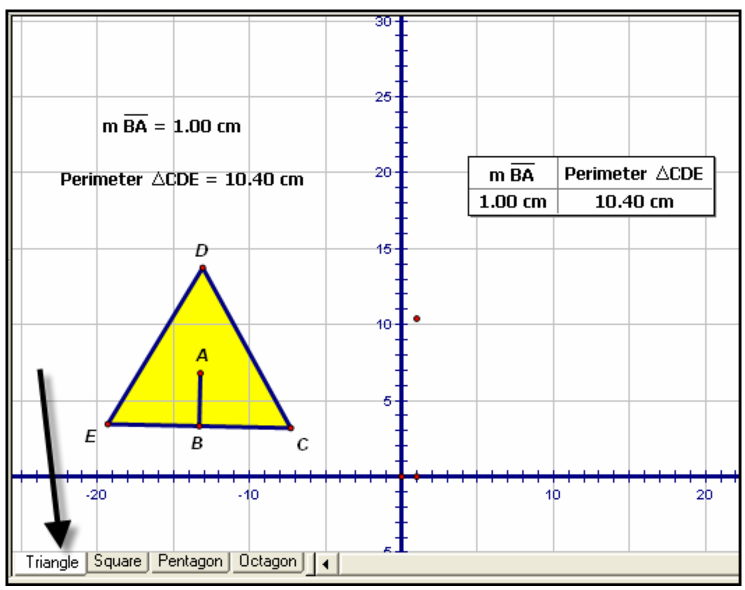
Polygarden Landscaping Company builds brick borders for flowerbeds that are always in the shape of regular polygons. To calculate the number of bricks necessary for a flowerbed, Brad, a bricklayer, needs to know the perimeter of the garden. On his last job Brad was not able to measure the perimeter of the flowerbed. He could only measure the distance from the center of the polygon to one side of the polygon. This distance is called the apothem. Is it possible for Brad to calculate the perimeter of the flowerbed if the only information he has is the length of the apothem and the number of sides of the garden?

### Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, you will use geometric sketches to collect and analyze data.

Open the sketch **Growing Polly's.**

Select the **Triangle** tab.




1. Double click on the table to add another row then click and drag point *C* away from point *B*. What do you observe?

2. Double click on the table again, and then move point *C* farther away from point *B*. Repeat this process until you have 10 rows in your table.

3. What patterns do you observe in the table?

4. What observations can you make about your graph?

5. Develop an algebraic rule that describes the relationship of the length of the apothem, $x$, to the perimeter, $y$.

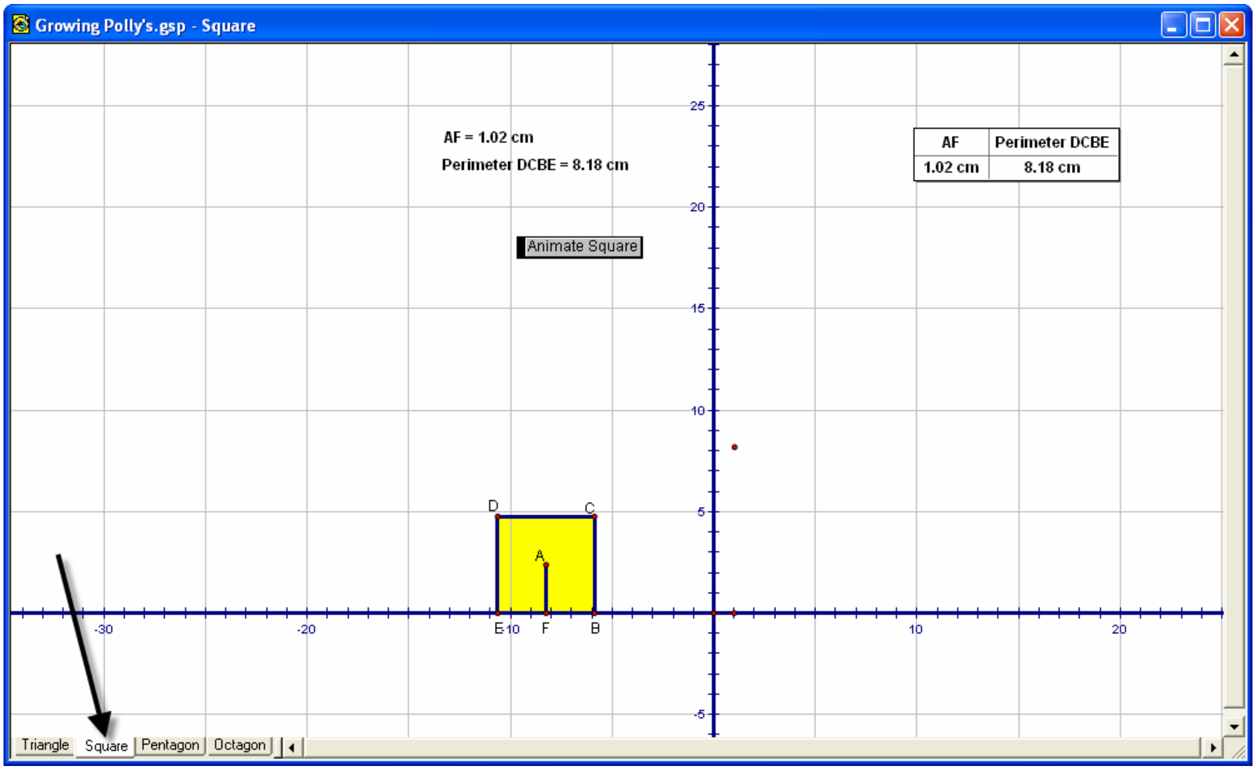
6. Verify that your function rule models your data. Explain your verification.

7. Write a verbal description of the relationship between the length of the apothem of an equilateral triangle and its perimeter.

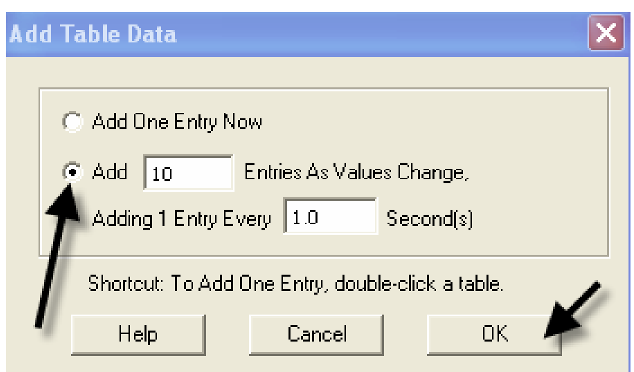
8. What is the approximate perimeter of a flowerbed that is in the shape of an equilateral triangle with an apothem of 7.23 centimeters?

9. What is the approximate length of the apothem of an equilateral triangle whose perimeter is 68.5 centimeters?

Select the **Square** tab.




1. ***Right*** click in the table and select the **Add Table Data** option. Select the **Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s)** and click **OK**.




2. Start the data collection process by clicking on the **Animate Square** button. After your table fills with data, stop the animation by clicking on the **Animate Square** button again. What happened?

3. What patterns do you observe in the table?

**4.** What observations can you make about your graph?

**5.** Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.

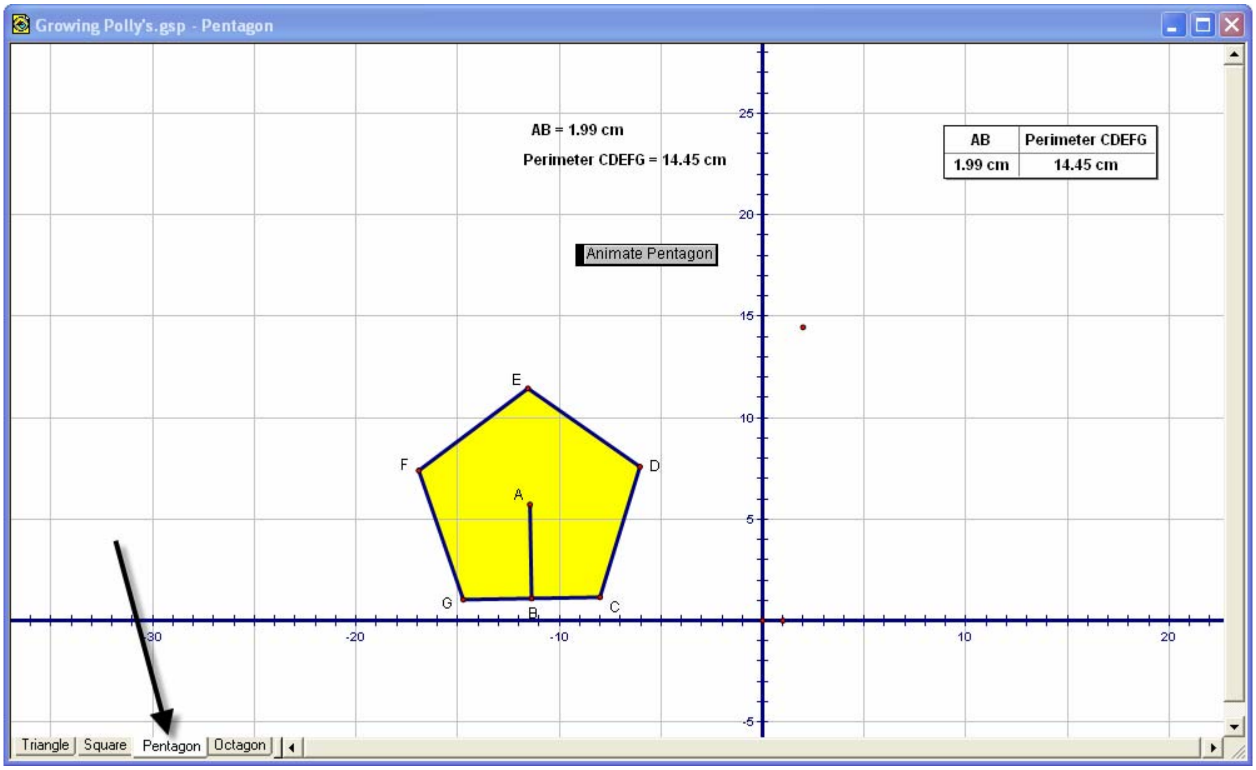
**6.** Verify that your function rule models your data. Explain your verification.

**7.** Write a verbal description of the relationship between the length of the apothem of square and its perimeter.

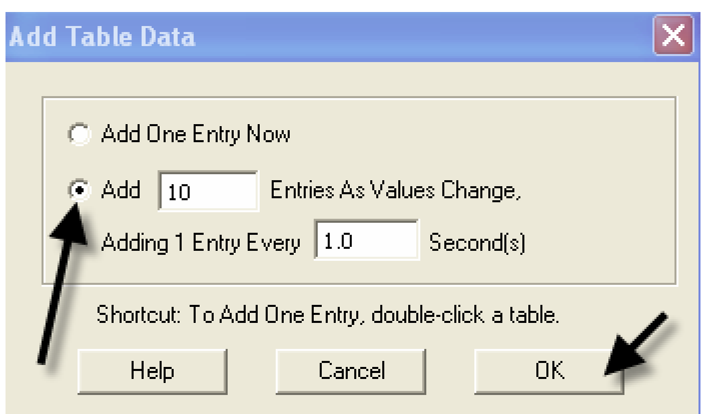
**8.** What is the approximate perimeter of a flowerbed that is in the shape of a square with an apothem of 7.23 centimeters?

**9.** What is the approximate length of the apothem of a square whose perimeter is 68.5 centimeters?

Select the **Pentagon** tab.




1. ***Right*** click in the table and select the **Add Table Data** option. Select the **Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s)** and click **OK**.




2. Start the data collection process by clicking on the **Animate Pentagon** button. After your table fills with data, stop the animation by clicking on the **Animate Pentagon** button again. What happened?

**3.** What patterns do you observe in the table?

**4.** What observations can you make about your graph?

**5.** Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.

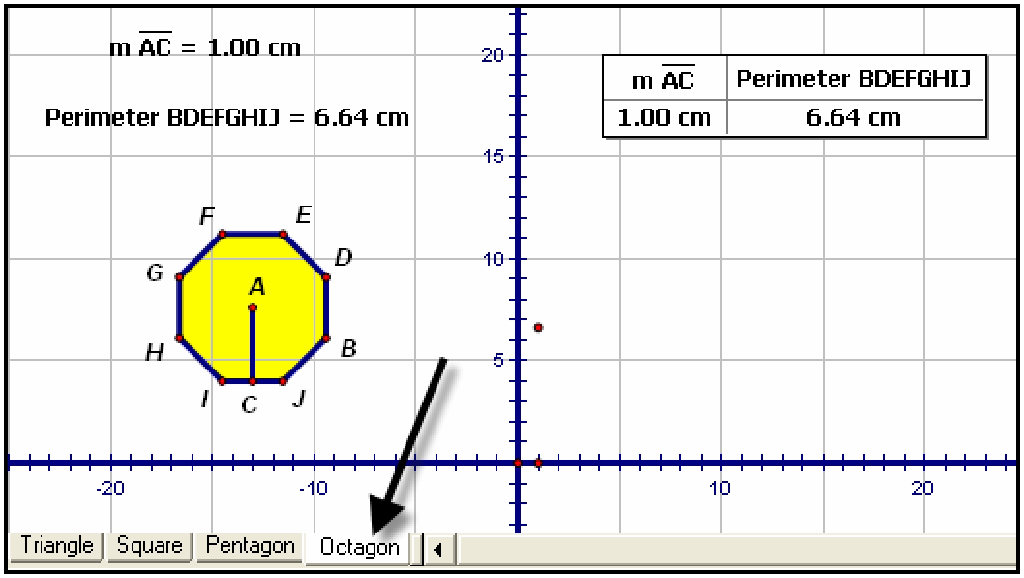
**6.** Verify that your function rule models your data. Explain your verification.

**7.** Write a verbal description of the relationship between the length of the apothem of a regular pentagon and its perimeter.

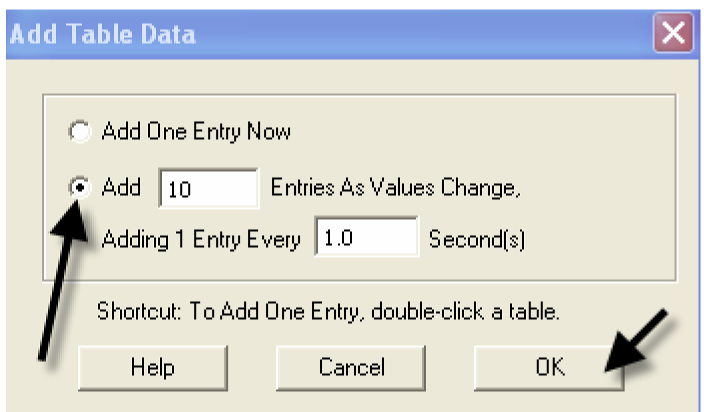
**8.** What is the approximate perimeter of a flowerbed that is in the shape of a regular pentagon with an apothem of 7.23 centimeters?

**9.** What is the approximate length of the apothem of a regular pentagon whose perimeter is 68.5 centimeters?

Select the **Octagon** tab.




1. ***Right*** click in the table and select the **Add Table Data** option. Select the **Add 10 Entries As Values Change, Adding 1 Entry Every 1.0 Second(s)** and click **OK**.




2. Start the data collection process by clicking on the **Animate Octagon** button. After your table fills with data, stop the animation by clicking on the **Animate Octagon** button again. What happened?

3. What patterns do you observe in the table?

4. What observations can you make about your graph?

**5.** Develop an algebraic rule that describes the relationship of the length of the apothem, *x*, to the perimeter, *y*.

**6.** Verify that your function rule models your data. Explain your verification.

**7.** Write a verbal description of the relationship between the length of the apothem of regular octagon and its perimeter.

**8.** What is the approximate perimeter of a flowerbed that is in the shape of a regular octagon with an apothem of 7.23 centimeters?

**9.** What is the approximate length of the apothem of a regular octagon whose perimeter is 68.5 centimeters?

## Putting It All Together

1. Complete the table.

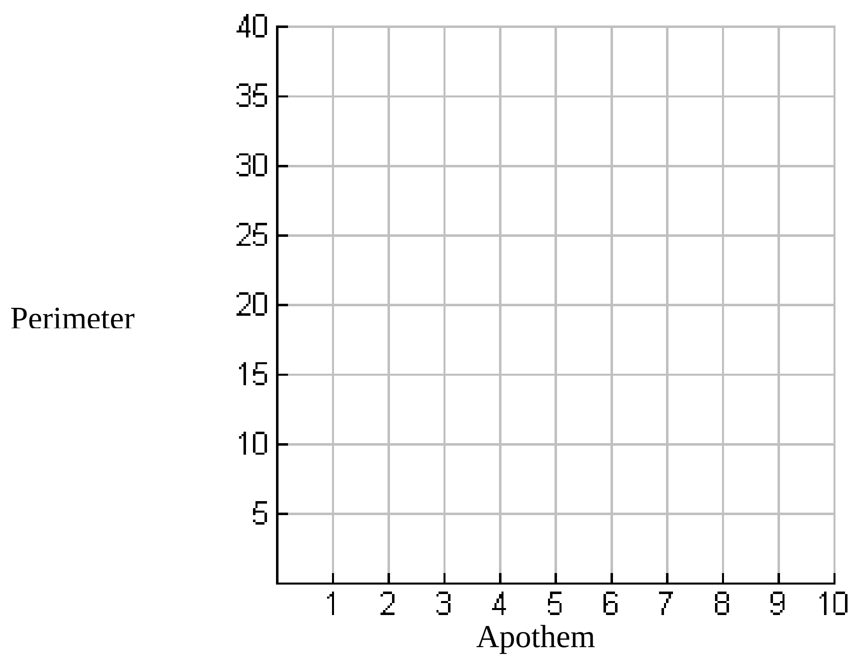
**Perimeter *versus* Apothem**

| Regular Polygon | Function Rule |
|---|---|
| Triangle | |
| Square | |
| Pentagon | |
| Octagon | |

2. In what ways are the function rules the same?

3. In what ways are the function rules different?

4. Graph all four-function rules on the same set of axes. Sketch your graph. Label each line with the name of the polygon.



5. What observations can you make about your graph? Connect your observations to geometric properties observed in this exploration.

6. Look back at Brad's problem. Is it possible for Brad to calculate the perimeter of the flowerbed if the only information he has is the length of the apothem and the number of sides of the garden? Why or why not?

7. Is there a general rule or trend you can develop using the information gathered? If so what is it?

8. If the length of the apothem remains constant, what is the effect on perimeter as the number of sides of the polygon increases?

9. If you continue to increase the number of sides of the polygon while keeping the length of the apothem constant, what value will the perimeter approach?

## Polygarden Landscaping Company
## Intentional Use of Data

| | | |
|---|---|---|
| TEKS | | |
| TEKS | | |
| Question(s) to Pose to Students | Math | |
| Question(s) to Pose to Students | Tech | |
| Cognitive Rigor | Knowledge | |
| | Understanding | |
| | Application | |
| | Analysis | |
| | Evaluation | |
| | Creation | |
| Data Source(s) | Real-Time | |
| | Archival | |
| | Categorical | |
| | Numerical | |
| Setting | Computer Lab | |
| | Mini-Lab | |
| | One Computer | |
| | Graphing Calculator | |
| | Measurement Based Data | |
| Bridge to the Classroom | | |

# Geometric Properties and Sketchpad Skills

## Explore Cycle II

### Purpose:

Provides participants the opportunity to use dynamic geometry technology to formulate and test conjectures about geometric properties and compare technology use to traditional teaching methods. This part of the training is designed for groups of two, three or four working with a computer station.

### Descriptor:

Participants will download pictures from the Internet and/or take digital photos with cameras and import them into Geometer's Sketchpad to explore geometric properties such as parallel and perpendicular lies and planes, congruence, similarity, etc. and make measurements of figures such as perimeter, area, volume. Participants will then use the collected information to formulate and test conjectures about geometric properties. They will then compare this activity with traditional methods of exploring print media with hand-held tolls such as compass, protractors, rulers, etc.

### Duration:

2 hours

### TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential I understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.7A Use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures.

G.7B Use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons.

G.7C Derive and use formulas involving length, slope, and midpoint.

G.8A Find areas of regular polygons, circles, and composite figures.

G.8B Find areas of sectors and arc lengths of circles using proportional reasoning.

G.8C Derive, extend, and use the Pythagorean Theorem.

G.8D Find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.

G.9A Formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models.

G.9B Formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models.

G.9C Formulate and test conjectures about the properties and attributes of circles and the lines that intersect the based on explorations and concrete models.

G.9D Analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

## TAKS Objectives:

- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of geometric relationships and shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

## Technology:

- Internet access
- Dynamic geometry software (Geometer's Sketchpad)
- Digital camera (optional)

## Materials:

### Advance Preparation:

- Participant access to computers with Geometer's Sketchpad (latest version update available from http://www.keypress.com/sketchpad) and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool.
- Sample sketches: Title.gsp, GeoPicExample1.gsp, GeoPicExample2.gsp, GeoPicExample3.gsp found on the CD.

**For each participant:**

- Ruler
- Protractor
- Copy of a magazine cover
- **Sketchpad Skills Investigation** activity sheet
- **Explore the World with Geometric Properties** activity sheet
- **Geometric Properties and Sketchpad Skills Intentional Use of Technology** activity sheet printed on green paper

**For each group of 2 participants:**

- Computers with Geometer's Sketchpad and Microsoft Excel
- Copy of the Technology Tutorial $T^2$

## Geometric Properties and Sketchpad Skills—Leader Notes

1. *Hand out a copy of the cover of a magazine, i.e. a copy of* Mathematics Teacher *from NCTM. Prompt participants to find an example of parallel lines and use measuring tools to prove the lines are parallel.*

2. *Show participants the same magazine cover pasted into Geometer's Sketchpad with parallel lines constructed on top of the letters and proved using the electronic measurement tools. For a sample see the **Title** sketch.*

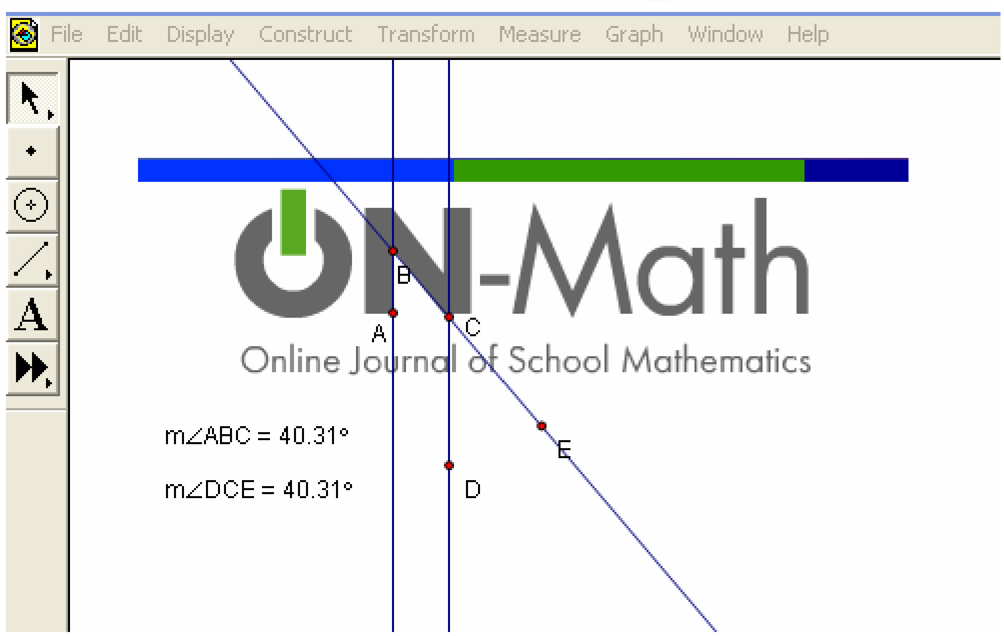
3. *Hand out the **Sketchpad Skills Investigation** activity sheet. Float among the participants to give assistance as needed. In particular, use facilitation questions to guide participants when they encounter the instructions to measure an angle and measure the area. Participants may use the Technology Tutorial $T^2$ if they need detailed instructions.*

4. *Hand out the **Explore the World with Geometric Properties** activity sheet.*

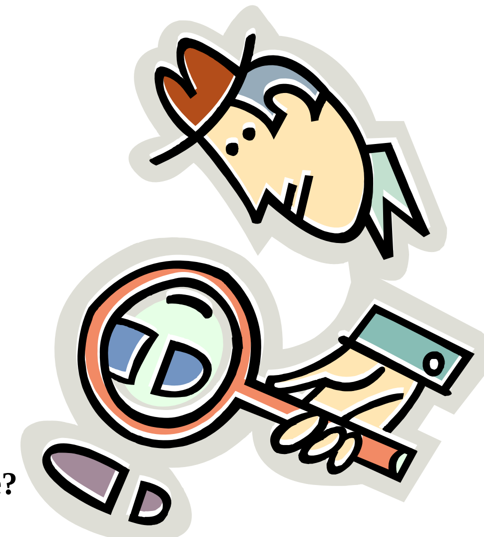
5. *Participants will search the Internet for pictures or take digital photos that represent geometric properties, import them into Geometer's Sketchpad, and then prove the properties using the skills they have just discovered in the Sketchpad Skills Investigation activity.*

# Sample Electronic Journal Cover with Constructions in Geometer's Sketchpad


See **Title** sketch.



# Sketchpad Skills Investigation



***For detailed instructions see Technology Tutorial T$^2$***

1. **Open a blank sketch in Geometer's Sketchpad.**

2. **Create some random points. What do you notice?**

3. **Select some points. Deselect them. How did you do this?**
   *Participants must use the Selection Tool and click in the blank white space to deselect items. Participants might confuse the Point Tool with the Selection Tool because it is shaped like an arrow or a "pointer." The Selection Tool is used often when working with Geometer's Sketchpad. Remind participants through out to be sure they have de-selected and/or selected items appropriately.*

4. **Label some points. What happens when you use the Label Tool? Why do you think this happens?**
   *When the cursor is lined up on the point, the hand turns black. This is so that I know I'm on the right point or item.*

5. **Make some circles. How can you deselect the last circle?**
   *De-selected the circle by clicking on the Selection Tool then clicking anywhere in the blank space.*

6. **Construct some segments, lines and rays. How do they differ from each other? Why?**
   *The segment has two endpoints; the ray shows one endpoint but the other end goes off the screen; the line goes off the screen in both directions and shows two points on the line. The representation of lines, rays and segments are true to their geometric definitions.*

7. **Label some of the segments, lines and rays you have created.**
   *Participants might inadvertently label a line when they really wanted to label a point.*

8. **Use the box feature of the** Selection **tool to quickly select some of your items. What happened?**
   *It selected every item it came in contact with even if it didn't surround it.*

**9. Use the Selection Tool to clear all objects from this page.**
*This allows the participants to clear a fresh start to explore the Menu Bar.*

**10. Click on the File menu and read the options. Slide the cursor across the menu bar and read the other options. What do you notice about some of the option choices?**
*Not all of the options are available. Things are "grayed" out. This is because nothing is selected.*

**11. Draw a segment and use the** Measurement **menu to measure it. Did you encounter any problems? If so, what were they?**
*Problems participants might encounter:*
- *Nothing highlighted under the measure bar so I had to be sure it was selected.*
- *Highlighting the segment and the endpoints will not allow measurements.*

**a. Did you measure a distance or a length? How could you have measured the other?**
*If participants get distance, then they only highlighted the two endpoints. If the participants get length, then they only highlighted the segment.*

**b. What is the difference between distance and length and how Geometer's Sketchpad interprets this?**
*While the value is the same, the distance measures the distance between the two endpoints and the length is the simply the length of the segment.*

**c. Create a line and measure it. Create a ray and measure it. What did you discover?**
*It is impossible to measure a ray and a line because they go off the page toward infinity.*

**12. Draw an angle and use the** Measurement **menu to measure it. Did you encounter any problems? If so, what were they?**
*Problems might be highlighting only the sides of the angle only allows length to be measured not the angle. If the entire angle is highlighted the measurement options are turned off.*

**Facilitation Question**
- How do you traditionally name an angle?
  *Name the point of the vertex, or name a point on the side, the vertex and a point on the other side.*

  *This will be the hint needed to measure their angle.*

a. **What was required in order for you to be able to measure your angle?**
*I had to select three points of the angle with the vertex in the middle.*

**13. Click on one side of your angle and adjust the size of your angle. What happens?**
*The angle measure changes with the angle.*

**14. Construct a circle and use the Measurement menu to explore the various measurement options. What measurements can be made?**
*Circumference, area, and radius.*

**15. Adjust the size of your circle by clicking on the control point on the circumference and dragging. What happens?**
*The measurements change as the size of the circle changes.*

**16. Draw a triangle and use the Measurement menu to explore the various measurement options. What measurements can be made?**
*We can measure each side and each angle.*

a. **Can you measure the perimeter? Is there another way?**
*We can add all the measurements of the sides together. The other way is to construct the interior of a triangle, which we will do next, but some participants might already know this.*

b. **How can you measure the area? Is there another way?**
*We can measure the height if we create a segment that represents the height and then compute the area using the formula. The other way is to construct the interior of a triangle, which we will do next, but some participants might already know this.*

**17. Construct the interior of your triangle. Measure the options that are now available. What were they?**
*Perimeter and Area.*

**18. Change the size of your triangle. What happens to the measurements?**
*All the measurements change as the triangle changes.*

**19. Draw a right triangle. Try to move it. Does it stay a right triangle? Why or why not?**
*Usually a right triangle that has been drawn will not stay a right triangle. To get a right triangle to stay a right triangle when moved, the right angle must be constructed.*

**20. Construct a 30-60-90 triangle.**

**21. Explore moving your triangle by clicking on various segments and angles. Which objects allow the triangle to stay the same size? Why?**
*Each segment will allow the triangle to slide on the page but the size stays the same. Also, the vertex that was constructed as an intersection of two lines will move the triangle but the size doesn't change. This has to do with how they were constructed.*

- **a. Which parts of the triangle allow it to adjust size? Why?**
  *The vertex at the right angle and the other vertex along the base of the triangle allow the triangle to change sizes.*

- **b. Will this triangle always stay a 30-60-90 degree triangle no matter how big or small it gets? How do you know?**
  *Yes, because we constructed a perpendicular and we rotated an angle to form the 30- or 60-degree angle.*

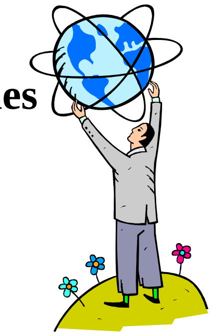
**22. Create a Hide/Show button to hide your extra construction pieces.**

**23. Reflect your triangle. What happens?**
*The entire triangle flipped over the line of reflection*

**24. How can you continue with this to make a tessellation? Try it.**
*Continue making transformations.*

- **a. Did you encounter any challenges? If so, what were they and how did you overcome them?**
  *Highlighting just one triangle was difficult; I selected the two legs and the vertex that were going to be reflected.*

**25. What other shapes appear in your tessellation?**
*Parallelograms, Quadrilaterals, Hexagons…etc.*



# Explore the World with Geometric Properties

- **Open a new sketch in Geometer's Sketchpad.**

  *For detailed instructions on opening a sketch, see the* ***Technology Tutorials T$^2$.***

- **Search the Internet for pictures or take digital photos that would demonstrate the following geometric properties: parallel lines, tangent to a circle, similar figures, congruent figures, and the central angle of a circle.**
  **Challenge: find other geometric concepts represented in the world.**

  *A search engine such as Google/Image works well with topics such as architecture pictures, kite pictures, bridge pictures, bicycle pictures, etc.*

- **Import your pictures into Geometer's Sketchpad, one picture per page.**

  *If participants need assistance on importing pictures, there are detailed instructions in the* ***Technology Tutorials T$^2$.***

- **Use the Geometer's Sketchpad tools to construct and prove the geometric properties represented in your picture. Use a Text Box to show the URL where your picture was found along with any additional information that would be helpful for other participants viewing your construction.**

  *Participants may find it difficult to see the default colors or lines as they are constructed on their photos. It may be necessary to remind them change the colors and thickness of figures. For detailed instructions see the* ***Technology Tutorials T$^2$.***

- **Report your findings to the rest of the participants via the method suggested by the facilitator.**

  *See the Explain section below.*

## Explain

*This Explain phase of the professional development provides each group of participants the opportunity to report their discoveries to their peers. This part of the training is designed for each group to present to the entire group while the facilitator encourages participants to see as many mathematical connections as possible. Use facilitation questions to lead the discussion.*

*Various methods of reporting out:*

- *Have each group save their sketch to a thumb drive. Re-open it at a computer connected to the presentation equipment in the room.*
- *Have participants do a gallery tour where one representative of the group stands by their computer to answer questions as other participants come by to look.*
- *Have entire groups rotate until they have viewed all the other sketches and return to their own station.*

*Use the facilitation questions as participants are reporting out or if they are floating from station to station. Ask the questions at the end with a sketch of your own or the example sketch* ***GeoPicExample***.

**Facilitation Questions**

- Did the perspective of the picture cause any problems?
- Which pictures were better then others? Why?
- What were the mathematical ideas that were explored? Were there any others?
- How did you verify the concepts were true?
- What other ways could students have explored the same concepts?
- Were there relationships that you hadn't thought of before? If so, what were they?
- Do you agree with the findings of the other groups?
- What else could you have done with the pictures?
- How will your students react to an activity like this?
- Could you make this more challenging for you students? How?
- What prior knowledge did you need to explore your picture geometrically?
- What questions could you ask students to help them focus on a specific geometric topic as they explored different pictures?
- How would this activity been different if you could import a picture you have taken yourself using a digital camera?
- Was there an underlying theme of mathematical topics in the pictures explored? If so, what was it? Were there sub categories into which the topic could be divided? If so, what are they?

**Geometric Properties and Sketchpad Skills**

**Intentional Use of Data—Leader Notes**

**1.** *At the close of the Explain phase, distribute the* ***Intentional Use of Data*** *activity sheet to each participant.*

**2.** *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.*

Facilitation Questions

- Which TEKS formed the primary focus of this activity?
- What additional TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

**3.** *As a whole group, share responses for two to three minutes.*

**4.** *As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.*

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

**5.** *As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.*

Facilitation Questions

- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- Why was technology withheld during the first part of this activity (the magazine cover)?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

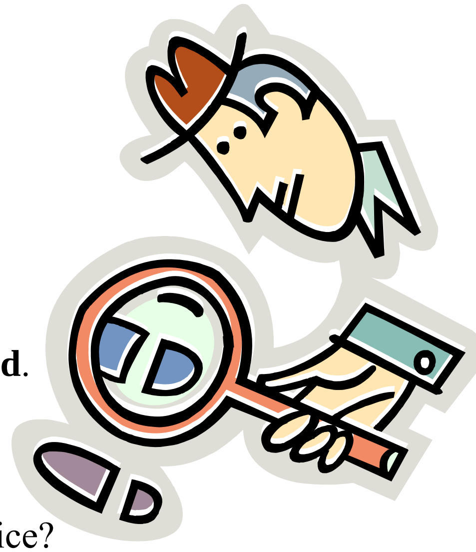
**6.** *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets  provide prompts for generating attributes of judicious users of technology during the elaborate pahse.*

## Geometric Properties and Sketchpad Skills
## Intentional Use of Data *(possible participant answers)*

| | | | |
|---|---|---|---|
| TEKS | | | *a(5), a(6),G.7A, G.7B, G.7C, G.8A, G.8B, G.8C, G.9A, G.9B, G.9C, G.9D* |
| Question(s) to Pose to Students | Math | | *How many different geometric properties were you able to identify in one picture?*<br>*What type of patterns did you discover? Are there others?* |
| | Tech | | *How did technology help you with the identification of geometric properties?* |
| Cognitive Rigor | | Knowledge | √ |
| | | Understanding | √ |
| | | Application | √ |
| | | Analysis | √ |
| | | Evaluation | √ |
| | | Creation | √ |
| Data Source(s) | | Real-Time | *none* |
| | | Archival | *none* |
| | | Categorical | *none* |
| | | Numerical | *none* |
| Setting | | Computer Lab | *Each student uses the computer.* |
| | | Mini-Lab | *In groups students take turns or groups switch out.* |
| | | One Computer | *A student operates the control as other students read directions, entire class records data.* |
| | | Graphing Calculator | *Could be used to enter data and find relationships.* |
| | | Measurement Based Data | *Could be done at stations or individually.* |
| Bridge to the Classroom | | | *This activity transfers directly to the classroom with the only modifications being the settings addressed above.* |

# Sketchpad Skills Investigation

***For detailed instructions see Technology Tutorial T²***



1. Open a blank sketch in **Geometer's Sketchpad**.

2. Create some random points. What do you notice?

3. Select some points. Deselect them. How did you do this?

4. Label some points. What happens when you use the **Label Tool**? Why do you think this happens?

5. Make some circles. How can you deselect the last circle?

6. Construct some segments, lines and rays. How do they differ from each other? Why?

7. Label some of the segments, lines and rays you have created.

8. Use the box feature of the **Selection** tool to quickly select some of your items. What happened?

9. Use the **Selection Tool** to clear all objects from this page.

10. Click on the **File** menu and read the options. Slide the cursor across the menu bar and read the other options. What do you notice about some of the option choices?

11. Draw a segment and use the **Measurement** menu to measure it. Did you encounter any problems? If so, what were they?

    a. Did you measure a distance or a length? How could you have measured the other?

    b. What is the difference between distance and length and how Geometer's Sketchpad interprets this?

    c. Create a line and measure it. Create a ray and measure it. What did you discover?

12. Draw an angle and use the **Measurement** menu to measure it. Did you encounter any problems? If so, what were they?

    a. What was required in order for you to be able to measure your angle?

13. Click on one side of your angle and adjust the size of your angle. What happens?

14. Construct a circle and use the **Measurement** menu to explore the various measurement options. What measurements can be made?

15. Adjust the size of your circle by clicking on the control point on the circumference and dragging. What happens?

16. Draw a triangle and use the **Measurement** menu to explore the various measurement options. What measurements can be made?

    a. Can you measure the perimeter? Is there another way?

    b. How can you measure the area? Is there another way?

17. Construct the interior of your triangle. What measurement options are now available?

18. Change the size of your triangle. What happens to the measurements?

19. Draw a right triangle. Try to move it. Does it stay a right triangle? Why or why not?

20. Construct a 30-60-90 triangle.

21. Explore moving your triangle by clicking on various segments and angles. Which objects allow the triangle to stay the same size? Why?

    a. Which parts of the triangle allow it to adjust size? Why?

    b. Will this triangle always stay a 30-60-90 degree triangle no matter how big or small it gets? How do you know?

22. Create a Hide/Show button to hide your extra construction pieces.

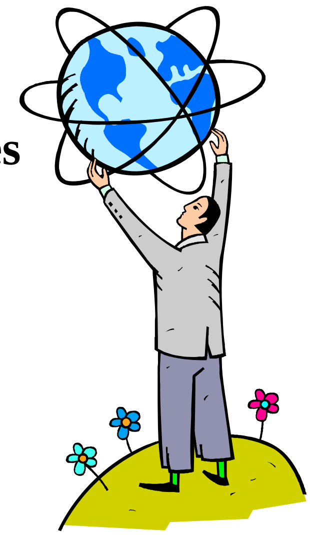
23. Reflect your triangle. What happens?

24. How can you continue with this to make a tessellation? Try it.

    a. Did you encounter any challenges? If so, what were they and how did you overcome them?

25. What other shapes appear in your tessellation?

# Explore the World with Geometric Properties



- Open a new sketch in Geometer's Sketchpad.

- Search the Internet for pictures or take digital photos that would demonstrate the following geometric properties:  parallel lines, tangent to a circle, similar figures, congruent figures, central angle of a circle. *Challenge:*  Find other geometric concepts represented in the world.

- Import your pictures into Geometer's Sketchpad, one picture per page.

- Use the Geometer's Sketchpad tools to construct and prove the geometric properties represented in your picture.  Use a Text Box to show the URL where your picture was found along with any additional information that would be helpful for other participants viewing your construction.

- Report your findings to the rest of the participants via the method suggested by the facilitator.

# Geometric Properties and Sketchpad Skills
## Intentional Use of Data

| | | | |
|---|---|---|---|
| TEKS | | | |
| | | | |
| Question(s) to Pose to Students | Math | | |
| | Tech | | |
| Cognitive Rigor | Knowledge | | |
| | Understanding | | |
| | Application | | |
| | Analysis | | |
| | Evaluation | | |
| | Creation | | |
| Data Source(s) | Real-Time | | |
| | Archival | | |
| | Categorical | | |
| | Numerical | | |
| Setting | Computer Lab | | |
| | Mini-Lab | | |
| | One Computer | | |
| | Graphing Calculator | | |
| | Measurement Based Data | | |
| Bridge to the Classroom | | | |

# Dome Floor Dilemma

## Explore/Explain Cycle III

### Purpose:

Provide participants the opportunity to use technology to explore relationships in geometric figures that yield quadratic data, such as change in area of a circle as the length of the radius changes. Participants will make connections between algebraic and geometric concepts that enhance their student's conceptual understanding of the Geometry TEKS.

### Descriptor:

In a guided exploration, participants will create a sketch using Geometer's Sketchpad. They will collect and analyze data collected from their sketch using a variety of technologies. They will use problem-solving strategies of breaking a large problem into smaller components and working backwards to facilitate the constructions and the development of geometry concepts.

### Duration:

2 hours

### TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.8A Find areas of regular polygons, circles, and composite figures.

G.8B Find areas of sectors and arc lengths of circles using proportional reasoning.

G.9C Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

G.11D Describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

## TAKS Objectives:

- Objective 3: Linear Functions
- Objective 4: Formulate and Use Linear Equations and Inequalities
- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of Geometric Relationships and Shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

## Technology:

- Spreadsheet technology
- Hand-held graphing calculator
- Dynamic geometry software (Geometer's Sketchpad)
- Graph link technology

## Materials:

**Advance Preparation:**

- Participant access to computers with Geometer's Sketchpad(latest version update available from http://www.keypress.com/sketchpad) and necessary sketches and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool
- Sketch **arcsegment.gsp** found on the CD (for leader's information).

**For each group of two:**

- Computer
- Copy of the **Technology Tutorial T$^2$**

**For each participant:**

- Dome Floor Dilemma activity sheets
- Analyze the Data activity sheets
- Explain activity sheet
- Dome Floor Dilemma Intentional Use of Data (printed on green paper)

## Leader Notes:

*In this exploration participants will use Geometer's Sketchpad to create a sketch. They will use the sketch to collect and analyze data to discover the relationship between the length of the radius of a circle and the area of a sector and segment with a 60° arc. Specific details for using Geometer's Sketchpad are found in the* ***Technology Tutorial*** $\boldsymbol{T^2}$***--Dome Floor Dilemma.***

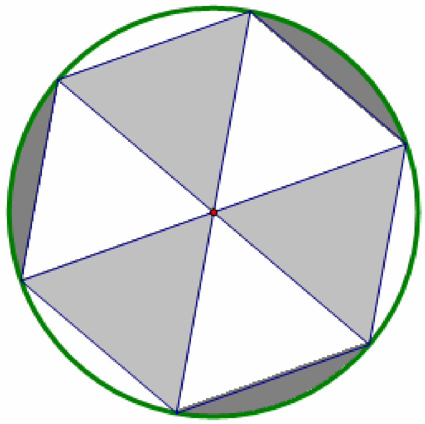
*Participants will gather the data and analyze it on their own using their choice of the Geometer's Sketchpad, graphing calculator, TI-Interactive, spreadsheet, etc. During the discussion of the Explain phase, they will discuss several methods of analyzing the data and identify comparative advantages and disadvantages of each method.*

# Dome Floor Dilemma

## Explore

### Posing the Problem:

**The diagram below represents the tile pattern on the circular floor of a domed building. Each shade, light, medium, and dark, represents a different color of floor tile. Each central angle is congruent to all others.**



**If you know the length of the radius of the circular floor, is it possible to calculate the area of each shaded region?**

*To answer this question participants will complete the explore activity.*

### Obtaining and Analyzing the Data:

**To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, you will construct a geometric figure. then collect and analyze data. You will determine three functional relationships: area of a sector of a circle versus the radius, area of a segment of a circle versus the radius, and the area of the triangle bound by the segment and the radii drawn to the endpoints of the arc of the segment.**

## The Sector Construction

**For detailed instructions on Geometer's Sketchpad see the Technology Tutorial T²—Dome Floor Dilemma.**

1. **Construct a circle with a radius.**

2. **Rotate the radius and the endpoint that lies on the circle 60°.**

3. **Construct the intercepted arc of the sector.**

4. **Construct the interior of the sector.**

5. **Measure the length of the radius and the area of the sector.**

6. **Create a table to compare the two measurements. Which one is the independent variable and which one is the dependent variable?**

   *The independent variable is the length of the radius and the dependent variable is the area of the sector.*

7. **Plot the two measurements on a graph and turn on the trace feature.**

## Collect the Data

6. **Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row. then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?**

   *The measures change. The points are plotted and traced to create a graph.*

7. **Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.**

8. **What patterns do you observe in the table?**

   *Participants may observe that there is not a constant rate of change.*

9. **What observations can you make about your graph?**

   *Participants may observe that the graph appears to be quadratic.*

## The Arc Segment Construction

1. Construct the arc segment.

2. Change the color of the segment.

3. Measure the area of the segment.

4. Create a table to compare the measure of the area of the arc segment and the length of the radius. Which one is the independent variable and which one is the dependent variable?

   *The independent variable is the length of the radius and the independent variable is the area of the arc segment.*

5. Plot the two measurements on the graph and turn on the trace feature.

   *Participants might have trouble with the trace feature if they have extra items highlighted on their screen when they choose the trace button. Remind them often to click in the blank white space.*

## Collect the Data

6. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

   *Possible answers might include: The measures change. The points are plotted and traced.*

7. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

8. What patterns do you observe in the table?

   *Participants may observe that there is not a constant rate of change.*

9. What observations can you make about your graph?

   *Participants may observe that the graph appears to be quadratic.*

## The Triangle Construction

1. Construct the triangle interior.

2. Measure the area of the triangle.

3. Create a table to compare the area of the triangle to the length of the radius. Which one is the independent variable? Which one is the dependent variable?

   *The independent variable is the length of the radius and the dependent variable is the area.*

4. Plot the two measurements on the graph and turn on the trace feature.

## Collect the Data

5. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

   *The measures change. The points are plotted and traced.*

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

   *Participants may observe that this is not a constant rate of change.*

8. What observations can you make about your graph?

   *Participants may observe that the graph appears to be quadratic.*

## Analyze the Data

1. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the sector, *y*.**

   $$y = \frac{\pi x^2}{6}$$

2. **Verify that your function rule models your data. Explain your verification.**

   *Participants may graph the function rule over the scatterplot or verify using a table.*

3. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the triangle, *y*.**

   $$y = \frac{x^2\sqrt{3}}{4}$$

4. **Verify that your function rule models your data. Explain your verification.**

   *Participants may graph the function rule over the scatterplot or verify using a table.*

5. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the segment, *y*.**

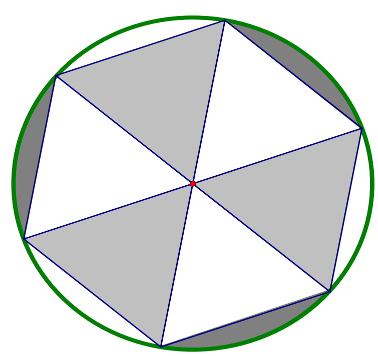
   $$y = \frac{\pi x^2}{6} - \frac{x^2\sqrt{3}}{4}$$

6. **Verify that your function rule models your data. Explain your verification.**

   *Participants may graph the function rule over the scatterplot or verify using a table.*

7. **Recall the floor design discussed earlier. The radius of the circle is 45 feet in length and the cost of tiling the different areas is listed below.**

   **Un-shaded areas - $10.50 per square foot,**
   **Medium shaded areas - $12.00 per square foot and the**
   **Darkest shaded areas - $17.45 per square foot.**

   **Approximately what will be the total cost of tiling the floor?**

   *The total cost will be $75,394.30.*

*As a segue into the Explain Phase use the following facilitation questions to discuss the data collection and data analysis of the Dome Floor Dilemma.*

**Facilitation Questions**
*Data Collection:*
- Why does the area not increase at the same rate as the radius?
  *Area involves squaring the radius.*
- What relationships are there among the 3 data sets?
  *For equal radii the area of the sector is always greatest, followed by the triangle, then the segment.*

*Data Analysis:*
- What type of function does this appear to be?
  *Quadratic*
- What is the parent function for this family?
  $y = x^2$
- How can you use geometric properties to determine the function rules?
  *Answers may vary. Using the concept of composite area, segment area + triangle area = sector area.*
- Why are the graphs of the functions similar?
  *They are all area functions and of the family* $y = x^2$.
- Why are the graphs of the functions different?
  *They are increasing at different rates.*

# Explain

*In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator and to some degree Geometer's Sketchpad. If none of the participant groups used a calculator, ask them how that method could have been used to analyze the data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.*

1. **What knowledge of geometric properties was necessary to complete the constructions?**
   *Answers may vary. Participants should discuss the properties of circles. For example the sum of the measures of the central angles of a circle is 360°.*

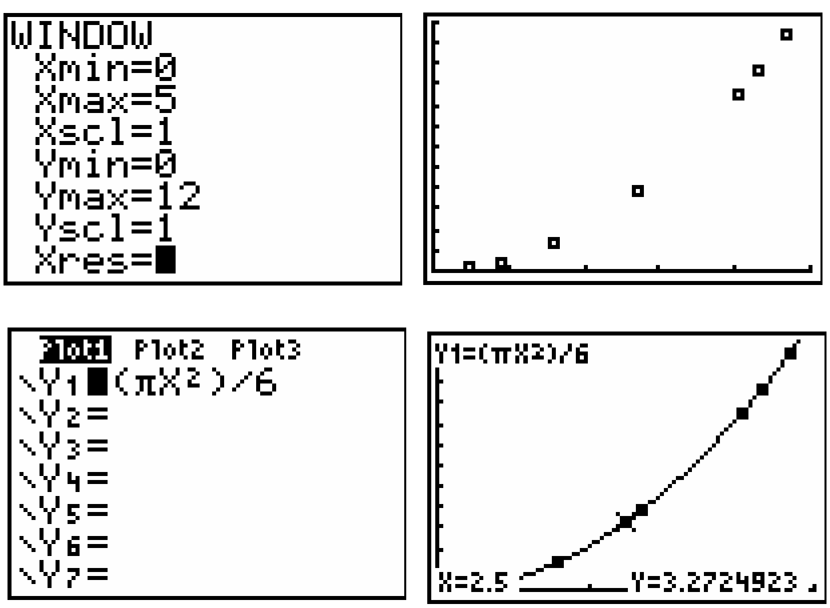
2. **What process did you use to develop your algebraic rules?**
*Participants should share their methods. Sample methods appear below. If participants do not discuss each of these methods, the leader will bring them into the discussion.*

***Area of a sector****: Since the area of the whole circle is* $A = \pi r^2$ *and in this case there are 6 sectors, the area of one sector is* $A_{sec} = \frac{\pi r^2}{6}$. *In this case the triangle is equilateral; therefore, its area can be expressed as* $A = \frac{s^2\sqrt{3}}{4}$ *where s represents the length of a side of the triangle. Since the triangle is equilateral s = r s,o the area can be expressed as* $A = \frac{r^2\sqrt{3}}{4}$. *To calculate the area of the segment, subtract the area of the triangle from the area of the sector. So the area of the segment is* $A_{seg} = \frac{\pi r^2}{6} - \frac{r^2\sqrt{3}}{4}$.

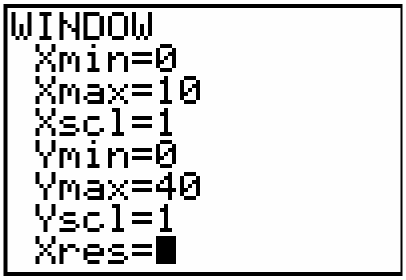
3. **How did you verify your function rules?**

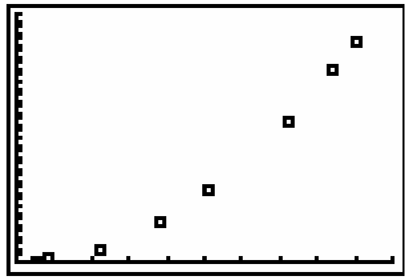
*Participants may have created a scatterplot using a graphing calculato,r then graphed the rule over the scatter plot.*

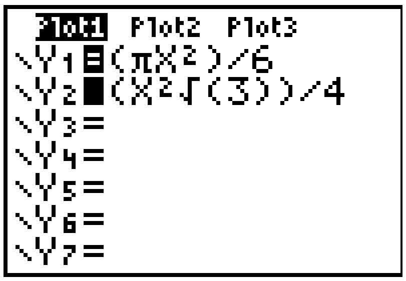
***Sector***

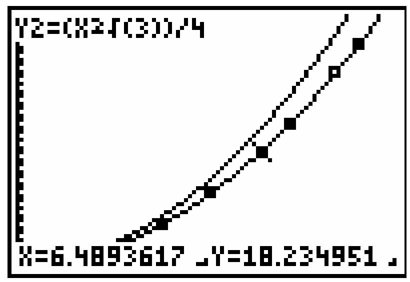


***Triangle***

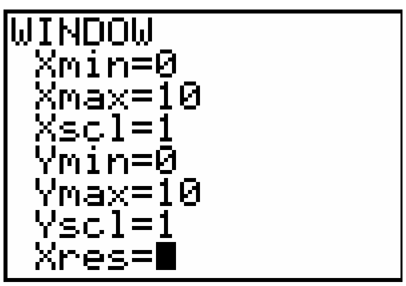


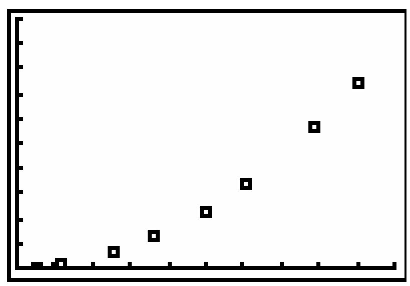



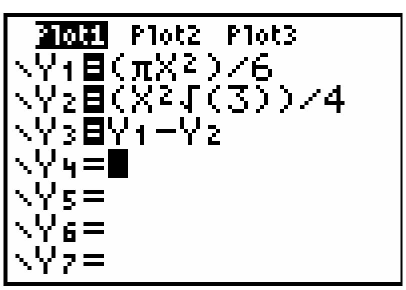


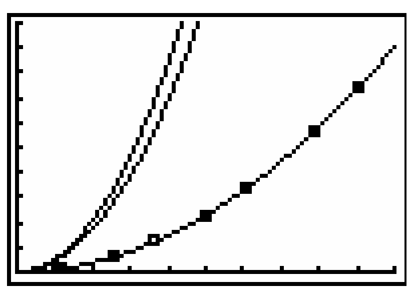




***Segment***





*Participants may have verified the area of the segment symbolically and graphically.*

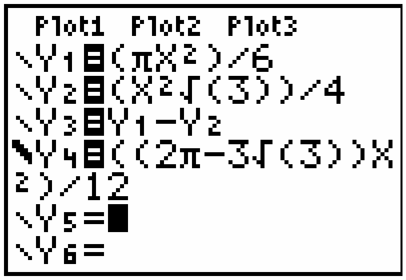
$$segment = sector - triangle$$

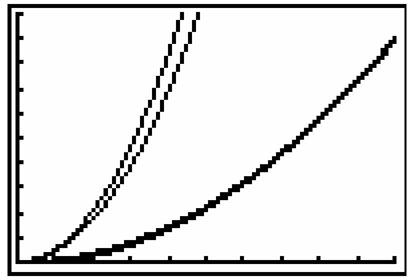
$$segment = \frac{\pi}{6}x^2 - \frac{\sqrt{3}}{4}x^2$$

$$segment = \frac{4\pi}{24}x^2 - \frac{6\sqrt{3}}{24}x^2$$

$$segment = \frac{4\pi - 6\sqrt{3}}{24}x^2$$

$$segment = \frac{2\pi - 3\sqrt{3}}{12}x^2$$



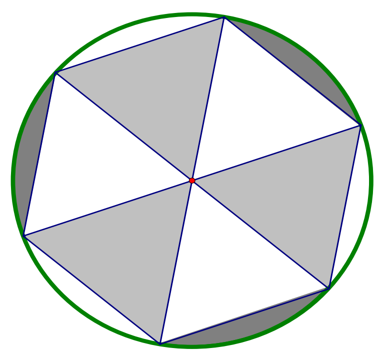



*Explain how to verify the function rule using Geometer's Sketchpad. (See* ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.****)*
*Explain how to verify the function rule using TI-Interactive. (See* ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.*** *)*
*Explain how to verify the function rule using Spreadsheet. (See* ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.****)*

4. **How did you solve the dome floor dilemma?**

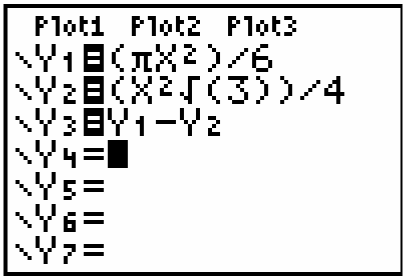
*Recall the floor design discussed earlier. The radius of the circle is 45 feet in lengt, and the cost of tiling the different areas is listed below.*

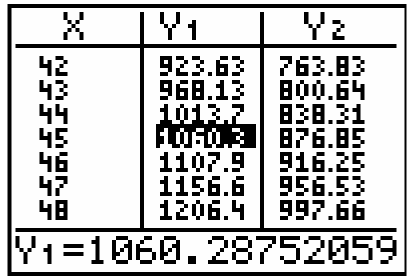
*Un-shaded areas - $10.50 per square foot,*
*Medium shaded areas - $12.00 per square foot and the*
*Darkest shaded areas - $17.45 per square foot.*

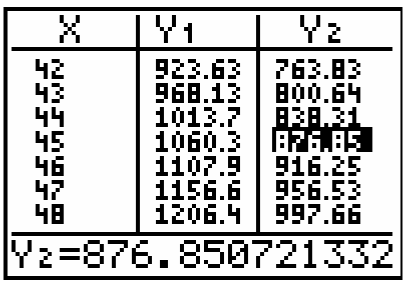


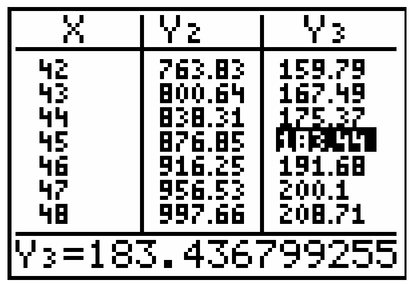
*Approximately what will be the total cost of tiling the floor?*

*Participants may have used the table feature of the calculator to determine the areas of the different regions.*




*The area of one sector is approximately 1,060.29 square feet. Since there are 3 medium-shaded sectors, multiply 3 times 1,060.29. This gives 3,180.87 square feet at a cost of $12.00 per square foot, for a total of $38,170.44.*

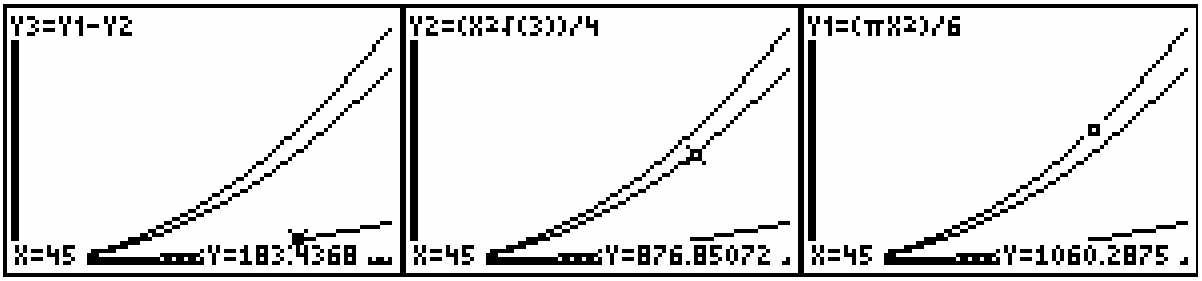
*The area of one triangle is approximately 876.85 square feet. Since there are 3unshaded triangles, multiply 3 times 876.85. This gives 2,630.55 square feet at a cost of $10.50 per square foot, for a total of $27,620.78.*

*The area of one dark shaded segment is approximately 183.44 square feet. Since there are 3 segments, multiply 3 times 183.44. This gives 550.32 square feet at a cost of $17.45 per square foot, for a total of $9,603.08.*

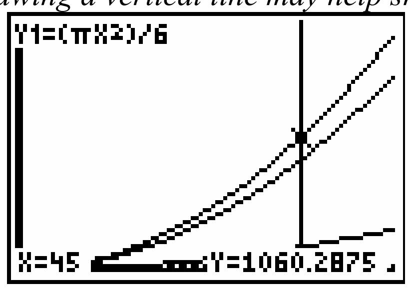
*The total cost will be $75,394.30.*

5. **How can you explain your graph of all three functions in a geometric context?**

*The graphs are all quadratic because we are calculating area. Each is a transformation of the other because as the radius changes the area of each region grows at a different rate. At any value for the radius the sum of the y values for the segment and triangle functions will equal the y value for the sector function. This verifies that the area of a segment equals the area of the sector minus the area of the triangle.*




*Drawing a vertical line may help show this relationship.*




**Note to Leader:** *Record or have a participant volunteer to record the responses to Questions 4, 5, and 6 on chart paper to use in the Elaborate phase of the professional development.*

6. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**

*Responses may vary.*

*The data analysis takes only a few keystrokes. The power to set your own parameters and graph the function rule empowers the participant to use numerical analysis to calculate meaningful parameters such as a constant of variation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines. However, the small screen is difficult to see, and the axes in the window cannot be labeled.*

7. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**

   *Responses may vary.*

   *The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.*

   *However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.*

8. **What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?**

   *Responses may vary.*

   *Like the graphing calculator, data analysis requires only a few keystrokes and clicks. The function editor enables participants to set their own rational function, empowering them to choose parameters that make physical sense in the context of the problem. The graphical analysis features of TI-Interactive make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.*

   *Like the spreadsheet, axes can be labeled numerically and with text. The graphs are cleaner and can be copied and pasted into other computer documents.*

9. **How will the use of these technologies promote a better understanding of the targeted mathematical concepts?**

   *Answers will vary. The technology makes data collection and analysis a less time-consuming process, allowing more time to explore and connect the geometric concepts.*

## Dome Floor Dilemma
## Intentional Use of Data—Leader Notes

1. *At the close of the Explain phase, distribute the* ***Dome Floor Dilemma Intentional Use of Data*** *activity sheet to each participant.*

2. *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to identify two key questions emphasized during this activity. Allow four minutes for discussion.*

> Facilitation Questions
> - Which TEKS formed the primary focus of this activity?
> - What additional TEKS supported the primary TEKS?
> - How do these TEKS translate into guiding questions to facilitate student exploration of the content?
> - How do your questions reflect the depth and complexity of the TEKS?
> - How do your questions support the use of technology?

3. *As a whole group, share responses for two to three minutes.*

4. *As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.*

> Facilitation Question
> - What attributes of the activity support the level of rigor that you identified?

5. *As a whole group, discuss how this activity might function in other settings. Allow five minutes for discussion.*

Facilitation Questions

- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

6. *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will provide prompts for generating attributes of judicious users of technology during the elaborate phase..*

**Dome Floor Dilemma Intentional Use of Data**
*(possible participant answers)*

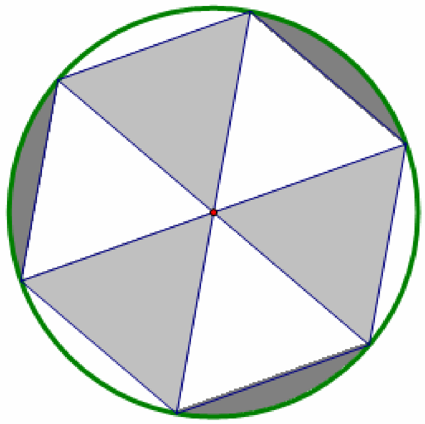
| | | |
|---|---|---|
| **TEKS** | | *a(5), a(6), G.8A, B.8B, G.9C, G.11D* |
| | | |
| **Question(s) to Pose to Students** | **Math** | *What type of relationships could be found among the measurements you gathered?* |
| | **Tech** | *How did technology help you solve the floor tile problem?* |
| **Cognitive Rigor** | **Knowledge** | √ |
| | **Understanding** | √ |
| | **Application** | √ |
| | **Analysis** | √ |
| | **Evaluation** | √ |
| | **Creation** | √ |
| **Data Source(s)** | **Real-Time** | *When using the computer sketch.* |
| | **Archival** | *none* |
| | **Categorical** | *none* |
| | **Numerical** | *none* |
| **Setting** | **Computer Lab** | *Each student uses the computer.* |
| | **Mini-Lab** | *In groups students take turns or groups switch out.* |
| | **One Computer** | *A student operates the control as other students read directions, entire class records data.* |
| | **Graphing Calculator** | *Could be used to enter data and find relationships.* |
| | **Measurement Based Data** | *Could be done at stations or individually.* |
| **Bridge to the Classroom** | | *This activity transfers directly to the classroom with the only modifications being the settings addressed above.* |

# Dome Floor Dilemma

## Explore

### Posing the Problem:

The diagram below represents the tile pattern on the circular floor of a domed building. Each shade, light, medium, and dark, represents a different color of floor tile. Each central angle is congruent to all others.



If you know the length of the radius of the circular floor, is it possible to calculate the area of each shaded region?

### Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of “solving a simpler problem.” To do so, you will construct a geometric figure then collect and analyze data. You will determine three functional relationships: area of a sector of a circle versus the radius, area of a segment of a circle versus the radius, and the area of the triangle bound by the segment and the radii drawn to the endpoints of the arc of the segment.

## The Sector Construction

For detailed instructions on Geometer's Sketchpad see the **Technology Tutorial $T^2$--Dome Floor Dilemma.**

1. Construct a circle with a radius.

2. Rotate the radius and the endpoint that lies on the circle $60^o$.

3. Construct the intercepted arc of the sector.

4. Construct the interior of the sector.

5. Measure the length of the radius and the area of the sector.

6. Create a table to compare the two measurements. Which one is the independent variable and which one is the dependent variable?

7. Plot the two measurements on a graph and turn on the trace feature.

## Collect the Data

8. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row, then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

9. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

10. What patterns do you observe in the table?

11. What observations can you make about your graph?

## The Arc Segment Construction

1. Construct the arc segment.
2. Change the color of the segment.
3. Measure the area of the segment.
4. Create a table to compare the measure of the area of the arc segment and the length of the radius. Which one is the independent variable and which one is the dependent variable?
5. Plot the two measurements on the graph and turn on the trace feature.

## Collect the Data

Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.
7. What patterns do you observe in the table?
8. What observations can you make about your graph?

**The Triangle Construction**

1. Construct the triangle interior.

2. Measure the area of the triangle.

3. Create a table to compare the area of the triangle to the length of the radius. Which one is the independent variable? Which one is the independent variable?

4. Plot the two measurements on the graph and turn on the trace feature.

**Collect the Data**

5. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

8. What observations can you make about your graph?

**Analyze the Data**

1. Develop an algebraic rule that describes the relationship of the length of the radius, $x$, to the area of the sector, $y$.

2. Verify that your function rule models your data. Explain your verification.

3. Develop an algebraic rule that describes the relationship of the length of the radius, $x$, to the area of the triangle, $y$.

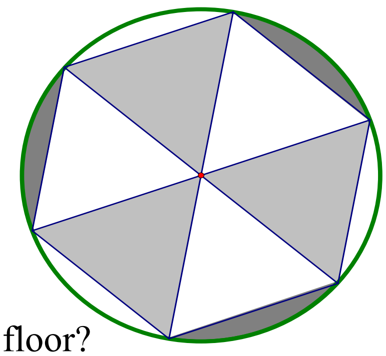
4. Verify that your function rule models your data. Explain your verification.

5. Develop an algebraic rule that describes the relationship of the length of the radius, $x$, to the area of the segment, $y$.

**6.** Verify that your function rule models your data. Explain your verification.

**7.** Recall the floor design discussed earlier. The radius of the circle is 45 feet in length and the cost of tiling the different areas is listed below.

Un-shaded areas - $10.50 per square foot,
Medium shaded areas - $12.00 per square foot and the
Darkest shaded areas - $17.45 per square foot.



Approximately what will be the total cost of tiling the floor?

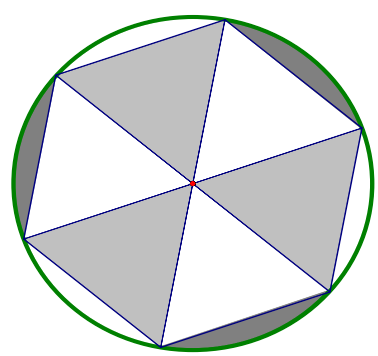
# Explain

**1.** What knowledge of geometric properties was necessary to complete the constructions?

**2.** What process did you use to develop your algebraic rules?

**3.** How did you verify your function rules?

**4.** How did you solve the dome floor dilemma?



**5.** How can you explain your graph of all three functions in a geometric context?

**6.** What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?

**7.** What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?

**8.** What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?

**9.** How will the use of these technologies promote a better understanding of the targeted mathematical concepts?

## Dome Floor Dilemma
## Intentional Use of Data

| | | | |
|---|---|---|---|
| TEKS | | | |
| TEKS | | | |
| Question(s) to Pose to Students | Math | | |
| Question(s) to Pose to Students | Tech | | |
| Cognitive Rigor | | Knowledge | |
| | | Understanding | |
| | | Application | |
| | | Analysis | |
| | | Evaluation | |
| | | Creation | |
| Data Source(s) | | Real-Time | |
| | | Archival | |
| | | Categorical | |
| | | Numerical | |
| Setting | | Computer Lab | |
| | | Mini-Lab | |
| | | One Computer | |
| | | Graphing Calculator | |
| | | Measurement Based Data | |
| Bridge to the Classroom | | | |

# Ring Around the Rose Window

## Elaborate

**Purpose:**
To acquire a deeper understanding of how technology can elevate the level of problem solving in the application of geometric concepts. Generate a list of attributes to guide judicious use of technology.

**Descriptor:**
Participants will utilize technology to plan, construct, and analyze a complex geometric figure. They will compare and contrast a pencil and paper approach and a technology based approach.

**Duration:**
2 hours

**TEKS:**

| | |
|---|---|
| G.1A | develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems; |
| G.2B | use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; |
| G.3B | construct and justify statements about geometric figures and their properties; |
| G.4 | select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems. |
| G.5C | use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and |
| G.9C | formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and |
| G.10A | use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and |
| G.11A | use and extend similarity properties and transformations to explore and justify conjectures about geometric figures; |
| G.11B | use ratios to solve problems involving similar figures; |

**TAKS Objectives:**

- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of geometric relationships and shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

## Technology:

- Dynamic geometry software such as Geometer's sketchpad

## Materials:

**Advance Preparation:**

- Participant access to computers with Geometer's Sketchpad (latest version update available from http://www.keypress.com/sketchpad) and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool
- Sketchpad sketch **Rose** (for leaders use)
- **Rose Hint Cards** copied on cardstock and cut out
- **Transparency: Rose Window**
- **Transparency 1: Looks Like—Sounds Like**
- **Transparency 2: Looks Like—Sounds Like**
- **Transparency: Teaching Strategies**
- **Transparency: Student Research**

**For each participant:**

- Ruler
- Protractor
- Compass
- Patty paper
- **Ring Around the Rose Window** activity sheets
- **Understanding the Problem and Planning the Solution** activity sheets
- **Constructing the Rose** activity sheet

**For each group of 2 participants:**

- Computers with Geometer's Sketchpad and Microsoft Excel
- Copy of the Technology Tutorial $T^2$

# Ring Around the Rose Window—Leader Notes

*This activity asks the participants to construct a complex two-dimensional figure that integrates a variety of geometric concepts. The determination of when, where and how to apply those concepts requires a great deal of problem-solving. This activity was designed for teachers. The construction with technology provides A successful backdrop for problem-solving. It is highly recommended that leaders work through the construction prior to presenting this activity*

## Posing the Problem:

*Use the transparency, Rose Window to pose the problem*

**A common architectural feature used in construction during the renaissance was the rose window. It can be found on palaces, cathedrals, and other buildings of that time. Originally made of stone and glass the windows consisted of a large circle with decorative features arranged like spokes of a wheel in the interior of the circle.**

**Attributes of the window:**

- **The window (figure 1) is made up of a central circle with twelve spokes**
- **The distance from *A* to *C* is three times the distance from *A* to *B***
- **The smaller circles are tangent to each other**
- **The arcs at the outer edge of the circle are tangent to each other and tangent to the smaller circle on its spoke**

**Your task is to use geometric tools to reproduce this window. The reproduction should be scalable with no visual defects.**

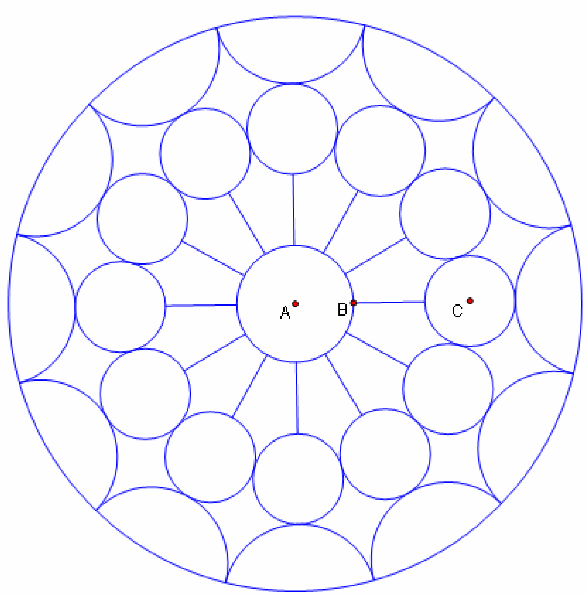
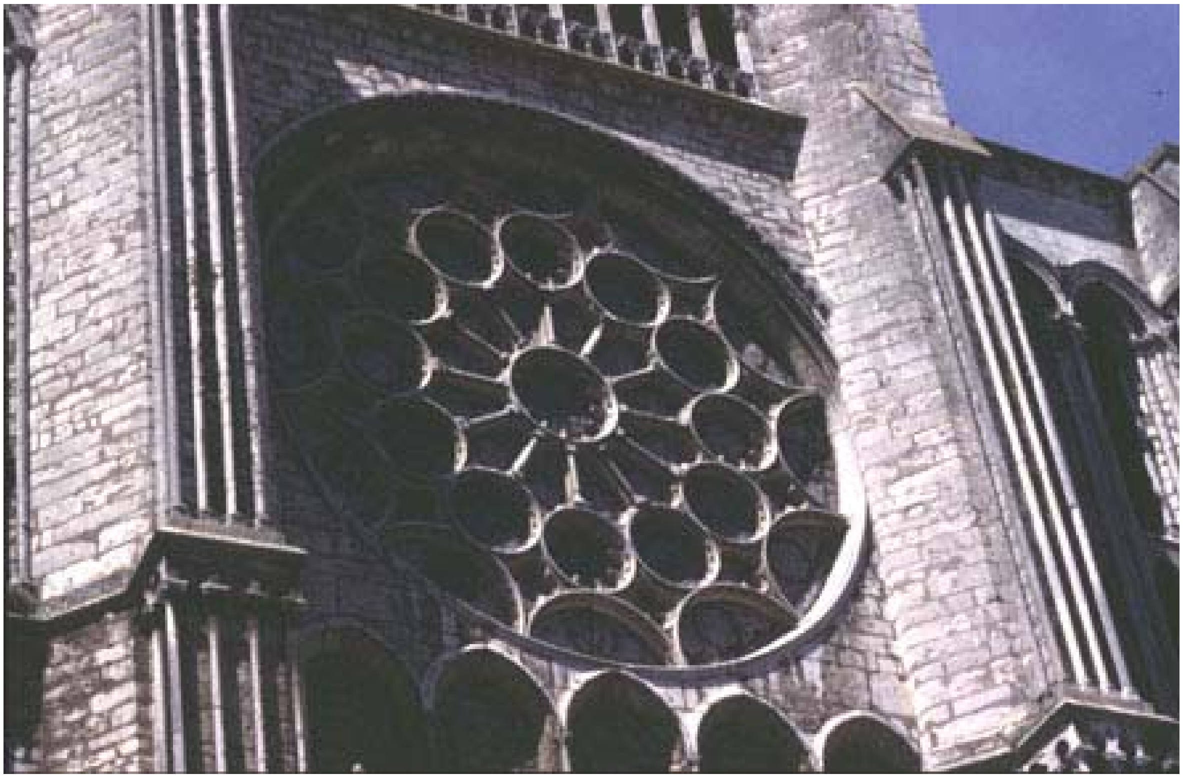
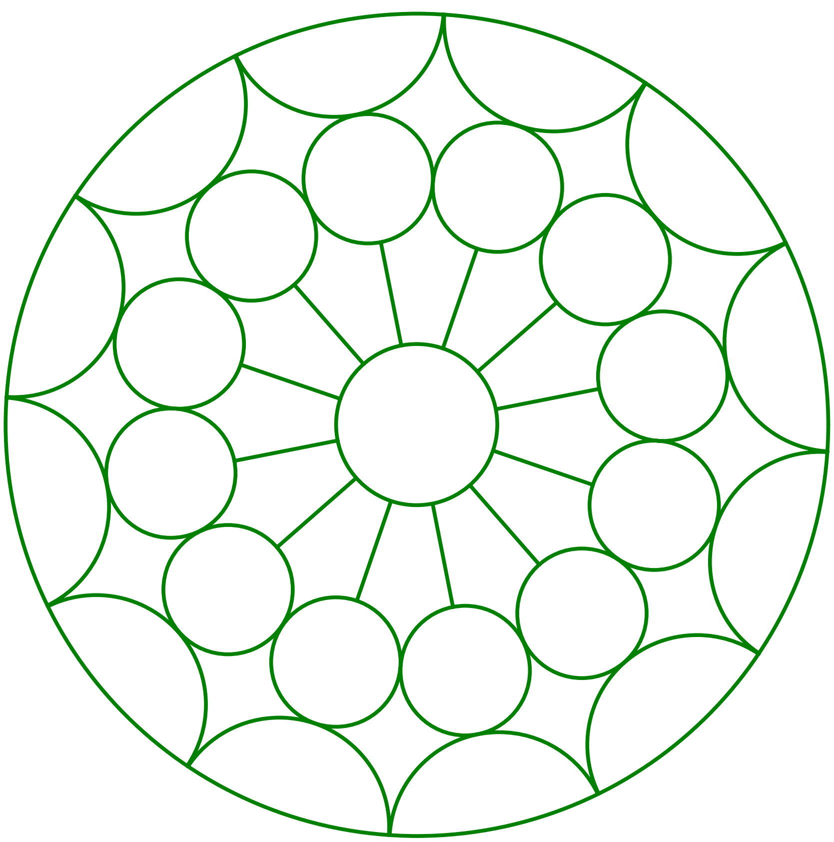


**Figure 1.**





**Your task is to use geometric tools to reproduce this window. The reproduction should be scalable with no visual defects.**

# Understanding the Problem and Planning the Solution

*(30 minutes)*

1. **Which geometric concepts and skills can be identified in the picture?**

   *Possible responses might include; rotational symmetry, transformation, rotation, dilation, tangent, proportion, circle angle properties, and circle arc properties.*

2. **List as many problem-solving strategies as you can recall. Which strategies will you use to perform your construction?**

   *Possible responses might include; simplify the problem, draw a picture or diagram, apply a rule, look for a pattern, write a number sentence, guess and check, make a model, act out the problem, make an organized list or table, divide into smaller simpler problems, and work backwards*

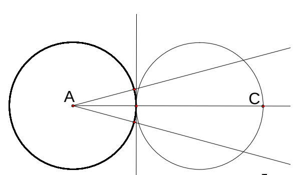
   *Participants might use divide into smaller simpler problems, draw a picture, simplify the problem, and apply a rule.*

3. **Determine a plan for your construction utilizing pencil and paper techniques. What will you do first, second, etc…? Write your plan below including any diagrams or rough sketches and justifications.**

   *The intention at this point is to have participants think through the construction not formally construct it by hand.*

**Facilitation Questions**

- What impact can enlarging or reducing the figure have on visual defects?
  *Often a defect is not visible until the figure is enlarged*
- How might you construct a circle whose center is one and one-half times the length of the radius of the center circle from the center of the original circle?
  *Reflect the original circle over a line perpendicular to the radius of the original circle.*




- How might you construct a circle tangent to a line?
  *Recall that a tangent and a radius of the circle are perpendicular. Construct a perpendicular to the desired line through the point that will serve as the center of the circle.*

4. **What geometric concepts will you utilize to carry out your plan?**

   *Possible responses might include; reflection, rotation, translation, dilation and scaling, tangent, proportion, circle angle properties, and circle arc properties.*

5. **How can you determine if your pencil and paper construction is scalable with no visual defects?**

   *Possible responses might include using a copy machine to enlarge or reduce the figure.*

6. **What are some of the challenges to constructing the figure with pencil and paper?**

   *Possible responses might include; accuracy, knowledge of construction techniques, etc…*

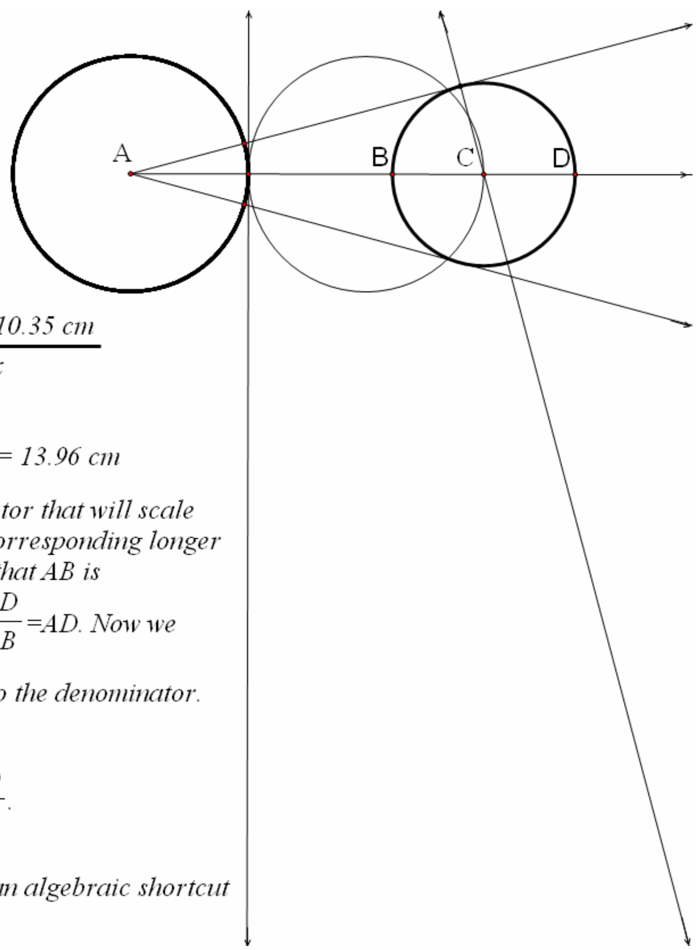
# Constructing the Rose

*(1 hour)*

**1. Construct the rose window using Geometer's Sketchpad**

*Leaders can find one possible solution in the Rose Technology Tutorial.*
*Leaders can provide assistance to participants by using the Rose Hint Cards. When participants are stuck or need a small amount of help just hand them a hint card for the part of the construction they are working on. If time is running short and participants are not finishing their construction, direct them to the tutorial. Leaders can demonstrate the tutorial to allow participants to see a completed construction. Participants will need to use proportional reasoning at some point during their construction. Many may choose to use cross products. If they do, ask them to explain how the cross product is connected to the geometric relationship, in other words, where does the cross product comes from.*

*We are trying to determine x, the distance from A, for the placement of the center of the circle used to create an arc that is tangent to circle C and to the rays that make up the central angle that contains the arc.*




$$\frac{AB = 6.09\text{ cm}}{AC = 8.22\text{ cm}} = \frac{AD = 10.35\text{ cm}}{x}$$

$$x = \frac{AC \cdot AD}{AB} = 13.96\text{ cm}$$

*We need to determine the factor that will scale the shorter lengths to their corresponding longer lengths. We can use the fact that AB is proportional to AD. So* $AB \cdot \frac{AD}{AB} = AD$*. Now we apply the scale factor* $\frac{AD}{AB}$ *to the denominator.*

$$\frac{AB \cdot \frac{AD}{AB}}{AC \cdot \frac{AD}{AB}} = \frac{AD}{x}.\ \text{So } x = \frac{AC \cdot AD}{AB}.$$

*The cross product is merely an algebraic shortcut for this process.*

**2. Did you have to alter your plan for constructing the figure? If so, how and why.**

*There are construction techniques that are specific to the technology software just as there are to ruler compass or paper folding. These techniques typically rely more heavily on transformations than pencil and paper methods*

# Technology Reflection

*(30 minutes)*

1. *Upon completion of their construction, prompt participants to work in pairs to brainstorm the role(s) technology played in the construction versus a pencil and paper approach*

2. *Repost the Venn diagram summaries from the Engage phase.*

3. *Prompt the participants to collect the "green sheets" from each Explore/Explain phases, the summaries about the intentional use of data that followed each Explore/Explain phase.*

4. *Display the* ***Transparency: Teaching Strategies*** *and prompt participants to reflect on the following question, "How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"*

Facilitation Questions

- How have the experiences in this professional development promoted careful decision-making about the appropriate use of technology?
  *Participant responses might include:*
  *Measuring the polygons and using the technology helped to see how students could use technology to explore different relationships in a limited amount of time.*
  *The sketchpad activities show that as a learning tool, technology should be available to students whenever possible.*
- How was technology used in the teaching and the learning of the TEKS?
  *Participant response might include:*
  *Technology was used as a tool to collect and explore data.*
  *Technology was used as a tool to explore algebraic relationships.*
- When was technology use promoted? Why?
  *Participant responses might include:*
  *Technology was promoted when we measured the polygon perimeters and radii so we could compare traditional measurement techniques to technological measurement techniques.*
  *Technology was promoted in the geometric properties exploration to allow a plethora of applications of geometry.*
  *Technology was promoted in the circle exploration to enhance the understanding of possible relationships within circles.*
  *Technology was promoted in the Rose construction to make a complicated construction more manageable.*

Facilitation Questions (continued)

- When was technology use restricted? Why?
  *Participant responses might include:*
  *Technology was restricted when we measured the polygon perimeters and radii so we could compare traditional measurement techniques to technological measurement techniques.*
  *Technology was restricted when we constructed parallel line on the magazine cover so we could compare traditional construction techniques to technological construction techniques.*
- How did the technology support anticipatory, or "what if…," thinking about "algebraic and geometric insight?"
  *Participant responses might include:*
  *Technology addressed the "what if" the polygon is bigger question.*
  *Technology allowed an unlimited number of applications of geometric properties.*
  *Technology answered the relationship question for circles.*
  *Technology allowed a difficult construction to become more manageable.*

5. *Prompt the participants to respond to the following statement and question: "A successful teacher is one who uses technology judiciously. What does this ideal teacher look like and sound like?" as described on* ***Transparency 1: Looks Like—Sounds Like*** *Record the participants' responses on sentence strips Post the sentence strips randomly so that they are visible to the entire group Use participants as scribes as needed to facilitate the recording process.*

6. *Prompt the participants to respond to the following statement and question: "A successful student is one who uses technology judiciously. What does this ideal student look like and sound like during the completion of this activity?" as described on* ***Transparency 2: Looks Like – Sounds Like.*** *Record the participant responses on sentence strips. Post the sentence strips so that they are visible to the entire group.*

7. *Direct the participants to work in small groups to brainstorm categories for classifying the "looks like" and "sounds like" responses.*

**Facilitation Questions**

- Do any of these responses require the teacher or the student to make decisions about technology use? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect decision making about how to best integrate technology? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect decision making about when to use or when not to use technology? Is this important? Should we add some responses?
  *Answers may vary.*
- Do any of these responses reflect the need for thinking about how the technology provides "geometric insight?" Is this important? Should we add some responses?
  *Answers may vary.*

8. *As a whole group, debrief the categories created by small groups. Reorganize the sentence strips into broad categories. As a whole group, create titles for each of these categories. Record each title on a separate sheet of chart paper. Post the chart paper and reorganize the related sentence strips as shown below. Enlist participants to help with this process.*

   *Sample Category:*
   Student Choice

   | |
   |---|
   | The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide. |
   | The student chooses to use a scatterplot instead of a table to represent her data. |

9. *Prompt the participants to consider adding additional statements to any of the categories listed above that are not already posted. Reorganize "looks like, sounds like" sentence strips as needed*

10. *Distribute sentence strips to each group that are a different color than the previously used sentence strips. Prompt each group to generate two classroom suggestions for each category. Examples may include "The teacher should ask, 'Should we use the spreadsheet to make our prediction or verify our prediction? Why:?'", "Students monitor their own use and misuse of technology," "Include examples that require technology use," "Do not allow students to use technology until after prediction are made and justified."*

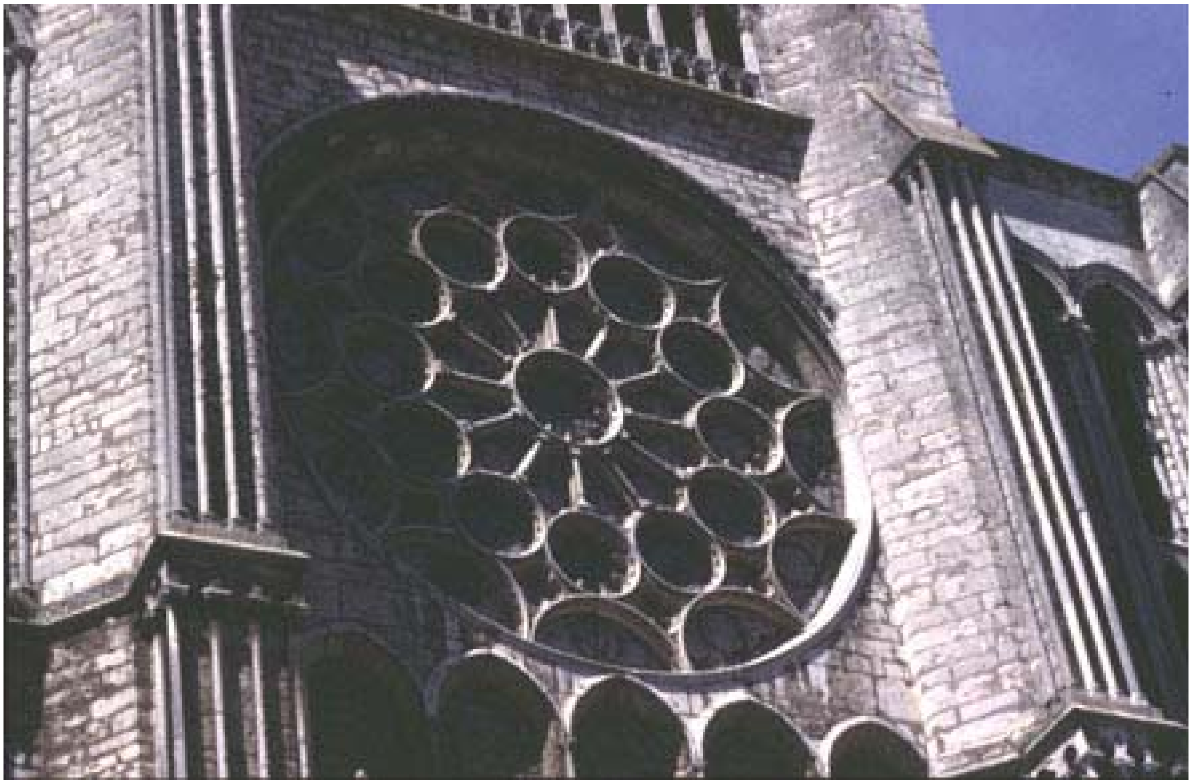
*11. Prompt participants to post their sentence strips as shown below.*

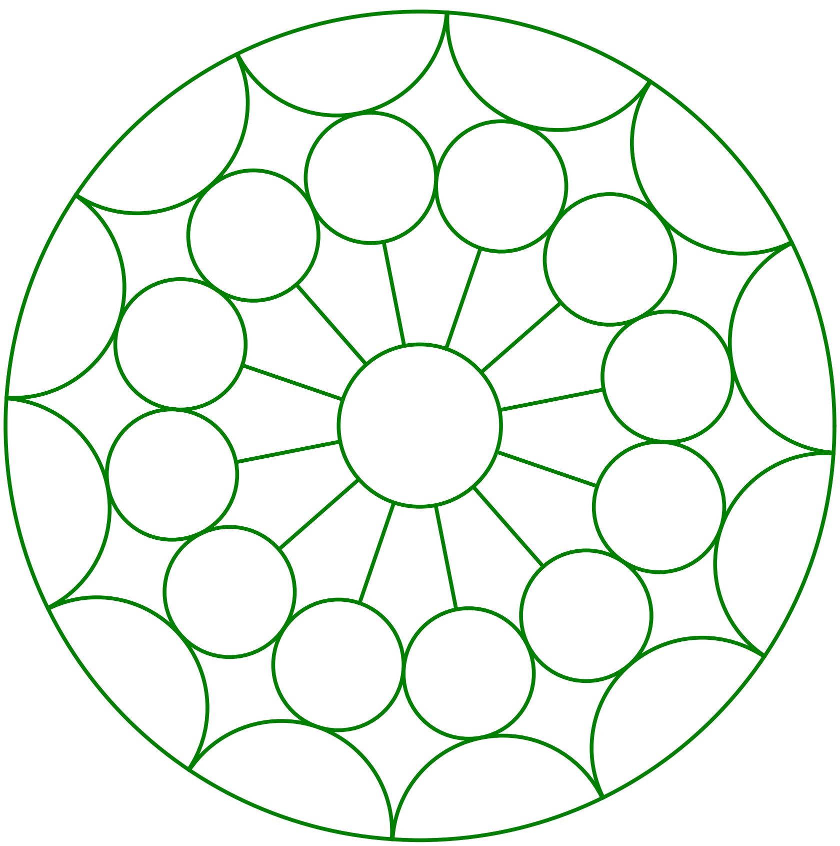
| |
|---|
| *Sample Category:* Student Choice |
| The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide. |
| The teacher posts a transparency that says, “Which tool will you use? Go there!” |
| The teacher provides students with a “pros and cons” chart to develop for the computer and the graphing calculator and then directs students to select a tool. |

*12. Ask the participants to summarize any trends or patterns observed in the classroom suggestions.*

*13. Read the statement by Ball and Stacey found on* ***Transparency: Student Research*** *as a closing thought to this phase of the professional development.*

# Transparency: Rose Window





# Transparency 1: Looks Like – Sounds Like

A successful teacher is one who uses technology judiciously.

What does this ideal teacher look like and sound like in this activity?

| Looks like… | Sounds like… |
|---|---|
| | |

# Transparency 2: Looks Like – Sounds Like

A successful student is one who uses technology judiciously.

What does this ideal student look like and sound like during the completion of this activity?

| Looks like… | Sounds like… |
|---|---|
| | |

# Transparency: Teaching Strategies

"How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect to the following four teaching strategies for developing judicious users of technology?"

Judicious users of technology:

a. Promote careful decision-making about the appropriate use of technology.

b. Integrate technology whenever relevant to the mathematical learning goals.

c. Promote and restrict the use of technology when appropriate for promoting mathematical learning.

d. Promote anticipatory thinking about "geometric insight."

# Transparency: Student Research

Research by Pierce (2002) indicates that some students are always judicious users and others persist with passive or random, unthinking use. However, she found that a large, middle group can be helped to learn to work judiciously.

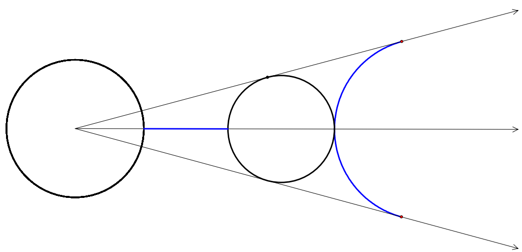
Ball & Stacey, 2005, p5

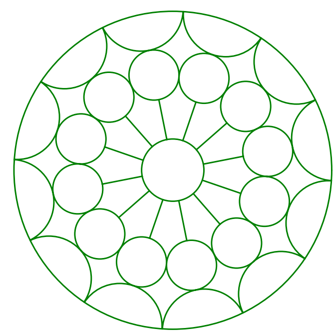
Ball, L., & Stacey, K(2005)Teaching strategies for developing judicious technology use. In Masalski, WJ., & Elliott, PC(Eds.), *Technology-supported mathematics learning environments, sixty-seventh yearbook*, pp3-16Reston, VA: National Council of Teachers of Mathematics.

# Activity Master

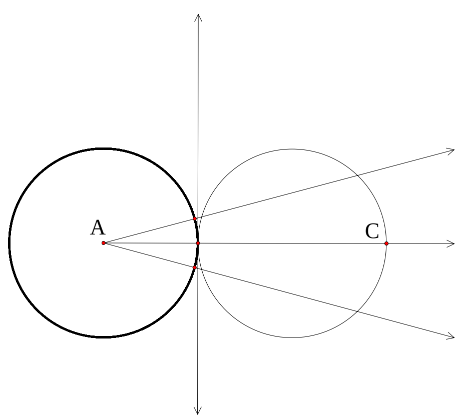
## Rose Hint Cards

**BREAK A LARGE PROBLEM INTO SMALLER PARTS**
Since there are 12 congruent spokes in the figure focus on one spoke, then rotate it around the circle.



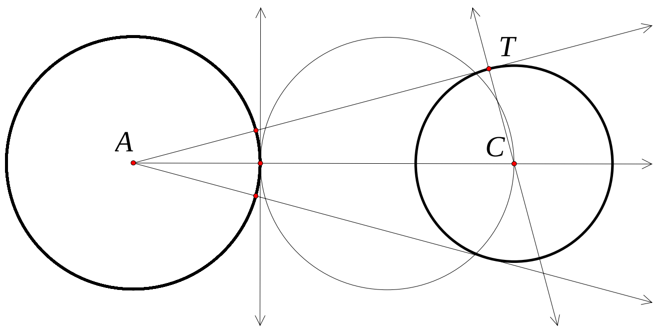


**USING TRANSFORMATIONS TO FIND A DISTANCE**
To find a point that is one and one-half times the radius of circle *A*, from *A* reflect circle *A* over a line tangent to circle *A.* In this example *C* is one and one-half times the radius of circle *A* from *A*.



**CONSTRUCTING A CIRCLE TANGENT TO A RAY**

Recall that a tangent is perpendicular to the radius of the circle. Construct a perpendicular to $\overrightarrow{AT}$ through *C*. Construct a circle with center *C* and radius $\overline{CT}$.

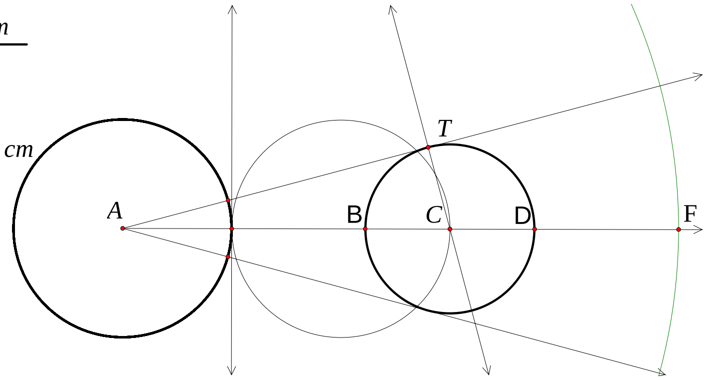



**DETERMINING A PROPORTIONAL DISTANCE**

Since we are dealing with a dilation, *AB* is proportional to *AD* and *AC* is proportional to *AF* or *x* in the equation. Solve the proportion and set the distance $\frac{AC \cdot AD}{AB}$ as a marked distance. Next translate *A* by that marked distance. It doesn't matter what direction. Finally construct a circle with center *A* and radius $\overline{AA'}$. The intersection of the circle and $\overrightarrow{AC}$ yields *F* for *AF*.

$$\frac{AB = 2.29\text{ cm}}{AC = 3.08\text{ cm}} = \frac{AD = 3.88\text{ cm}}{x}$$

$$x = \frac{AC \cdot AD}{AB} = 5.24\text{ cm}$$



# Ring Around the Rose Window

A common architectural feature used in construction during the renaissance was the rose window. It can be found on palaces, cathedrals, and other buildings of that time. Originally made of stone and glass the windows consisted of a large circle with decorative features arranged like spokes of a wheel in the interior of the circle.

**Attributes of the window:**

- **The window (figure 1) is made up of a central circle with twelve spokes.**
- **The distance from A to C is three times the distance for A to B.**
- **The smaller circles are tangent to each other.**
- **The arcs at the outer edge of the circle are tangent to each other and tangent to the smaller circle on its spoke.**

Your task is to use geometric tools to reproduce this window. The reproduction should be scalable with no visual defects.

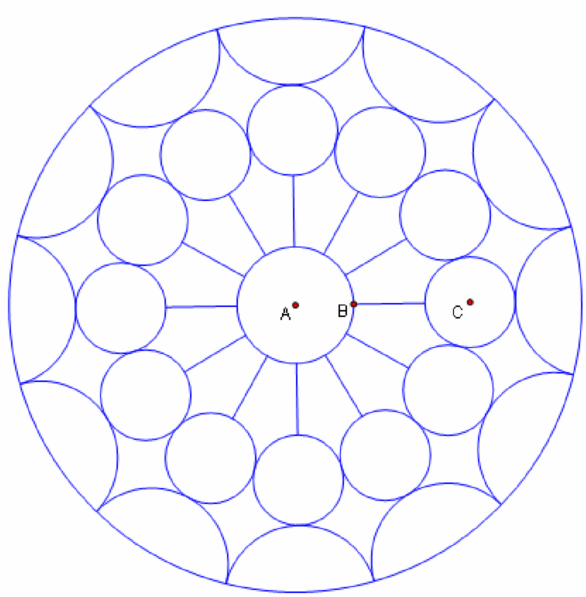
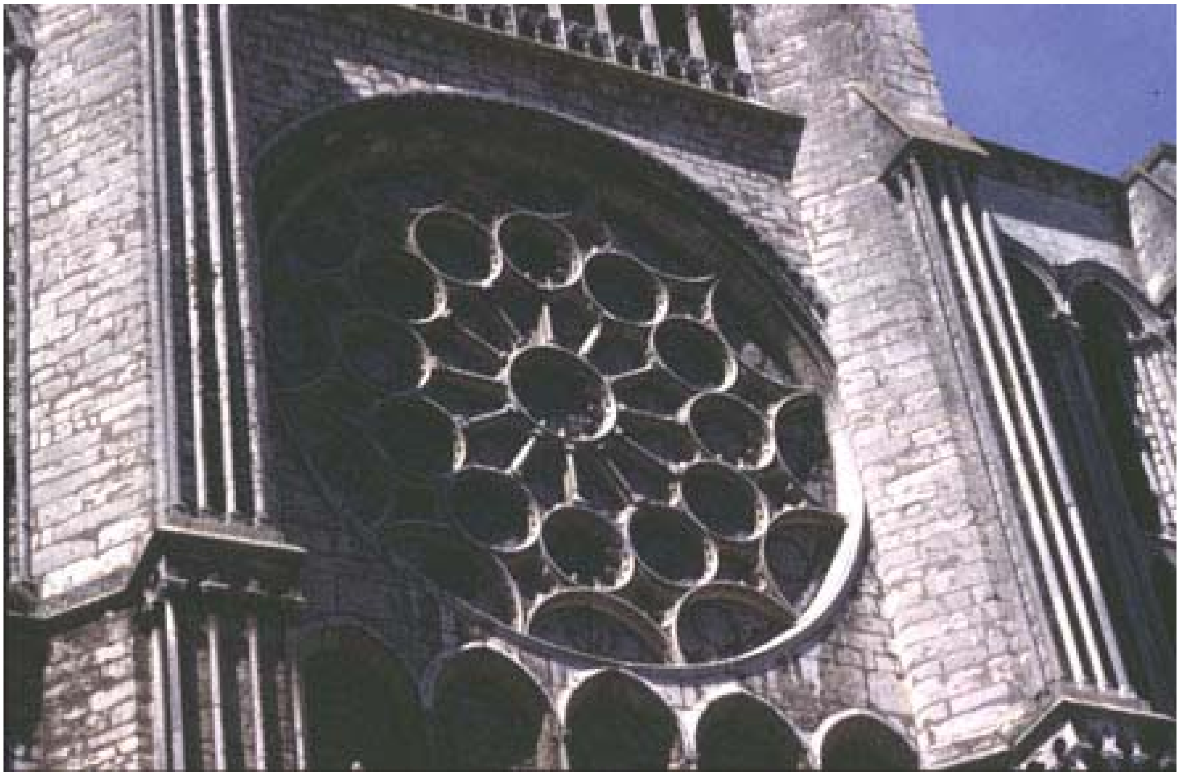
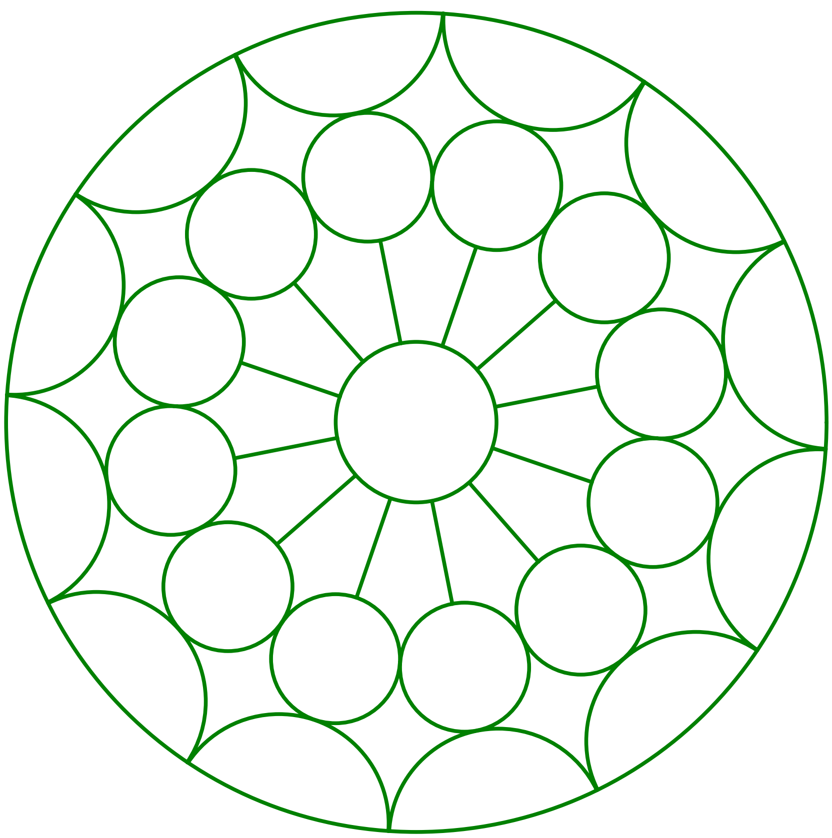


**Figure 1.**





## Understanding the Problem and Planning the Solution

1. Which geometric concepts and skills can be identified in the picture?

2. List as many problem-solving strategies as you can recall. Which strategies will you use to perform your construction?

3. Determine a plan for your construction utilizing pencil and paper techniques. What will you do first, second, etc…? Write your plan below including any diagrams or rough sketches and justifications.

4. What geometric concepts will you utilize to carry out your plan?

5. How can you determine if your pencil and paper construction is scalable with no visual defects?

6. What are some of the challenges to constructing the figure with pencil and paper?

# Constructing the Rose

1. Construct the rose window using Geometer's Sketchpad.

2. Did you have to alter your plan for constructing the figure? If so, how and why.

# The Who, What, When, Why, Where, and How

## Evaluate

### Purpose:

Evaluate judicious uses of technology in the mathematics classroom.

### Descriptor:

Participants will review the instructional phases of this professional development and the classroom-ready lessons according to the list of attributes generated in the elaborate phase of the professional development. Revisions to the list of attributes may occur. Participants will engage in discussion about how each lesson exhibits a judicious use of technology; i.e., participants will address the question, "How does the use of technology in this student lesson help me teach the concepts and skills more effectively and efficiently?"

### Duration:

2 hours

### Materials:

**Advance Preparation*:***

- Transparency: **Encouraging Judicious Use of Technology**

**For each participant:**

- **Gallery Walk Observation** activity sheets

**For each group of 2 participants:**

- Small (1" x 1.5") restickable notes
- Chart paper
- Markers
- Tape to adhere chart paper to the wall

## The Who, What, When, Why, Where, and How—Leader Notes

*The Evaluate phase allows participants to reflect upon their experiences and apply their knowledge to a new situation. The facilitator can deduce from the participants' actions how well they have been able to develop a sense of the judicious use of technology.*

1. *Distribute small restickable notes to each participant.*

2. *Assign different phases of this professional development to pairs of participants.*

3. *Prompt each pair of participants to use the restickable notes to highlight locations in each phase of the professional development that make judicious use of technology, according to the criteria on the* ***Transparency: Encouraging Judicious Use of Technology****. The restickable notes should be used to highlight those attributes of the teaching strategies outlined during the Elaborate Phase of this professional development.*

4. *After each pair has had time to evaluate the given phase of the professional development, prompt each pair of participants to create a summary of its findings on chart paper.*

   *Sample response might be:*

   *Having students first measure with a handmade measuring tool helps develop the concept of measurement by hand in contrast to measurement using computer software.*

   *Data collection via technology allows students to focus on the relationship concept instead of getting bogged down in non-technology data collection.*

   *Technology use is thoroughly integrated into this phase of the lesson.*

   *Was the graph of the data what we expected? Why?*

5. *Identify a location in the room for each phase of the professional development. Direct participants to post their summaries in the appropriate location.*

6. *Perform a gallery walk through each phase, asking participants to determine which teaching strategies for judicious use of technology seemed to have the greatest impact on the given phase.*

7. *Prompt participants to share any new thoughts that should be added to the classroom suggestions for each teaching strategy.*

8. *Distribute the classroom-ready lessons to each participant. Prompt each participant to continue the evaluation process for judicious use of technology, using the classroom-ready lessons as the context for evaluation. The participants should use the restickable notes to highlight those parts of each lesson that reflect the four teaching strategies for developing judicious use of technology.*

9. *As time allows, offer small-group and whole-group opportunities for participants to share what participants highlighted.*

10. *Redirect participants' attention to the four statements made at the beginning of the professional development session. Ask the participants if they would "shift" the placement of their sticky dots. If they respond with a "Yes," ask the participants why they would shift the placement of their sticky dots.*

11. *Draw an end to the professional development session with a parting thought rather than a closing thought so that participants leaving thinking "How will I use what I learned?" rather than "That was a good session." Examples of such parting thoughts include:*
    a. *As you leave, please consider ways that you might include the use of data and technology in your classroom next week.*

    b. *As you leave, please consider how you might best make use of the computer or computers available for your classroom use.*

    c. *As you leave, please consider how students might be equipped to ask better questions about what they are learning when they have graphing calculators in their hands.*

# Transparency: Encouraging Judicious Use of Technology

- How did the activity promote careful decision-making about the use of technology?
- How did the activity integrate technology into the learning of mathematics?
- Was technology use ever restricted for the purpose of enhancing learning? Why?
- How did the technology facilitate discussion about "geometric sense"?

# Gallery Walk Observations

| | |
|---|---|
| Polygarden Landscaping Company | How did the activity promote careful decision-making about the use of technology?<br><br>How did the activity integrate technology into the learning of mathematics?<br><br>Was technology use ever restricted for the purpose of enhancing learning? Why?<br><br>How did the technology facilitate discussion about “geometric sense”? |

| | |
|---|---|
| Sketchpad Skills Investigation and Exploring the World | How did the activity promote careful decision-making about the use of technology?<br><br>How did the activity integrate technology into the learning of mathematics?<br><br>Was technology use ever restricted for the purpose of enhancing learning? Why?<br><br>How did the technology facilitate discussion about “geometric sense”? |

| | |
|---|---|
| Dome Floor Dilemma | How did the activity promote careful decision-making about the use of technology?<br><br>How did the activity integrate technology into the learning of mathematics?<br><br>Was technology use ever restricted for the purpose of enhancing learning? Why?<br><br>How did the technology facilitate discussion about "geometric sense"? |

| | |
|---|---|
| Ring Around the Rose Window | How did the activity promote careful decision-making about the use of technology?<br><br>How did the activity integrate technology into the learning of mathematics?<br><br>Was technology use ever restricted for the purpose of enhancing learning? Why?<br><br>How did the technology facilitate discussion about “geometric sense”? |

# Circles, Angle Measures and Arcs

| | |
|---|---|
| A.5C | Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions |
| A.6G | Relate direct variation to linear functions and solve problems involving proportional change. |
| A.7A | Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems; |
| G.2A | Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships. |
| G.2B | Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic. |
| G.3D | Use inductive reasoning to formulate a conjecture. |
| G.9C | Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models. |

## Materials

Advance Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use Geometer's Sketchpad as a class demonstration tool.

For each student:

- Graphing calculator
- **Create an "Arc Measuring Tool"** activity sheet
- **Angles Formed by Chords Intersecting Inside a Circle** activity sheet
- **Angles Formed by Secants Intersecting Outside a Circle** activity sheet
- **Other Intersecting Lines and Segments** activity sheet
- **Quad-Tri Incorporated** activity sheet

For each student group of 3 - 4 students:

- Compasses
- Protractors
- Patty paper or tracing paper
- Rulers
- Scissors

## ENGAGE

*The Engage portion of the lesson is designed to create student interest in the relationships among the measures of angles formed by segments in circles and related arc measures. This part of the lesson is designed for groups of three to four students.*

1. Distribute two sheets of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on one sheet of patty paper. Then have them construct a second circle, congruent to the first circle on the second sheet of patty paper.
3. Distribute the **Create an Arc Measuring Tool** activity sheet. Students should follow the directions on the sheet.
4. On their second circle, students should draw two intersecting chords that do not intersect in the center of the circle.
5. Students should use the available measuring tools to find angle measures and estimate arc measures.
6. Students will record their individual results, share results with their group, and discuss observations.
7. Debrief the activity using the Facilitation Questions.

*Facilitation Questions – Engage Phase*

1. When you fold a diameter, how many degrees are in each semi-circle?
   *180° semi means half; one-half of 360° is 180°.*
2. When you fold a second diameter perpendicular to the first, how many degrees are in each quarter-circle?
   *90° one quarter means one-fourth, one-fourth of 360° is 90°.*
3. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*
4. How did you use your "Arc Measuring Tool" to estimate the measures of the arcs in your circle? *Answers may vary. Students should be able to explain how they used known "benchmarks" like 90°.*
5. What other method could you use to determine the measures of the arcs on your second circle?
   *Answers may vary. Students should realize they can draw central angles that intercept the arc they are trying to measure and the measure of the central angle is equal to the measure of the intercepted arc.*
6. What similarities do your measurements have with measurements taken by other members of your group?
   *Answers may vary. Students may notice, vertical angles are congruent; the sum of the measures of all arcs of the circle is 360° etc.*
7. How can you determine if your observations will be true for any circle?
   *Answers may vary. Students should realize that data for several circles could be collected and analyzed to verify conjectures.*

## EXPLORE

*The Explore portion of the lesson provides the student with an opportunity to participate actively in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.*

1. Distribute the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet.
2. Students should open the sketch **Twochords-in.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
   Note: If students are not familiar with the operation of Geometer's Sketchpad, they will need the necessary instruction at this time.

***Facilitation Questions – Explore Phase***

1. What patterns do you notice in the table?
   *Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles, etc.*

2. Where do you see proportional relationships in your table?
   *Properties of proportional relationships can be explored at this time. Remind students of scale factors and constant of proportionality.*

3. How did you use your table to develop an algebraic rule for this relationship?
   *Answers may vary. Students may have used the process column, constant of proportionality, finite differences, etc.*

## EXPLAIN

*The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.*

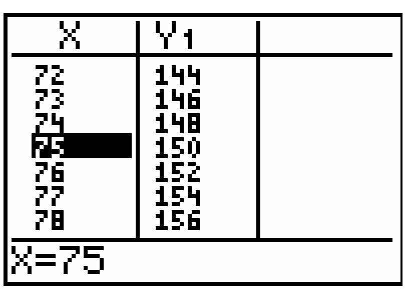
1. Debrief the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet. Use the Facilitation Questions to help students make connections among methods that can be used to calculate the measure of the angle or intercepted arc.
2. Have each student group present the way they found the algebraic rule and give a verbal description of the relationship.
3. Be sure students understand how to use the Geometer's Sketchpad sketches.

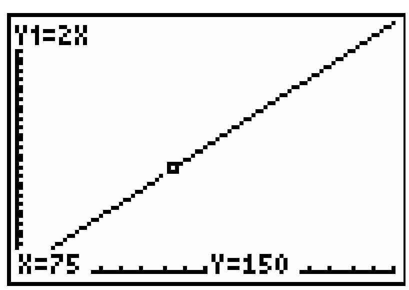
***Facilitation Questions – Explain Phase***

1. What is the meaning of your algebraic rule in this relationship?
   *Two times the angle measure equals the sum of the intercepted arcs.*
2. If you know the measure of the angle, how can you find the sum of the measures of the intercepted arcs?
   *Multiply the angle measure by 2.*
3. If you know the measure of each intercepted arc, how can you find the angle measure?
   *Find the sum of the arcs and then divide by 2.*
4. If you know the measure of one angle and one intercepted arc, how could you find the measure of the other intercepted arc?
   *Double the angle measure then subtract the known arc from that value.*
5. If you know the measure of one angle and one intercepted arc, what algebraic equation could you write to calculate the measure of the other intercepted arc?

$$2(angle) = arc1 + arc2$$

6. How could you use the table or graph feature of your graphing calculator to determine the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?



## ELABORATE

*The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for groups of three to four students.*

1. Distribute the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
2. Students should open the sketch **Twosecants-out.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
4. Debrief the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
5. Distribute the **Other Intersecting Lines and Segments** activity sheet.
6. Prompt students to open the sketches as directed and explore the relationships.
7. Debrief the **Other Intersecting Lines and Segments** activity sheet.

***Facilitation Questions – Elaborate Phase***

1. What patterns do you notice in the table?
   *Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles etc.*
2. Where do you see proportional relationships in your table?
   *Properties of proportional relationships can be explored at this time. Remind students of Scale factors and constant of proportionality.*
3. How did you use your table to develop an algebraic rule for this relationship?
   *Answers may vary. Students may have used the process column, constant of proportionality, finite differences etc.*
   After completing the summary table for this activity, what general statements can you make about angles formed by lines and segments that intersect circles?

## EVALUATE

*The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.*

1. Distribute the Mathematics Chart.
2. Provide each student with the **Quad-Tri Incorporated** activity sheet.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

*Answers and Error Analysis for selected response questions:*

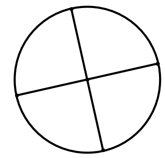
| *Question Number* | *TEKS* | *Correct Answer* | *Conceptual Error* | *Conceptual Error* | *Procedural Error* | *Procedural Error* | *Guess* |
|---|---|---|---|---|---|---|---|
| **1** | G.9(c) | D | B | C | A | | |
| **2** | G.9(c) | D | B | C | A | | |
| **3** | G.9(c) | A | C | D | B | | |
| **4** | G.9(c) | A | C | B | D | | |

## Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

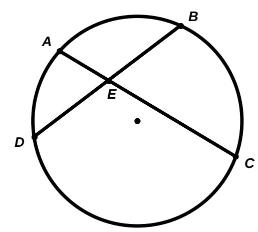
3. Fold a diameter in the second circle. Unfold the circle then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?
   *The point is the center of the circle. It is the midpoint of the diameters so it must be the center.*

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



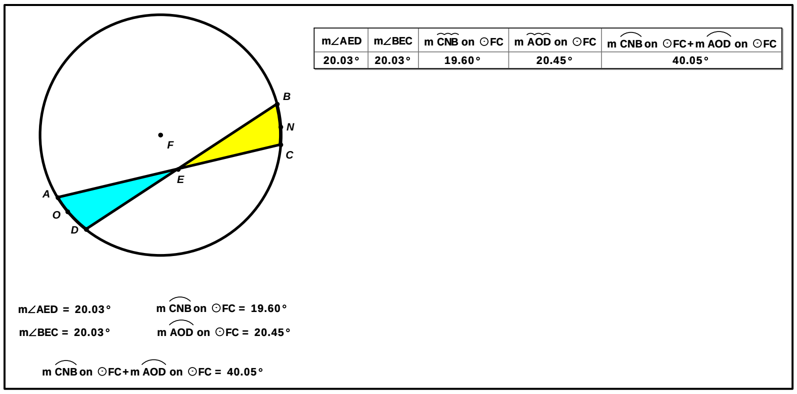

7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

| Name | $m\angle AED$ | $m\angle BEC$ | $m\overset{\frown}{BC}$ | $m\overset{\frown}{AD}$ |
|---|---|---|---|---|
| | *50°* | *50°* | *60°* | *40°* |
| | *65°* | *65°* | *70°* | *60°* |
| | *43°* | *43°* | *40°* | *46°* |
| | *124°* | *124°* | *82°* | *166°* |
| | *130°* | *130°* | *100°* | *160°* |

8. What patterns do you observe in the table?
   *Answers may vary. Students should observe that the* $m\angle AED = m\angle BEC$ *or that the sum of the measures of the arcs is twice the measure of each angle.*

## Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**




1. **Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?**
   *The measures change.*
2. **Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.**
3. **Double click again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.**
4. **Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.**
5. **Record the data from the computer in the table below.**

| $m\angle AED$ | $m\angle BEC$ | $m\overset{\frown}{BC}$ | $m\overset{\frown}{AD}$ | $m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ |
|---|---|---|---|---|
| *20.03* | *20.03* | *19.60* | *20.45* | *40.05* |
| *35.53* | *35.53* | *50.61* | *20.45* | *71.06* |
| *42.57* | *42.57* | *64.69* | *20.45* | *85.14* |
| *56.60* | *56.60* | *64.69* | *48.51* | *113.20* |
| *68.98* | *68.98* | *64.69* | *73.28* | *137.97* |
| *79.68* | *79.68* | *86.09* | *73.28* | *159.37* |
| *96.54* | *96.54* | *119.79* | *73.28* | *193.07* |
| *125.02* | *125.02* | *119.79* | *130.24* | *250.03* |
| *144.07* | *144.07* | *119.79* | *168.35* | *288.14* |
| *170.00* | *170.00* | *171.65* | *168.35* | *340.00* |

6. **What patterns do you observe in the table?**
*Answers may vary. Students should observe the $m\angle AED = m\angle BEC$ and the sum of the measures of the arcs is twice the measure of each angle.*

7. **To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.**

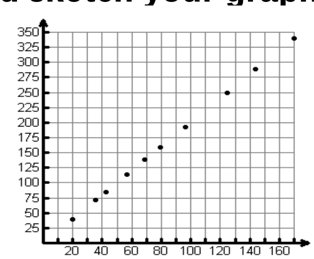
| $m\angle AED$ ($x$) | PROCESS | $m\widehat{CNB} + m\widehat{AOD}$ ($y$) |
|---|---|---|
| *20.03* | *(2) 20.03* | *40.05* |
| *35.53* | *(2) 35.53* | *71.06* |
| *42.57* | *(2) 42.57* | *85.14* |
| *56.60* | *(2) 56.60* | *113.20* |
| *68.98* | *(2) 68.98* | *137.97* |
| *79.68* | *(2) 79.68* | *159.37* |
| *96.54* | *(2) 96.54* | *193.07* |
| *125.02* | *(2) 125.02* | *250.03* |
| *144.07* | *(2) 144.07* | *288.14* |
| *170.00* | *(2) 170.00* | *340.00* |
| $x$ | $2x$ | $y$ |

8. **Use the process column to develop an algebraic rule that describes this relationship.**
*$y = 2x$*

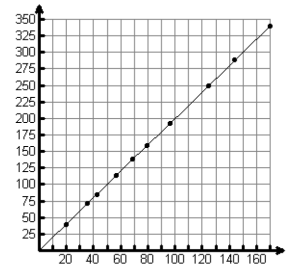
9. **Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.**
*Two times the measure of the angle is equal to the sum of the measures of the intercepted arcs. The sum of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.*

10. **Create a scatterplot of the sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.**

$x$-min = *0*
$x$-max = *170*
$y$-min = *0*
$y$-max = 350



11. **Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.**




12. **Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

13. **What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?**
*75°*

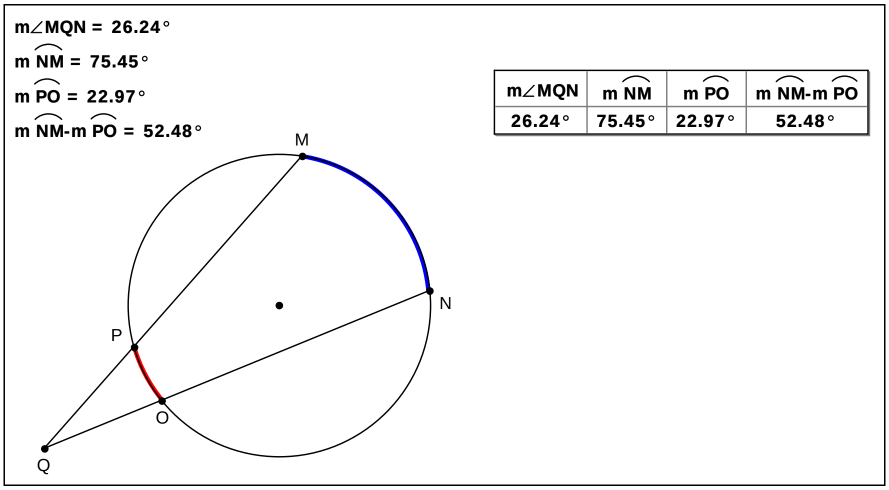
14. **What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?**
*112°*

15. **Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.**
*To determine the measure of the angle, add the two intercepted arcs then divide by 2.*

16. **Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.**
*To determine the sum of the measures of the intercepted arcs, multiply the measure of the angle by 2*

## Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecants-out**.




1. **Double click on the table to add another row, then click and drag point *M*. What do you observe?**
   *The measures change.*
2. **Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.**

3. **You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.**
4. **Record the data from the computer in the table below.**

| $m\angle MQN$ | $m\overset{\frown}{MN}$ | $m\overset{\frown}{PO}$ | $m\overset{\frown}{MN} - m\overset{\frown}{PO}$ |
|---|---|---|---|
| *26.24* | *75.45* | *22.97* | *52.48* |
| *29.84* | *85.92* | *26.24* | *59.68* |
| *35.90* | *99.89* | *28.09* | *71.80* |
| *40.58* | *113.21* | *32.05* | *81.16* |
| *46.22* | *130.52* | *38.09* | *92.43* |
| *50.68* | *143.71* | *42.35* | *101.36* |
| *55.99* | *163.39* | *51.40* | *111.99* |
| *58.91* | *172.42* | *54.60* | *117.82* |
| *64.63* | *192.27* | *63.01* | *129.25* |
| *73.05* | *241.94* | *95.84* | *146.10* |

5. **What patterns do you observe in the table?**
*Answers may vary. Students should observe the measure of the angle is one-half the difference of the measures of the intercepted arcs.*

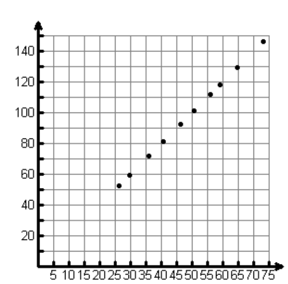
6. **To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.**

| $m\angle MQN$ ($x$) | PROCESS | $m\overparen{MN} - m\overparen{PO}$ ($y$) |
|---|---|---|
| *26.24* | *(2) 26.24* | *52.48* |
| *29.84* | *(2) 29.84* | *59.68* |
| *35.90* | *(2) 35.90* | *71.80* |
| *40.58* | *(2) 40.58* | *81.16* |
| *46.22* | *(2) 46.22* | *92.43* |
| *50.68* | *(2) 50.68* | *101.36* |
| *55.99* | *(2) 55.99* | *111.99* |
| *58.91* | *(2) 58.91* | *117.82* |
| *64.63* | *(2) 64.63* | *129.25* |
| *73.05* | *(2) 73.05* | *146.10* |
| $x$ | $2x$ | y |

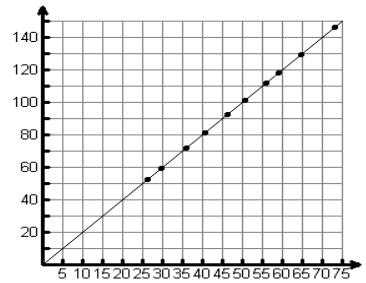
7. **Use the process column to develop an algebraic rule that describes this relationship.**
*$y = 2x$*

8. **Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.**
*Two times the measure of the angle is equal to the difference of the measures of the intercepted arcs. The difference of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.*

9. **Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window.**
   $x$-min = *0*
   $x$-max = *75*
   $y$-min = *0*
   $y$-max = *150*



10. **Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.**




11. **Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

12. **What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?**
*45°*

13. **What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants 43°?**
*86°*

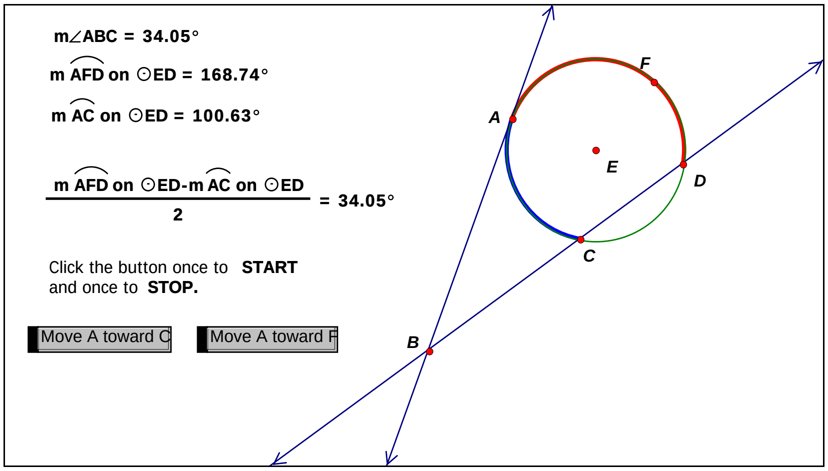
14. **Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.**
*To determine the measure of the angle, subtract the measures of the two intercepted arcs then divide by 2.*

15. **Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.**
*To determine the difference of the measures of the intercepted arcs, multiply the measure of the angle by 2.*

## Other Intersecting Lines and Segments

### 1. Tangent and a Secant that intersect in the exterior of a circle

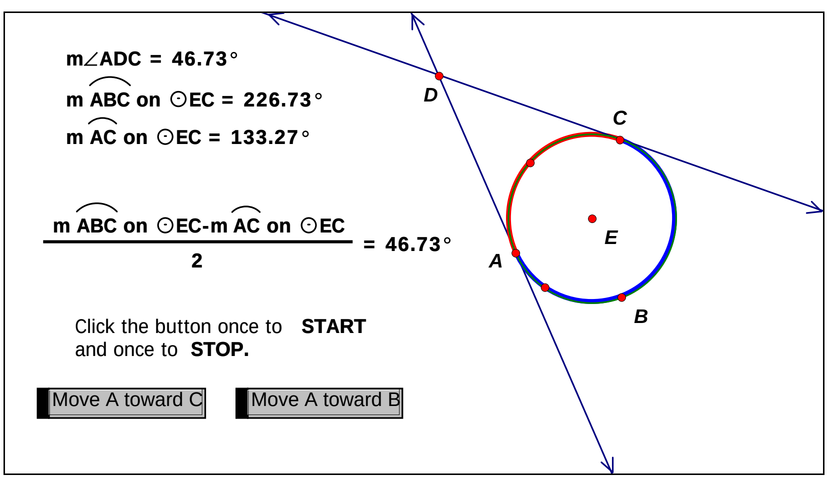
**a. Open the sketch,** "Tansecant-out.".




**b. Click a button to move point A. What do you observe about the angle and arc relationships?**
*The measure of the angle is one-half the difference in the measures of the intercepted arcs.*

### 2. Two tangents that intersect in the exterior of a circle

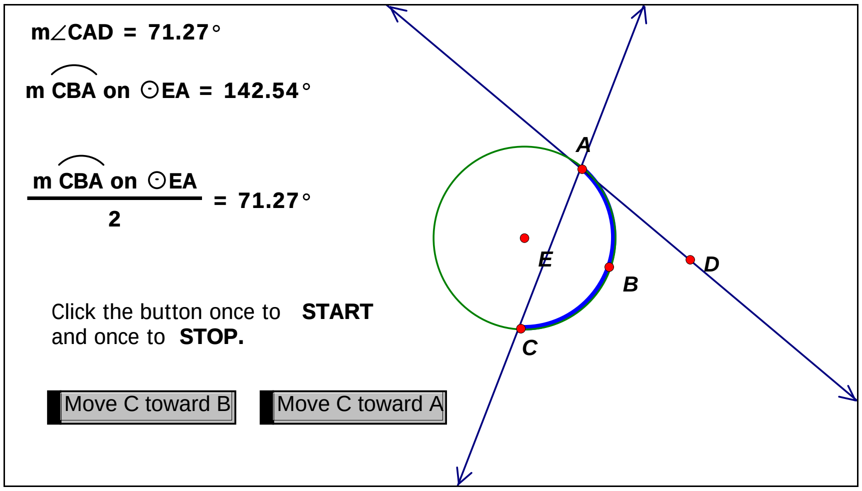
**a. Open the sketch,** "Twotangents-out."




**b. Click a button to move point A. What do you observe about the angle and arc relationships?**
*The measure of the angle is one-half the difference in the measures of the intercepted arcs.*

**3.** Tangent and a Secant that intersect on a circle

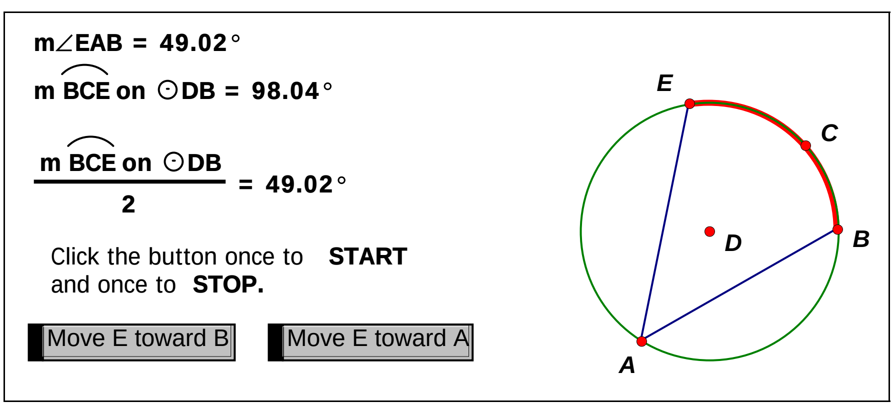
**a. Open the sketch** "Tansecant-on."




**b. Click a button to move point C. What do you observe about the angle and arc relationships?**
*The measure of the angle is one-half the measure of the intercepted arc.*

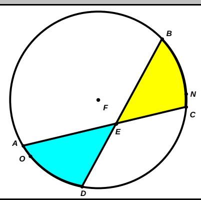
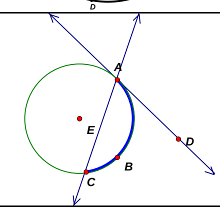
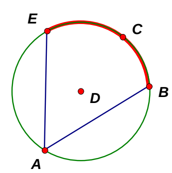
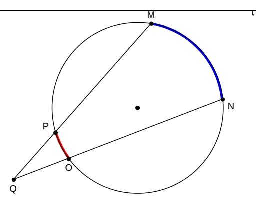
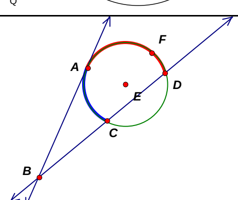
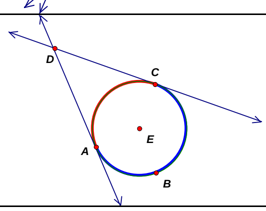
**4.** Two chords that intersect on a circle

**a. Open the sketch** "Twochords-on."




**b. Click a button to move point *E*. What do you observe about the angle and arc relationships?**
*The measure of the angle is one-half the measure of the intercepted arc.*

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

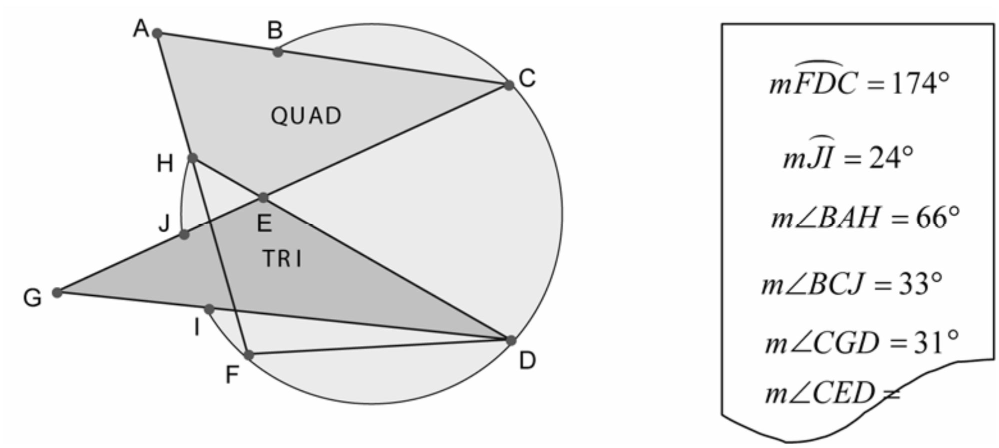
| Diagram | Is the vertex of the angle inside, outside or on the circle? | How to calculate the measure of the angle |
|---|---|---|
|  | *Inside the circle* | *The measure of the angle is one-half the sum of the measures of the intercepted arcs.* |
|  | *On the circle* | *The measure of the angle is one-half the measure of the intercepted arc.* |
|  | *On the circle* | |
|  | *Outside the circle* | *The measure of the angle is one-half the difference in the measures of the intercepted arcs.* |
|  | *Outside the circle* | |
|  | *Outside the circle* | |

Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, _add_ the measures of the intercepted arcs then _divide by 2_.
2. When the vertex is **outside** the circle, _subtract the_ measures of the intercepted arcs then _divide by 2_.
3. When the vertex is **on** the circle, _divide the measure of the intercepted arc by 2_.

## Quad-Tri Incorporated

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.




**Pierre thinks the measure of angle CED must be 60°. Is he correct? Justify your answer.**

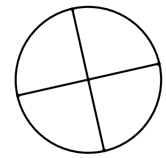
*Answer: Pierre is not correct. Based on the known information, the measure of angle CED must be 55°.*

# Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

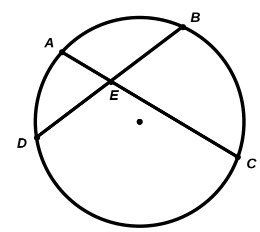
3. Fold a diameter in the second circle. Unfold the circle, then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool?

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



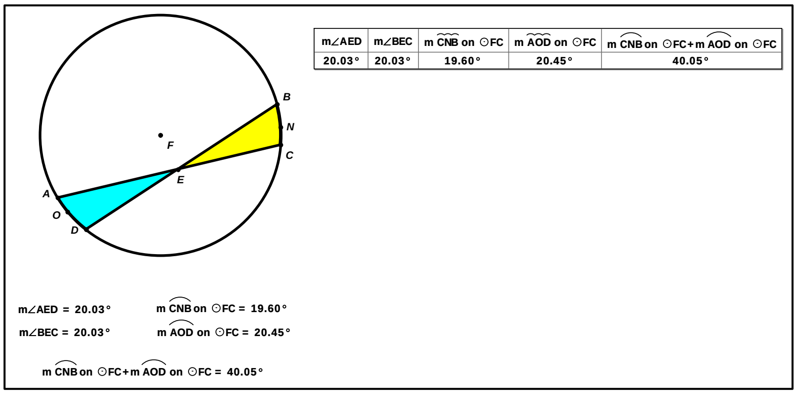

7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

| Name | $m\angle AED$ | $m\angle BEC$ | $m\overset{\frown}{BC}$ | $m\overset{\frown}{AD}$ |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

8. What patterns do you observe in the table?

# Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**




1. Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?

2. Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.
3. Double click on the table again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.
4. Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.
5. Record the data from the computer in the table below.

| $m\angle AED$ | $m\angle BEC$ | $m\overparen{BC}$ | $m\overparen{AD}$ | $m\overparen{CNB} + m\overparen{AOD}$ |
|---|---|---|---|---|
| | | | | |
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| | | | | |
| | | | | |
| | | | | |

6. What patterns do you observe in the table?

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

| $m\angle AED$ ($x$) | PROCESS | $m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ ($y$) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| $x$ | | $y$ |

8. Use the process column to develop an algebraic rule that describes this relationship.

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

10. Create a scatterplot of sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

$x$-min =
$x$-max =
$y$-min =
$y$-max **=**

11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

12. Does the graph verify your function rule? Why or why not?

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?

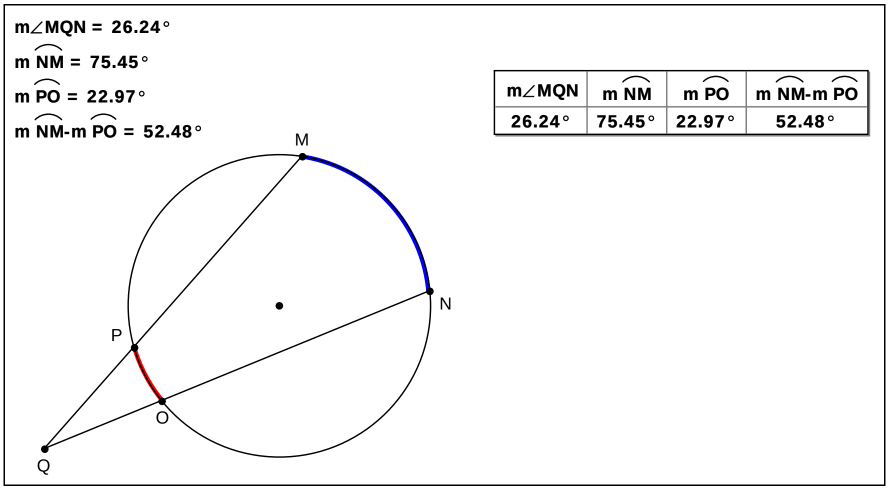
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.

## Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecant-out**.




1. Double click on the table to add another row, then click and drag point *M*. What do you observe?

2. Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.

3. You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.

4. Record the data from the computer in the table below.

| $m\angle MQN$ | $m\overset{\frown}{MN}$ | $m\overset{\frown}{PO}$ | $m\overset{\frown}{MN} - m\overset{\frown}{PO}$ |
|---|---|---|---|
| | | | |
| | | | |
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| | | | |
| | | | |
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| | | | |
| | | | |
| | | | |
| | | | |

5. What patterns do you observe in the table?

6. To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.

| $m\angle MQN$ ($x$) | PROCESS | $m\overset{\frown}{MN} - m\overset{\frown}{PO}$ ($y$) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| $x$ | | y |

7. Use the process column to develop an algebraic rule that describes this relationship.

8. Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.

9. Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window

$x$-min =
$x$-max =
$y$-min =
$y$-max =

10. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

11. Does the graph verify your function rule? Why or why not?

12. What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?

13. What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants is 43°?

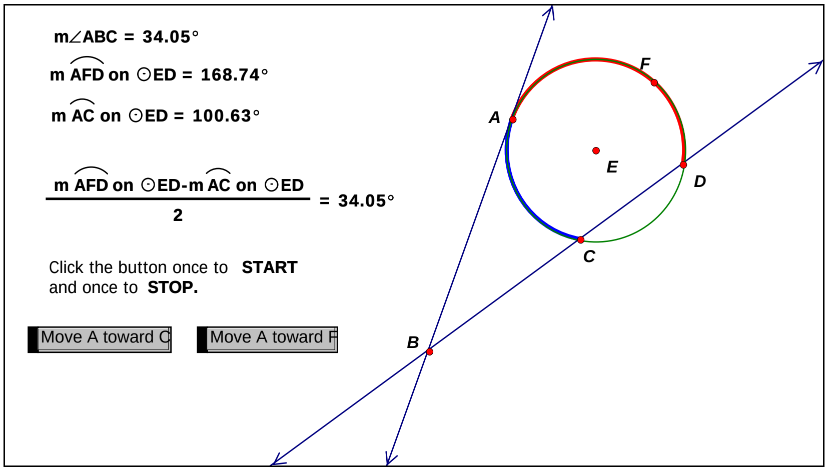
14. Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.

15. Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.

# Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

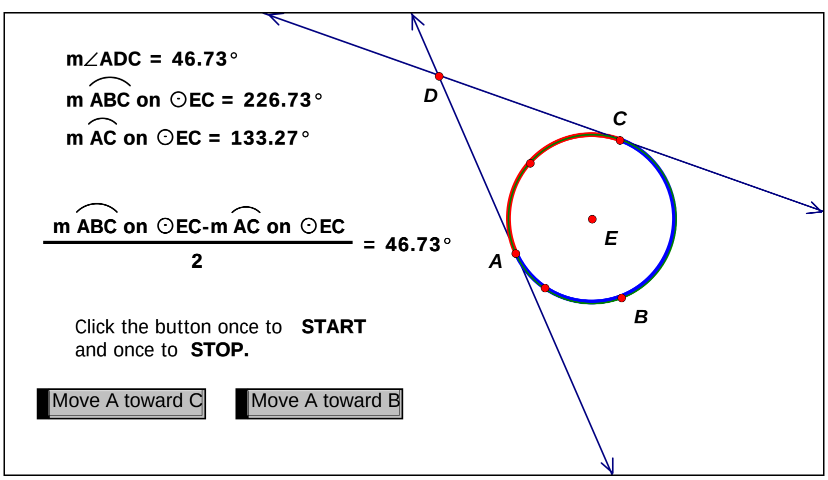
   a. Open the sketch, "**Tansecant-out.**"




   b. Click a button to move point A. What do you observe about the angle and arc relationships?

2. Two tangents that intersect in the exterior of a circle

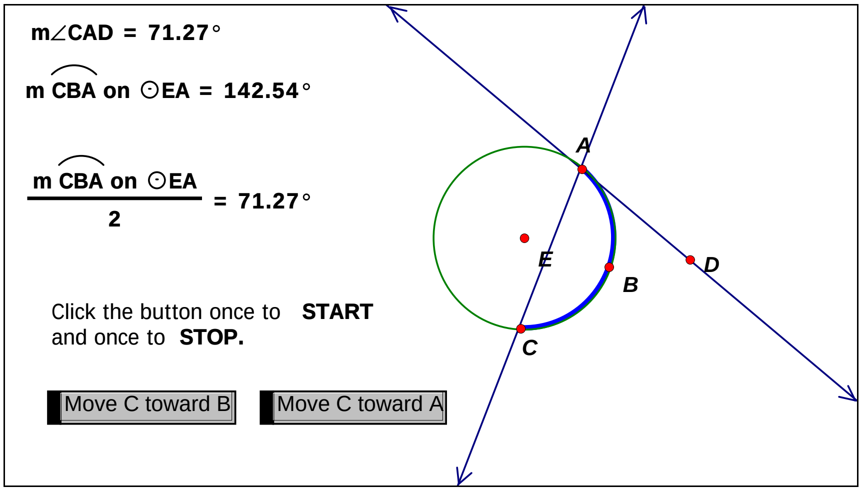
   a. Open the sketch, "**Twotangents-out.**"




   b. Click a button to move point A. What do you observe about the angle and arc relationships?

3. Tangent and a Secant that intersect on a circle

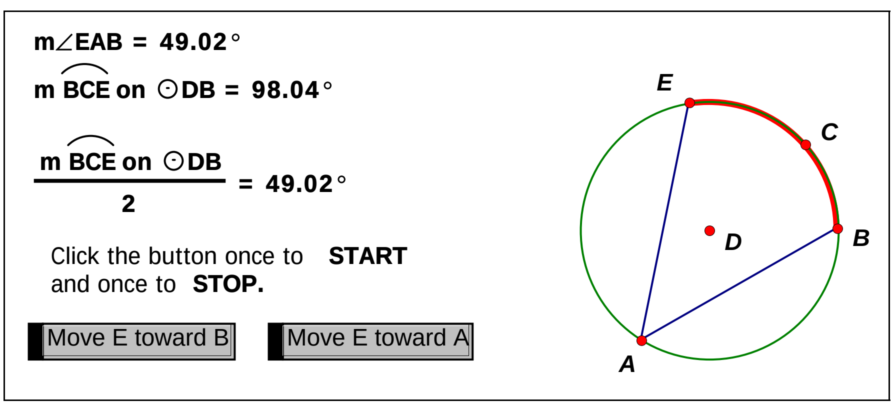
   a. Open the sketch "**Tansecant-on.**"




   b. Click a button to move point C. What do you observe about the angle and arc relationships?

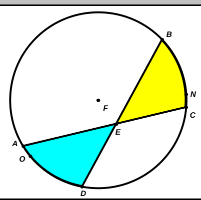
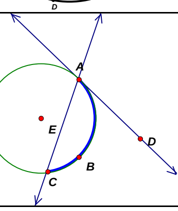
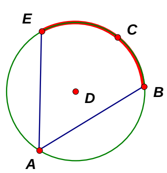
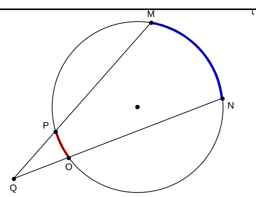
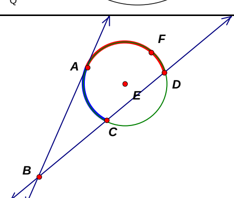
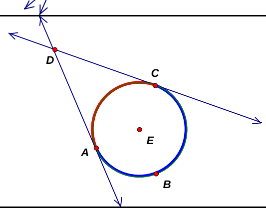
4. Two chords that intersect on a circle

   a. Open the sketch "**Twochords-on.**"




   b. Click a button to move point E. What do you observe about the angle and arc relationships?

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

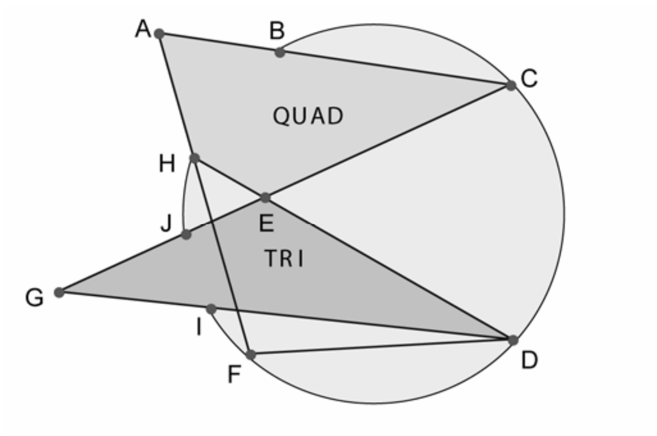
| Diagram | Is the vertex of the angle inside, outside or on the circle? | How to calculate the measure of the angle |
|---|---|---|
|  | | |
|  | | |
|  | | |
|  | | |
|  | | |
|  | | |

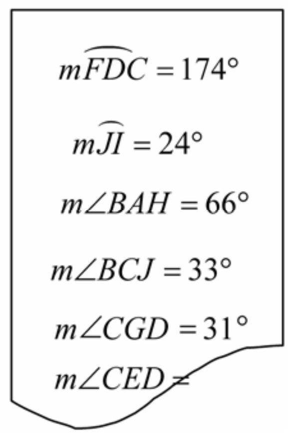
Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, ______ the measures of the intercepted arcs then ________________.
2. When the vertex is **outside** the circle, ______ the measures of the intercepted arcs then ________________.
3. When the vertex is **on** the circle,____________________________.

**Quad-Tri Incorporated**

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.

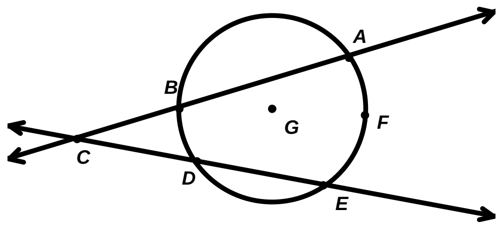



Pierre thinks the measure of angle *CED* must be 60°. Is he correct? Justify your answer.

Circles, Angle Measures and Arcs

**1** In the diagram $m\angle BCD = 25°$ and $m\overset{\frown}{BD} = 33°$.




Find $m\overset{\frown}{AFE}$.

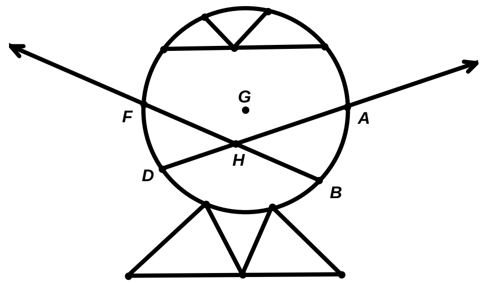
A 17°

B 50°

C 58°

D 83°

**2** The metal sculpture shown was found in a recent archeological dig. $m\overset{\frown}{AB} = 46°$ and $m\overset{\frown}{FD} = 38°$




What is $m\angle DHB$?

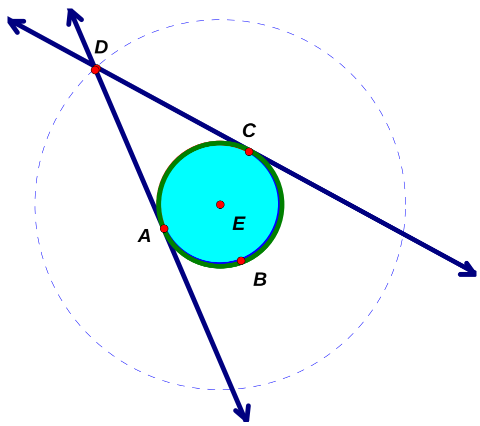
A 4°

B 42°

C 84°

D 138°

**3** In the diagram, Point *D* represents a spacecraft as it orbits the Earth.




At this location 220° of the Earths surface is not visible from the spacecraft. What must be the $m\angle ADC$ ?

A 40°

B 80°

C 110°

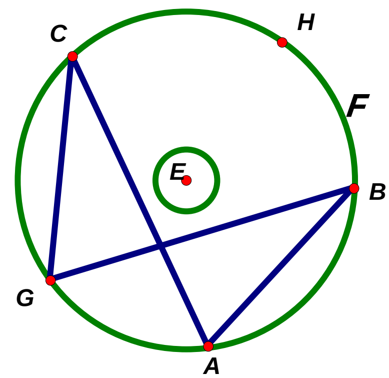
D 140°

**4** Pablo created the sketch below.

**m $\overset{\frown}{AB}$ on ⊙EF = 80°**

**m $\overset{\frown}{CG}$ on ⊙EF = 84°**

**m∠GBA = 31°**




Based on the measurements he took, what must be $m\overset{\frown}{CHB}$ ?

A 134°

B 82°

C 67°

D 33.5°

## Area of Regular Polygons

| | |
|---|---|
| A.1B | Gather and record data and use data sets to determine functional relationships between quantities. |
| A.5C | Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions. |
| A.9A | Determine the domain and range for quadratic functions in given situations. |
| A.9D | Analyze graphs of quadratic functions and draw conclusions. |
| G.2A | Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships. |
| G.2B | Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic. |
| G.3D | Use inductive reasoning to formulate a conjecture. |
| G.8A | Find areas of regular polygons, circles, and composite figures. |
| G.11C | Develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods. |

## Materials

Advance Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use Geometer's Sketchpad as a class demonstration tool.

For each student:

- Graphing calculator
- **Create an Area of Regular Polygons** activity sheet
- **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet
- **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet
- **Equilateral Triangles, Squares and Regular Octagons** activity sheet
- **Kick It Incorporated** activity sheet

For each student group of 3 - 4 students:

- Compasses
- Protractors
- Patty paper or tracing paper
- Rulers
- Scissors

## ENGAGE

*The Engage portion of the lesson is designed to create student interest in the relationships between the area of regular polygons and the area of the triangles that compose them. This part of the lesson is designed for groups of three to four students.*

1. Distribute a sheet of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on the sheet of patty paper. Distribute the **Area of Regular Polygons** activity sheet. Students should follow the directions on the sheet.
3. Students will use paper folding to construct an octagon then cut it into 8 congruent triangles.
4. Students should use the available measuring tools to measure critical attributes then calculate the area of the octagon.
5. Students will share their method of calculating the area with their group, and then with the whole class.

*Facilitation Questions – Engage Phase*

1. When you folded your circle into 8 equal sectors, what was the measure of each central angle?
   *45°, 360° divided by 8.*
2. What do you observe about the 8 triangles that you cut out?
   *Answers may vary. Students may observe that the triangles are congruent.*
3. How do you know the triangles are congruent?
   *Answers may vary. Students may stack the triangles or offer an informal proof using SAS since all radii and all central angles of the octagon are congruent.*
4. How did you use your triangles to determine the area of your octagon?
   *Answers may vary. Students should be able to explain their method. Students may have measured the base and height of one triangle, calculated the area of the triangle then multiplied it by 8. The may have arranged the triangles into the shape of a parallelogram or two trapezoids and a parallelogram, measured the critical attributes then found the area.*

   *This activity sets the stage for exploring the relationship between the apothem of a regular polygon (height of the triangle) and the area of the polygon.*
5. How can you determine a method that could be used to calculate the area of any regular polygon?
   *Answers may vary. Students should realize that data for several polygons could be collected and analyzed to verify conjectures.*

## EXPLORE

*The Explore portion of the lesson provides the student with an opportunity to participate actively in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.*

1. Distribute the **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet.
2. Students should open the sketch **HEXAGO.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between the length of the apothem of a regular polygon and its area.
   Note: If students are not familiar with the operation of Geometer's Sketchpad, they will need the necessary instruction at this time. Also you may need to remind students to set their calculator MODE to Degrees,

***Facilitation Questions – Explore Phase***

1. What patterns do you notice in the table?
   *Students should notice that as the apothem length increases, the area of the polygon increases, but not at a constant rate.*

2. How do you know this is a non-linear relationship?
   *Answers may vary. Students should notice that there is not a constant rate of change and that the graph is not linear.*

3. In right triangle trigonometry, which trigonometric ratio should you use when you know the length of the leg adjacent to the reference angle and you want to find the length of the leg opposite the angle?
   *The Tangent Ratio.*

## EXPLAIN

*The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.*

1. Debrief the **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet. Use the Facilitation Questions to help students make connections among central angles, length of apothem, The Tangent Ratio, area of triangles and area of the polygon
2. Have each student group demonstrate how to use the graph and table features of the calculator to find the area given the apothem and find the apothem given the area.
3. Be sure students understand how to use the Geometer's Sketchpad sketches.

***Facilitation Questions – Explain Phase***

1. How did you determine the measure of the central angle of the hexagon?
   *Divide 360° by 6'.*
2. How did you determine the measure of the angle formed by the radius of the hexagon and its apothem?
   *Divide the measure of the central angle by 2.*
3. Why was the Tangent ratio used to find the measure of the leg of the right triangle?
   *Tangent is used because the apothem is adjacent to the angle and the short leg of is opposite the angle.*
4. Why was the length of the short leg of the right triangle multiplied by 2?
   *Multiplying by 2 gives the length of the base of the triangle.*
5. What is it about the relationship between apothem length and area that makes it a quadratic relationship?
   *When a linear measure in a polygon is changed by a scale factor, the area of the polygon is changed by the square of that scale factor.*
6. How did you determine the area of a hexagon with an apothem of 6.5?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the area.*
7. How could you use your calculator to find the area of any regular hexagon when you know the length of the apothem?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the area of any regular hexagon.*
8. How could you use your calculator to find the apothem of any regular hexagon when you know the area?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the apothem of any regular hexagon.*

## ELABORATE

*The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for groups of three to four students.*

1. Distribute the **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet.
2. Students should open the sketch **PENTA.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between apothem length and area of a pentagon.
4. Debrief the **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet.
5. Distribute the **Equilateral Triangles and Regular Octagons** activity sheet.
6. Prompt students to open the sketches as directed and explore the relationships.
7. Debrief the **Equilateral Triangles and Regular Octagons** activity sheet.

***Facilitation Questions – Elaborate Phase***

1. How did you determine the area of a pentagon with an apothem of 8.5?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the area.*
2. How could you use your calculator to find the area of any regular pentagon when you know the length of the apothem?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the area of any regular pentagon.*
3. How could you use your calculator to find the apothem of any regular pentagon when you know the area?
   *Students should be able to demonstrate how to use the graph and table features of the calculator to find the apothem of any regular hexagon.*
4. What would be the function rule for determining the area of a 12-sided polygon given its apothem?
   $y = 12x^2(tan(15))$
5. Explain how to find the area of any regular polygon when you know the length of its apothem.
   *First find half the measure of the central angle. Find the tangent of that measure, then multiply by the number of sides and the square of the length of the apothem.*

## EVALUATE

*The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.*

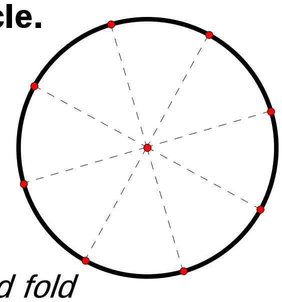
1. Provide each student with the **Kick It Incorporated** activity sheet.
2. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

*Answers and Error Analysis for selected response questions:*

| *Question Number* | *TEKS* | *Correct Answer* | *Conceptual Error* | *Conceptual Error* | *Procedural Error* | *Procedural Error* | *Guess* |
|---|---|---|---|---|---|---|---|
| **1** | G.8(A) | B | D | | A | C | |
| **2** | G.8(A) | C | A | B | D | | |
| **3** | G.8(A) | B | A | C | | | D |
| **4** | G.8(A) | D | C | | A | B | |

# Area of Regular Polygons

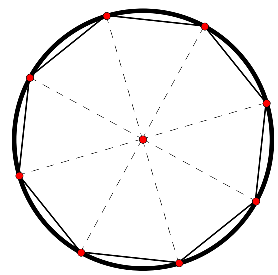
1. **On a sheet of patty paper construct a large circle.**



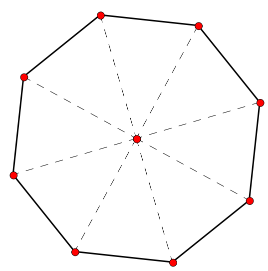
2. **Cut out the circle.**

3. **Use paper folding to divide the circle into 8 congruent sectors.**
   *Students should fold a diameter, then fold a second diameter perpendicular to the first. Next, they should fold the bisectors of the 90° angles*

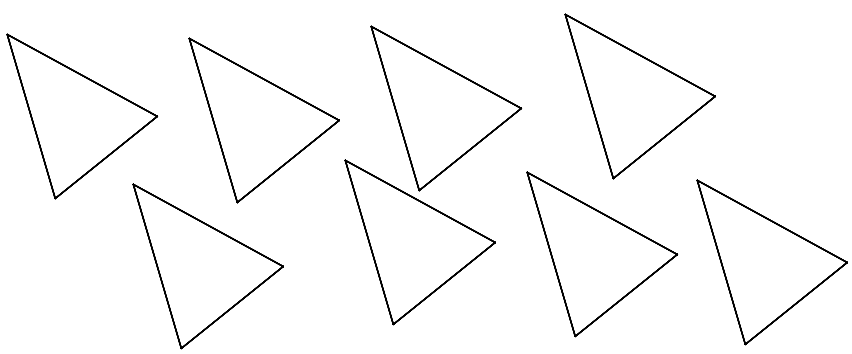
4. **Use a straight edge to connect the endpoints of the folded radii.**



5. **Cut out the polygon.**



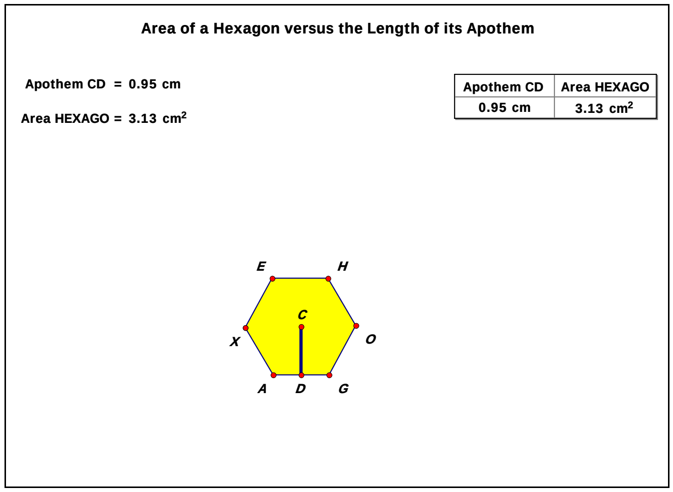
6. **Cut the polygon along each fold.**



7. **Determine the area of your original polygon.**
   *Students may arrange the triangles to form familiar polygons, take measurements, then calculate the area.*

## Area of a Regular Hexagon versus the Length of its Apothem

Open the sketch **HEXAGO.**




1. **Double click on the table to add another row, then click and drag point *G* a short distance to the right. What do you observe?**
   *The measures change.*
2. **Double click on the table again, then move point *G* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.**
3. **Record the data from the computer into the table below.**

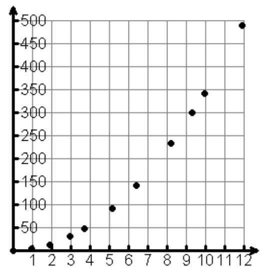
| *Apothem CD* | *Area HEXAGO* |
|---|---|
| *0.95* | *3.13* |
| *1.89* | *12.32* |
| *2.97* | *30.51* |
| *3.70* | *47.44* |
| *5.17* | *92.48* |
| *6.41* | *142.49* |
| *8.19* | *232.45* |
| *9.29* | *299.06* |
| *9.93* | *341.79* |
| *11.89* | *490.10* |

4. **What patterns do you observe in the table?**
   *Answers may vary. Students should observe that as the length of the apothem increases the area increases, but not at a constant rate.*

**5. What is a reasonable domain and range for your data?**
*A reasonable domain includes values between 0 and 12. A reasonable range includes values between 0 and 500.*

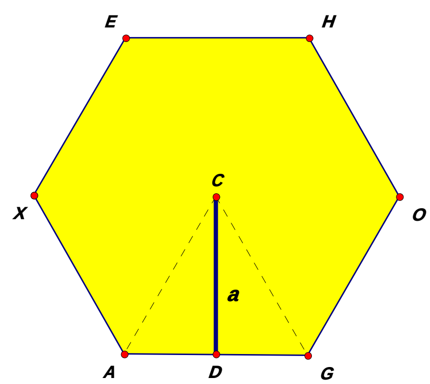
**6. Create a scatterplot of Area of a Regular Hexagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.**

$x$-min = *0*
$x$-max = *12*
$y$-min = *0*
$y$-max = *500*




**7. What observations can you make about your graph?**
*The graph appears to be a non-linear function, possibly quadratic.*

**8. To help develop a function rule for this situation use Hexagon *HEXAGO* to complete the following.**




**a. Since *HEXAGO* is a regular hexagon, $m\angle ACG = 60°$. What is $m\angle ACD$?** *30°*

**b. Using $\angle ACD$ as the reference angle, the trigonometric ratio "Tangent" can be used to find *AD* in terms of the apothem length, (*a*).**

$$tan\,30° = \frac{AD}{a} \text{ or } AD = a(tan\,30°)$$

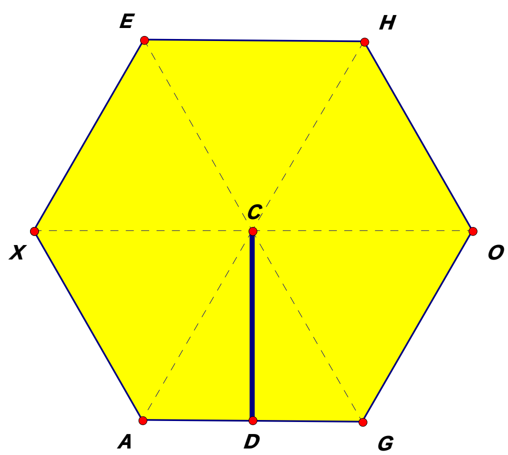
**c. Write an expression for *AG* in terms of *a* and tan 30°.**
*Since AG = 2(AD) then AG = 2a tan 30°.*

**d. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem (*a*) and your answer to question (c) above, write and simplify an expression for the area of $\triangle ACG$.**

$$Area = \frac{2a(\tan 30°)a}{2}$$
$$Area = \frac{2a^2(\tan 30°)}{2}$$
$$Area = a^2(\tan 30°)$$

**e. Draw the radius to each vertex of Hexagon *HEXAGO*. How many congruent isosceles triangles are formed?**

*6*



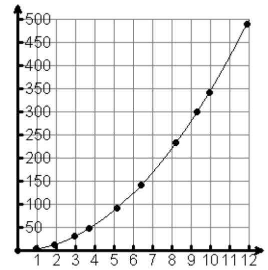

**f. Use your answer to questions (d) and (e) above to write an expression for the area of a hexagon.**

$$Area = 6a^2(\tan 30°)$$

**g. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.**

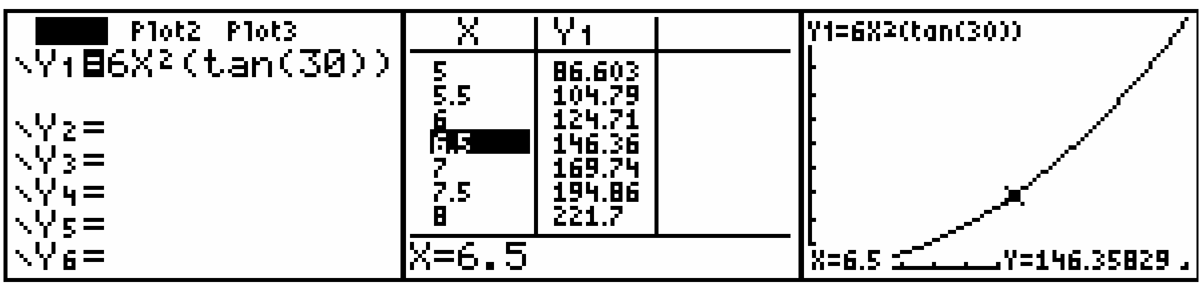
$$y = 6x^2(\tan(30))$$

**9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.**

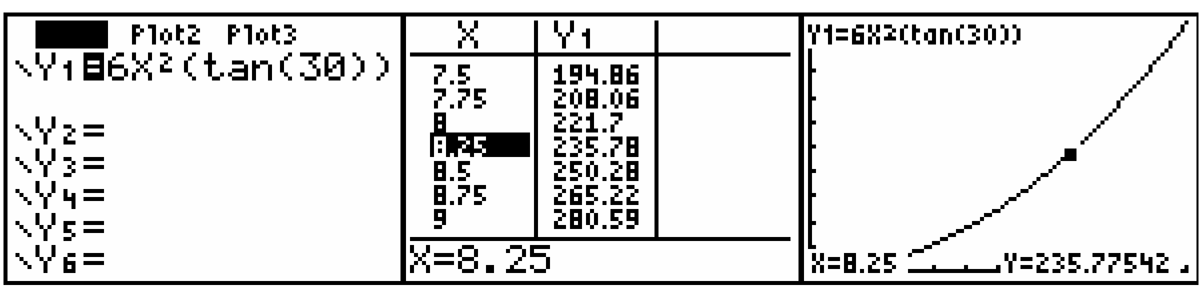



**10. Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

**11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular hexagon with an apothem of 6.5 centimeters.**
*Area ≈146.36 square centimeters.*

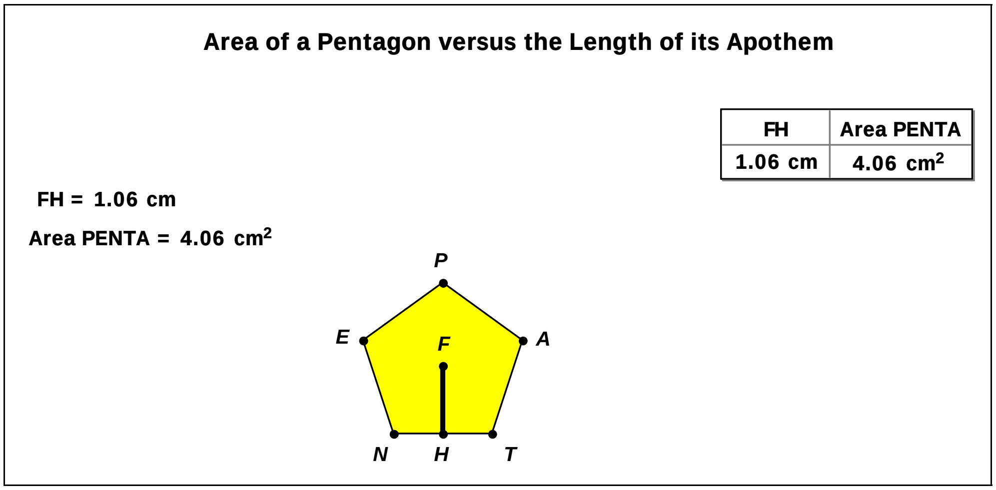



**12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular hexagon with an area of 235.78 square centimeters.**
*Apothem ≈8.25 centimeters.*



## Area of a Regular Pentagon versus the Length of its Apothem

Open the sketch **PENTA.**




1. **Double click on the table to add another row then click and drag point *T* a short distance to the right. What do you observe?**
   *The measures change.*
2. Double click on the table again, and then move point *T* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.
3. Record the data from the computer in the table below.

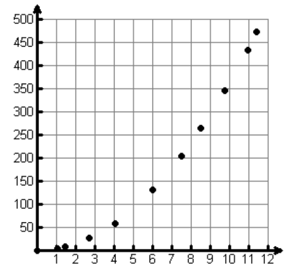
| *Apothem FH* | *Area PENTA* |
|---|---|
| *1.06* | *4.06* |
| *1.43* | *7.39* |
| *2.71* | *26.63* |
| *4.04* | *59.20* |
| *6.02* | *131.53* |
| *7.49* | *204.06* |
| *8.52* | *263.79* |
| *9.75* | *345.27* |
| *10.93* | *434.23* |
| *11.40* | *472.22* |

4. **What patterns do you observe in the table?**
   *Answers may vary. Students should observe that as the length of the apothem increases the area increases, but not at a constant rate.*

5. **What is a reasonable domain and range for your data?**
   *A reasonable domain includes values between 0 and 12. A reasonable range includes values between 0 and 500.*

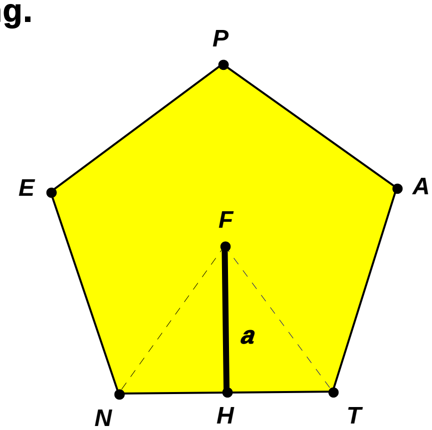
6. **Create a scatterplot of Area of a Regular Pentagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.**

   *x*-min = *0*
   *x*-max = *12*
   *y*-min = *0*
   *y*-max = *500*




7. **What observations can you make about your graph?**
   *The graph appears to be a non-linear function, possibly quadratic.*

8. **To help develop a function rule for this situation use Pentagon *PENTA* to complete the following.**




   h. **Since *PENTA* is a regular pentagon, what is** $m\angle NFH$ **?** *36°*

   i. **Using** $\angle NFH$ **as the reference angle, the trigonometric ratio, tangent, can be used to find *NH* in terms of the apothem length,** *a*.

   j. **Complete the expression** ***NH*** = ___$a(\tan 36°)$___.

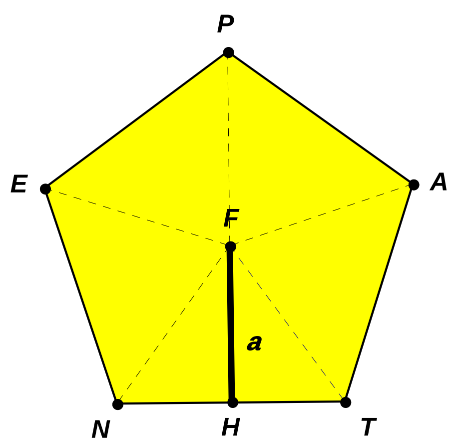
   k. **Write an expression for *NT* in terms of *a and* tan 36°.**
   *Since NT = 2(NH) then NT= 2a tan 36°.*

**l. Recall the formula for area of a triangle,** $Area = \frac{bh}{2}$**. Using the length of the apothem *a* and your answer to question (c) above, write and simplify an expression for the area of** $\Delta NFT$ **.**

$$Area = \frac{2a(tan36°)a}{2}$$
$$Area = \frac{2a^2(tan36°)}{2}$$
$$Area = a^2(tan36°)$$

**m. Draw the radius to each vertex of Pentagon *PENTA.* How many congruent isosceles triangles are formed?**

*5*



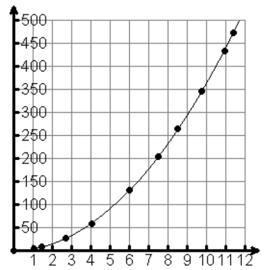

**n. Use your answer to questions (d) and (e) above to write an expression for the area of a regular pentagon.**

$Area = 5a^2(tan36°)$

**o. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.**

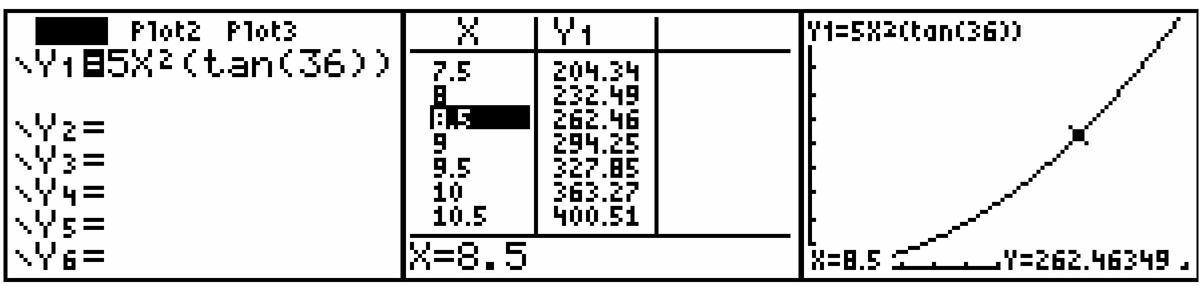
$y = 5x^2(tan(36))$

**9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.**

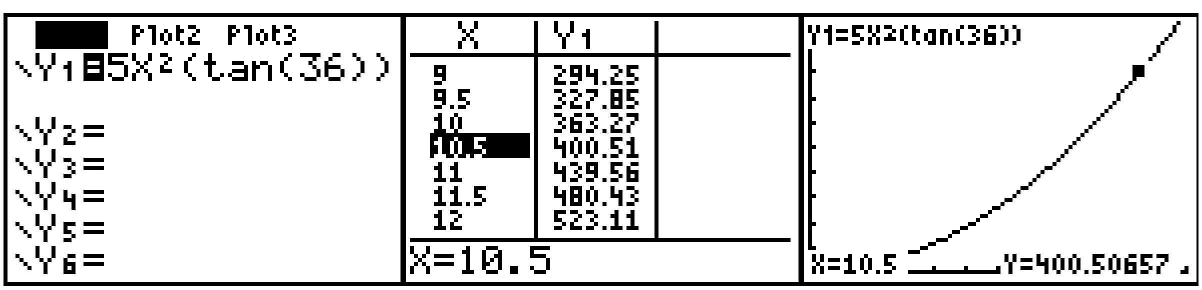



**10. Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

**11.** Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular pentagon with an apothem of 8.5 centimeters. Sketch your graph and table.
*Area ≈262.46 square centimeters.*




**12.** Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular pentagon with an area of 400.51 square centimeters. Sketch your graph and table.
*Apothem ≈10.5 centimeters.*



## Equilateral Triangles and Regular Octagons

In the previous investigations you developed two function rules.

To determine the area, *y*, of a regular **hexagon** given the length of its apothem, *a*, the function rule is:

$$y = 6x^2(tan(30))$$

To determine the area, *y*, of a regular **pentagon** given the length of its apothem, *a*, the function rule is:

$$y = 5x^2(tan(36))$$

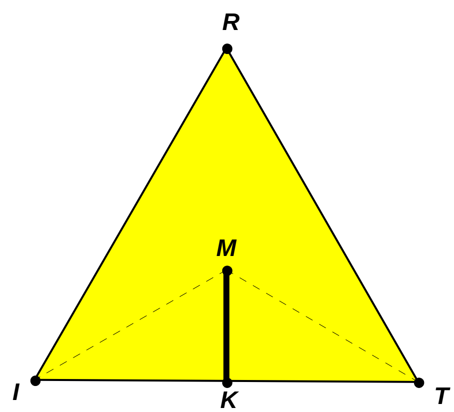
**1. How are the function rules alike? What accounts for the similarities?**

*Both contain $x^2$ and tangent. The $x^2$ is because we multiply a·a in each case. In each case tangent used to find the length of the base of the triangle.*

**2. How are the function rules different? What accounts for the differences?**

*The rule for a hexagon has a 6 because a hexagon has 6 sides and tan 30 because one-half the measure of the central angle is 30°. The rule for a pentagon has a 5 because a pentagon has 5 sides and tan 36 because one-half the measure of the central angle is 36°.*

**3. Examine $\Delta TRI$. What is $m\angle TMI$?** *120°* **What is $m\angle IMK$?** *60°*




**4. Based on your answers to questions 1, 2 and 3 above, write what you think will be the function rule to determine the area, *y*, of a regular triangle (equilateral) given the length of its apothem, *a*.**

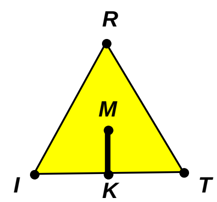
$y = 3x^2(tan(60))$

**5. Open the sketch,** "TRI."

**Area of an Equilateral Triangle versus the Length of its Apothem**

**Apothem MK = 0.62 cm**
**Area △TRI = 2.01 cm$^2$**

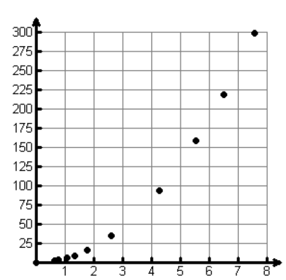
| Apothem MK | Area △TRI |
|---|---|
| 0.62 cm | 2.01 cm$^2$ |



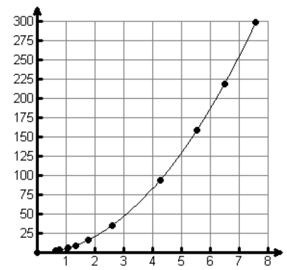

**6. Click and drag point *T*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.**

| *Apothem MK* | *Area Triangle TRI* |
|---|---|
| *0.62* | *2.01* |
| *0.78* | *3.13* |
| *1.05* | *5.73* |
| *1.33* | *9.16* |
| *1.77* | *16.20* |
| *2.60* | *35.19* |
| *4.26* | *94.13* |
| *5.53* | *159.08* |
| *6.49* | *218.62* |
| *7.57* | *298.13* |

**7. Create a scatterplot of Area of △*TRI* versus Apothem *MK*.**

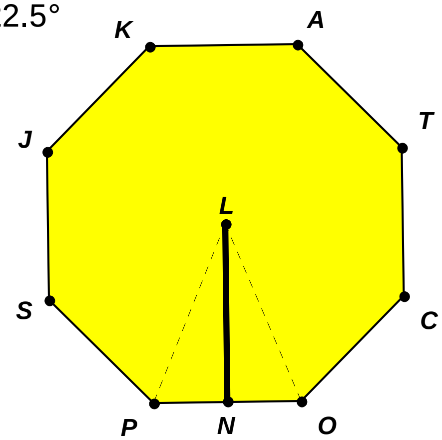


8. **Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.**




9. **Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

10. **Examine regular octagon *OCTAKJSP*. What is** $m\angle PLO$**?** *45°*
**What is** $m\angle PLN$**?** 22.5°



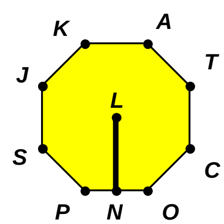

11. **Write what you think will be the function rule to determine the area, *y*, of a regular** octagon **given the length of its apothem, *a*.**
$y = 8x^2(tan(22.5))$

12. Open the sketch, "**OCTAGONS.**"

**Area of a Regular Octagon versus the Length of its Apothem**

**Apothem LN = 1.06 cm**

**Area Octagon = 3.73 cm$^2$**

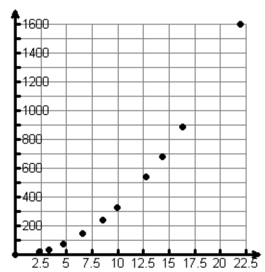



| Apothem LN | Area Octagon |
|---|---|
| 1.06 cm | 3.73 cm$^2$ |

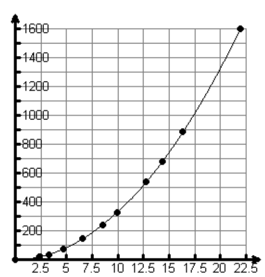
13. **Click and drag point *O*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.**

| *Apothem LN* | *Area Octagon* |
|---|---|
| *2.32* | *17.88* |
| *3.25* | *35.08* |
| *4.71* | *73.54* |
| *6.59* | *144.12* |
| *8.52* | *240.51* |
| *9.91* | *325.44* |
| *12.73* | *537.21* |
| *14.31* | *678.19* |
| *16.33* | *884.16* |
| *21.97* | *1598.84* |

14. **Create a scatterplot of Area of the Octagon versus Apothem *LN*.**




15. **Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.**




16. **Does the graph verify your function rule? Why or why not?**
*Yes. The graph of the function rule passes through each data point.*

**17. In the previous activities you investigated relationships between area of regular polygons and the length of their apothems. The table below includes function rules for triangles, pentagons, hexagons, and octagons. Fill in any missing information then develop a general function rule that can be used to find the area of any regular polygon.**

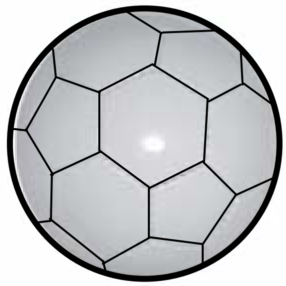
| Regular Polygon | Number of Sides | Measure of the Central Angle | Function Rule |
|---|---|---|---|
| Triangle | *3* | *120°* | $y = 3x^2(tan(60))$ |
| Square | *4* | *90°* | $y = 4x^2(tan(45))$ |
| Pentagon | *5* | *72°* | $y = 5x^2(tan(36))$ |
| Hexagon | *6* | *60°* | $y = 6x^2(tan(30))$ |
| Heptagon | *7* | *51.43* | $y = 7x^2(tan(25.715))$ |
| Octagon | *8* | *45°* | $y = 8x^2(tan(22.5))$ |
| Any | *n* | $\frac{360}{n}$ | $y = nx^2(tan\left(\frac{central\ angle}{2}\right))$ |

**18. Use words to describe how to calculate the area of any regular polygon when you know the length of its apothem.**

*First find half the measure of the central angle. Find the tangent of that measure, then multiply by the number of sides and the square of the length of the apothem.*

**Kick It Incorporated**

Banish's company, "Kick It Incorporated," manufactures soccer balls. To construct the covering for each ball 20 regular hexagons and 12 regular pentagons cut from synthetic leather are sewn together. The length of the apothem of each hexagon is 1.5 inches, and the length of the apothem of each pentagon is 1.2 inches.



The shipping manager needs to ship six inflated balls to a customer. He has a box with dimensions 22 inches by 15 inches by 8 inches. Can he fit 2 rows of 3 balls in the box? Justify your answer.

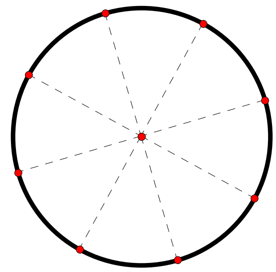
*Answer: No. The surface area of a ball is approximately 208 square inches so the diameter of one ball is about 8 inches. Since the width of the box is only 15 inches, 2 balls will not fit. Since the length of the box is only 22 inches, 3 balls will not fit lengthwise. The height of the box may work, but the box could only hold 2 balls at most.*

# Area of Regular Polygons

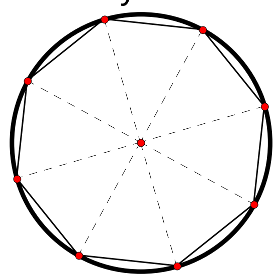
1. On a sheet of patty paper construct a large circle.

2. Cut out the circle.

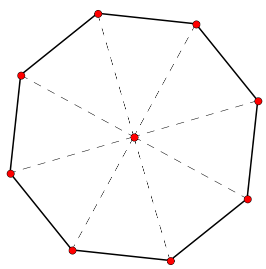
3. Use paper folding to divide the circle into 8 congruent sectors.



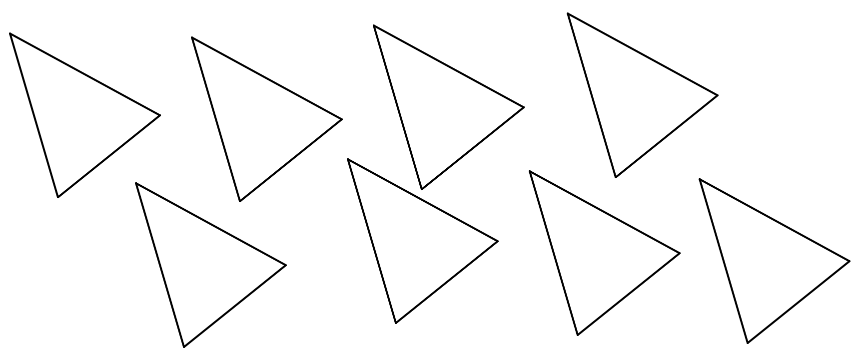
4. Use a straight edge to connect the endpoints of the radii you folded.



5. Cut out the polygon.



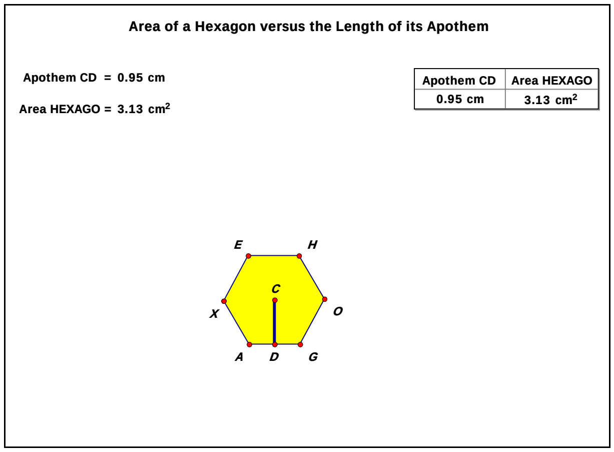
6. Cut the polygon along each fold.



7. Determine the area of your original polygon.

# Area of a Regular Hexagon versus the Length of its Apothem

Open the sketch **HEXAGO.**




1. Double click on the table to add another row, then click and drag point *G* a short distance to the right. What do you observe?

2. Double click on the table again, then move point *G* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer into the table below.

| *Apothem CD* | *Area HEXAGO* |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

4. What patterns do you observe in the table?

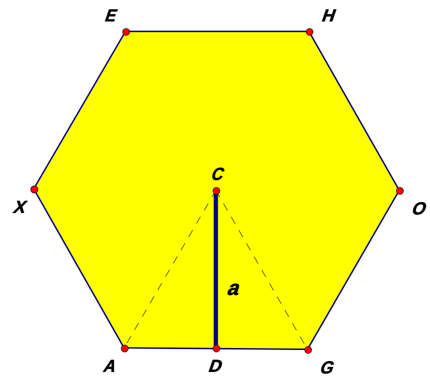
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Hexagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

$x$-min =
$x$-max =
$y$-min =
$y$-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation use Hexagon *HEXAGO* to complete the following.




a. Since *HEXAGO* is a regular hexagon, $m\angle ACG = 60°$. What is $m\angle ACD$?

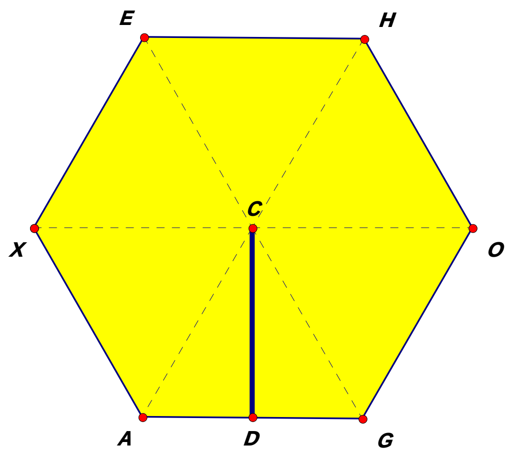
b. Using $\angle ACD$ as the reference angle, the trigonometric ratio "tangent" can be used to find *AD* in terms of the apothem length, *a*.

$$tan\,30° = \frac{AD}{a} \text{ or } AD = a(tan\,30°)$$

c. Write an expression for $AG$ in terms of $a$ and tan 30°.

d. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem $a$ and your answer to question (c) above, write and simplify an expression for the area of $\Delta ACG$.

e. Draw the radius to each vertex of Hexagon *HEXAGO*. How many congruent isosceles triangles are formed?




f. Use your answer to questions (d) and (e) above to write an expression for the area of a hexagon.

g. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

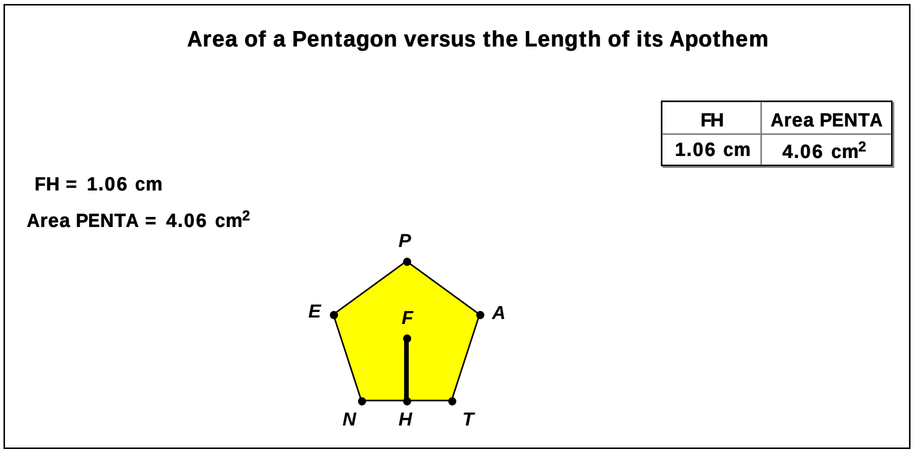
10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular hexagon with an apothem of 6.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular hexagon with an area of 235.78 square centimeters. Sketch your graph and table.

## Area of a Regular Pentagon versus the Length of its Apothem

Open the sketch **PENTA.**




1. Double click on the table to add another row, then click and drag point *T* a short distance to the right. What do you observe?

2. Double click on the table again, and then move point *T* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer in the table below.

| *Apothem FH* | *Area PENTA* |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

4. What patterns do you observe in the table?

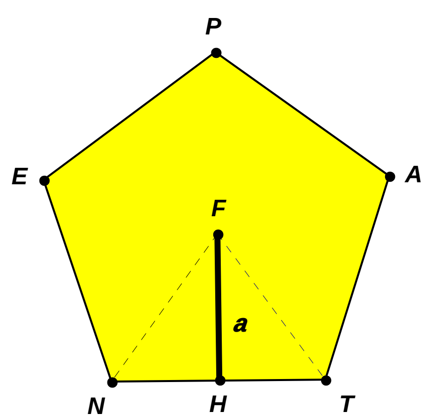
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Pentagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

   $x$-min =
   $x$-max =
   $y$-min =
   $y$-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation, use Pentagon *PENTA* to complete the following.




   a. Since *PENTA* is a regular pentagon, what is $m\angle NFH$ ?

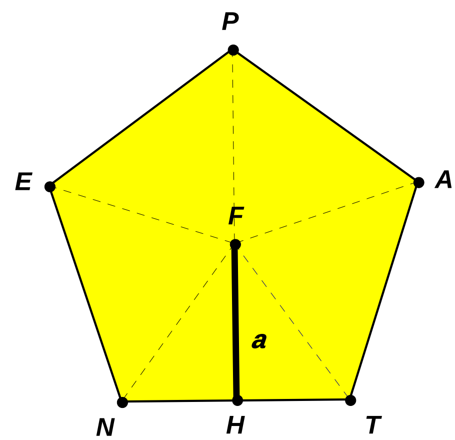
   b. Using $\angle NFH$ as the reference angle, the trigonometric ratio, tangent, can be used to find *NH* in terms of the apothem length, *a*.

   c. Complete the expression *NH =* ________________.

   d. Write an expression for *NT* in terms of, *a, and* tan 36°.

e. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem, *a*, and your answer to question (c) above, write and simplify an expression for the area of $\Delta NFT$ .

f. Draw the radius to each vertex of Pentagon *PENTA.* How many congruent isosceles triangles are formed?




g. Use your answer to questions d and e above to write an expression for the area of a regular pentagon.

h. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular pentagon with an apothem of 8.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular pentagon with an area of 400.51 square centimeters. Sketch your graph and table.

## Equilateral Triangles and Regular Octagons

In the previous investigations you developed two function rules.

To determine the area, *y*, of a regular **hexagon** given the length of its apothem, *a*, the function rule is:

$$y = 6x^2(tan(30))$$

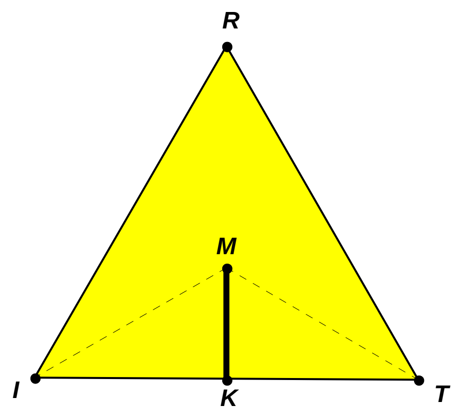
To determine the area, *y*, of a regular **pentagon** given the length of its apothem, *a*, the function rule is:

$$y = 5x^2(tan(36))$$

1. How are the function rules alike? What accounts for the similarities?

2. How are the function rules different? What accounts for the differences?

3. Examine $\Delta TRI$. What is $m\angle TMI$? What is $m\angle IMK$?



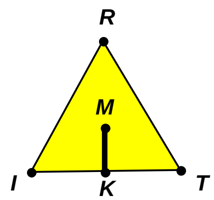

4. Based on your answers to questions 1, 2 and 3 above, write what you think will be the function rule to determine the area, *y*, of a regular **triangle** (equilateral) given the length of its apothem, *a*.

5. Open the sketch, "**TRI.**"

**Area of an Equilateral Triangle versus the Length of its Apothem**

**Apothem MK = 0.62 cm**
**Area △TRI = 2.01 cm²**

| Apothem MK | Area △TRI |
|---|---|
| 0.62 cm | 2.01 cm² |




6. Click and drag point *T*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

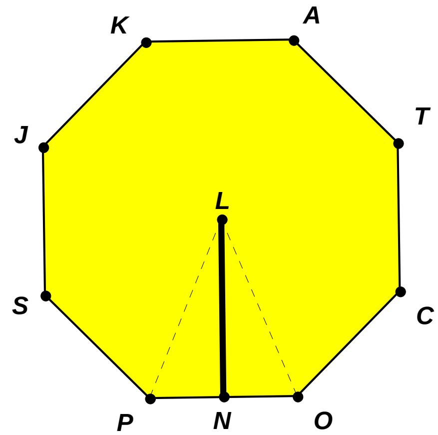
| *Apothem MK* | *Area Triangle TRI* |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

7. Create a scatterplot of Area of $\triangle TRI$ versus Apothem *MK.*

8. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

9. Does the graph verify your function rule? Why or why not?

10. Examine regular octagon *OCTAKJSP*. What is $m\angle PLO$? What is $m\angle PLN$?



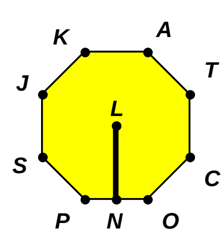

11. Write what you think will be the function rule to determine the area, *y*, of a regular **octagon** given the length of its apothem, *a*.

12. Open the sketch, "**OCTAGONS.**"

**Area of a Regular Octagon versus the Length of its Apothem**

Apothem LN = 1.06 cm

Area Octagon = 3.73 cm$^2$




| Apothem LN | Area Octagon |
|---|---|
| 1.06 cm | 3.73 cm$^2$ |

13. Click and drag point *O*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

| *Apothem LN* | *Area Octagon* |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

14. Create a scatterplot of Area of the Octagon versus Apothem *LN.*

15. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

16. Does the graph verify your function rule? Why or why not?

17. In the previous activities you investigated relationship between area of regular polygons and the length of their apothems. The table below includes function rules for triangles, pentagons, hexagons, and octagons. Fill in any missing information, then develop a general function rule that can be used to find the area of any regular polygon.

| Regular Polygon | Number of Sides | Measure of the Central Angle | Function Rule |
|---|---|---|---|
| Triangle | *3* | *120°* | $y = 3x^2(tan(60))$ |
| Square | | | |
| Pentagon | *5* | *72°* | $y = 5x^2(tan(36))$ |
| Hexagon | *6* | *60°* | $y = 6x^2(tan(30))$ |
| Heptagon | | | |
| Octagon | *8* | *45°* | $y = 8x^2(tan(22.5))$ |
| Any | *n* | | |

18. Use words to describe how to calculate the area of any regular polygon when you know the length of its apothem.

## Kick It Incorporated

Banish's company, "Kick It Incorporated," manufactures soccer balls. To construct the covering for each ball 20 regular hexagons and 12 regular pentagons cut from synthetic leather are sewn together. The length of the apothem of each hexagon is 1.5 inches, and the length of the apothem of each pentagon is 1.2 inches.

The shipping manager needs to ship six inflated balls to a customer. He has a box with dimensions 22 inches by 15 inches by 8 inches. Can he fit 2 rows of 3 balls in the box? Justify your answer.

# Composite Area

**1** The floor of a room is in the shape of a regular hexagon. If the area of the room is 200 square feet, what is the approximate length of the apothem of the hexagon?

A 4.39 feet

B 7.60 feet

C 8.78 feet

D 38.11 feet

**3** The table below was generated by a function rule that calculates the area of a regular polygon (*y*) given the length of its apothem (*x*).

| X | Y1 | |
|---|---|---|
| 1 | 3.6327 | |
| 1.5 | 8.1736 | |
| 2 | 14.531 | |
| 2.5 | 22.704 | |
| 3 | 32.694 | |
| 3.5 | 44.501 | |
| 4 | 58.123 | |

X=1

Which polygon was it?

A triangle

B pentagon

C octagon

D decagon

**2** The length of the apothem of the STOP sign at the corner of Ashcroft Drive and Ludington Street is 12 inches.

What is the area of the STOP sign?

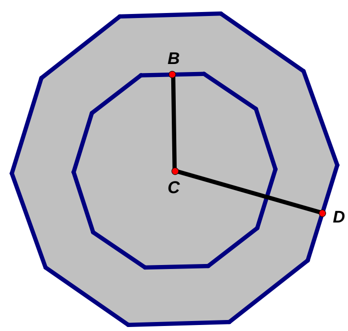
A 96

B 144 square inches

C 477.17

D 498.83

**4** The drawing shows a cement walkway around a swimming pool. The walkway and the pool are in the shape of regular polygons. The length of $\overline{BC}$ is 18 feet and the length of $\overline{CD}$ is 29 feet.




What is the area of the walkway?

A 1052.7 square feet

B 2732.6 square feet

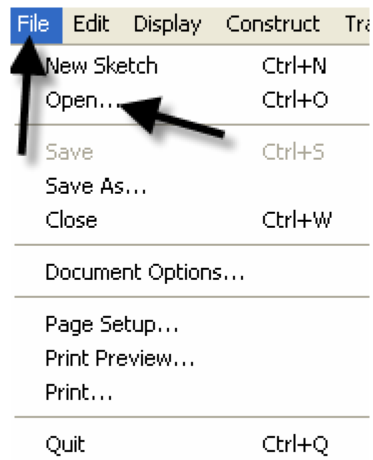
C 1873.2 square feet

D 1679.9 square feet

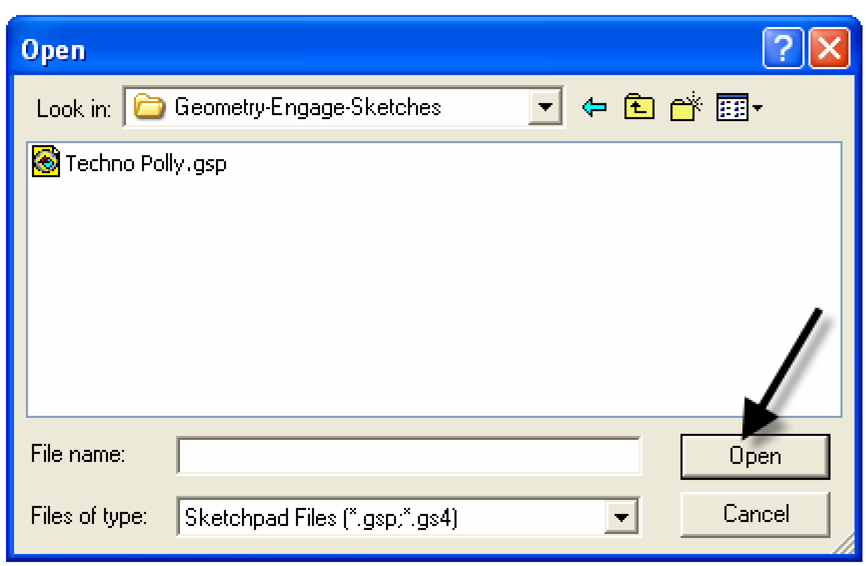
# Geometer's Sketchpad—Techno Polly

## Opening an Existing Sketch

1. To open an **existing sketch** in Geometer's Sketchpad, first click on the icon on your desktop then when the program opens click on **File**, **Open**.

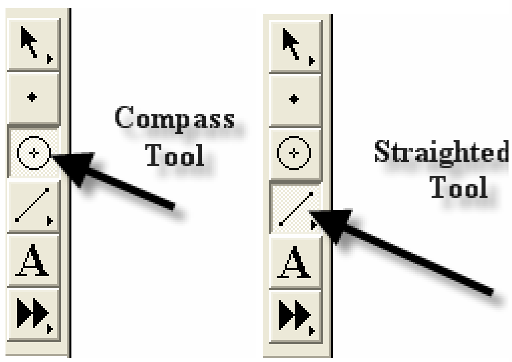



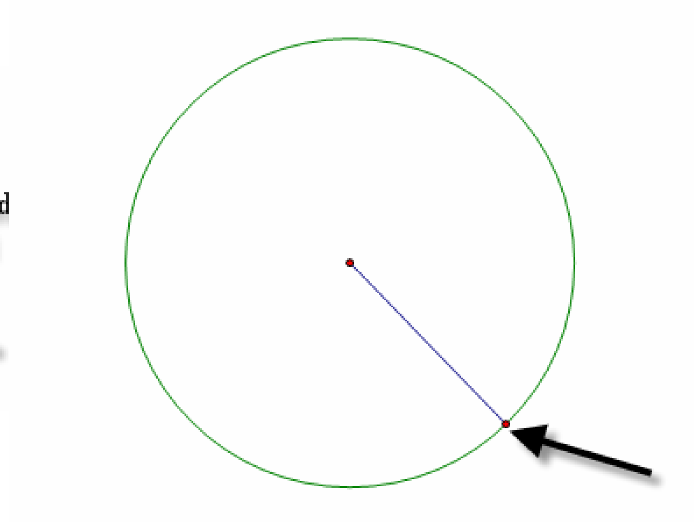

2. A pop-up window will appear. Follow the directions for your particular computer system to get to the file where the existing sketches are stored. Select the desired file by clicking on it, then click the **Open** button.



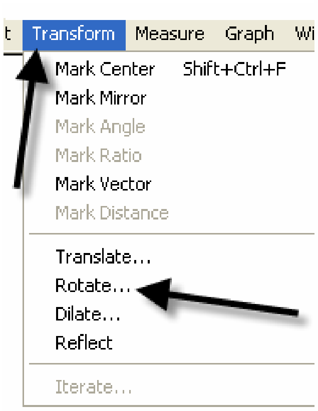
# Polygarden Landscaping Company
# Equilateral Triangle with Graph

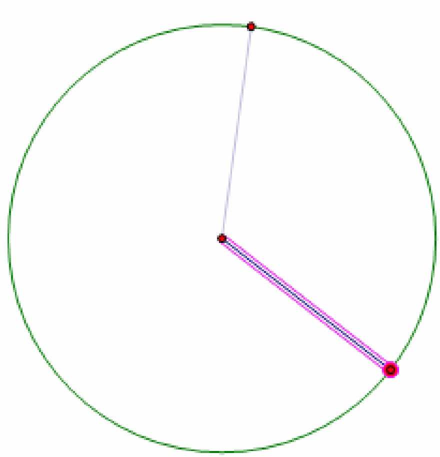
Using the **Compass tool** and the **Straightedge tool**, construct a circle and its radius. Be sure the radius is attached to the point that is constructed on the side of the circle—this will later allow all vertices of the triangle to act as control points to adjust the size of triangle.

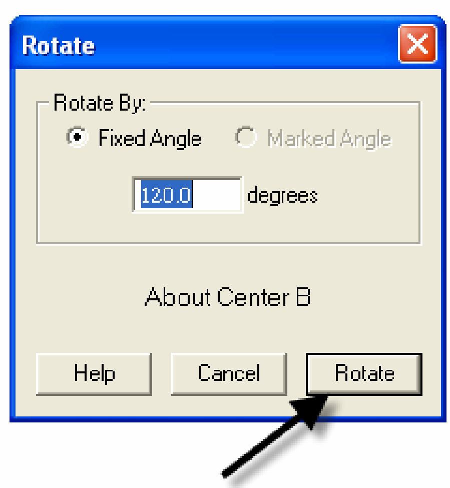


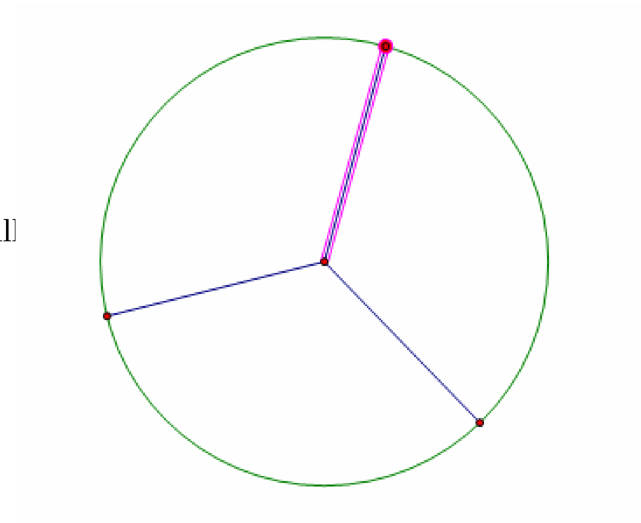
With the selection tool, highlight the radius and the point of intersection on the circle. Double click on the center point of the circle to mark it as a point of rotation. There will be a flash of concentric circles around the point as it is marked. From the Menu Bar use the **Transform** option and choose **Rotate**. A window will pop up with a box to enter the number of degrees of rotation desired. In this case enter 120 degrees, the number of degrees of the central angle of an equilateral triangle inscribed in a circle. Click the **Rotate** button to complete the rotation.



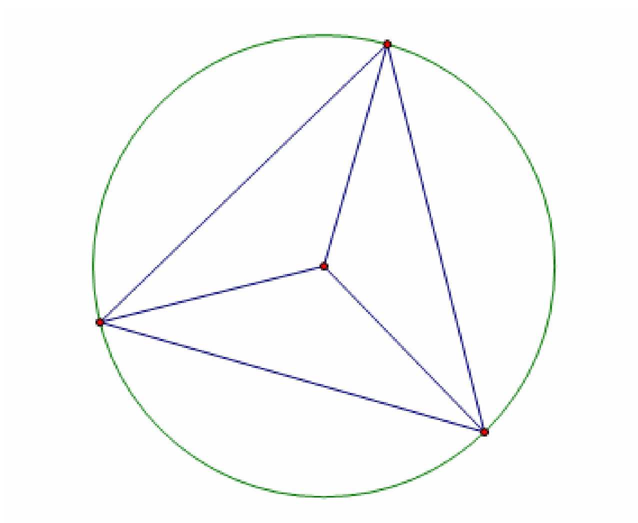





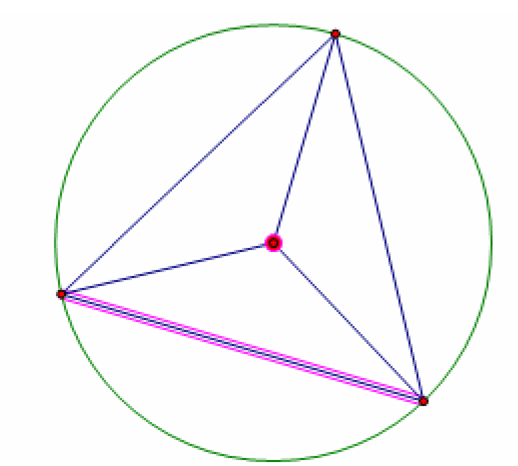
After the rotation, the new radius and its point of intersection are highlighted. The center point is still “marked” as the point of rotation. To rotate the radius, simply use the **Transform** and **Rotate** options again. The 120 degrees should still in the pop-up window; so to complete the rotation, click on the **Rotate** button.

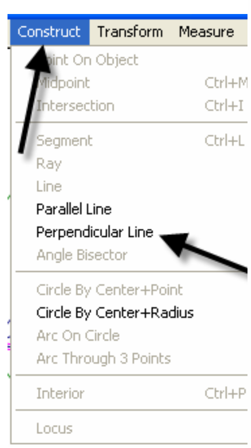


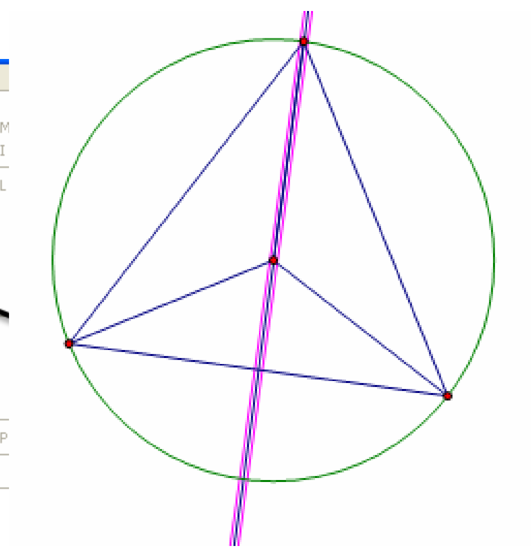
Use the **Segment** tool to connect the points on the circle forming the equilateral triangle.



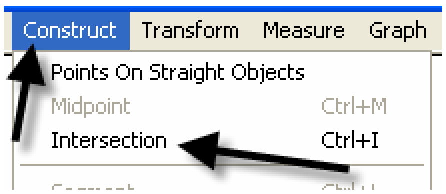
Construct a line perpendicular to one side of the triangle through the center by first highlighting the side and the center, then using the Construct and Perpendicular options from the Menu Bar.

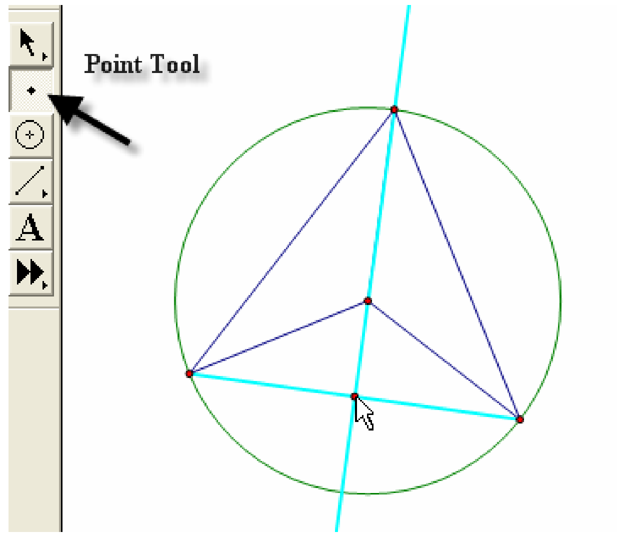




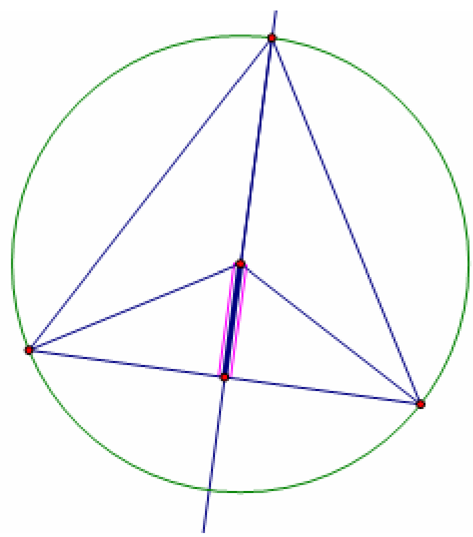



Construct the point of intersection between the side of the triangle and its perpendicular either by using the **Point** tool and clicking at the intersection when both lines turn blue OR by highlighting the side and the perpendicular and using the **Construct** and **Intersection** options from the Menu Bar.

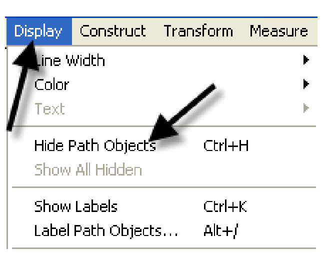


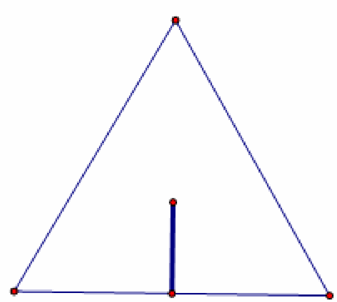




To construct the apothem, the only part of the perpendicular needed is the segment connecting the center to the side. Use the **Straightedge tool** to construct a segment (on top of the perpendicular line) that joins the two points.

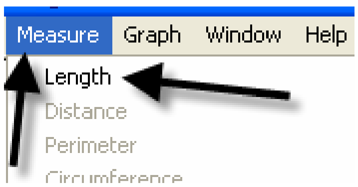


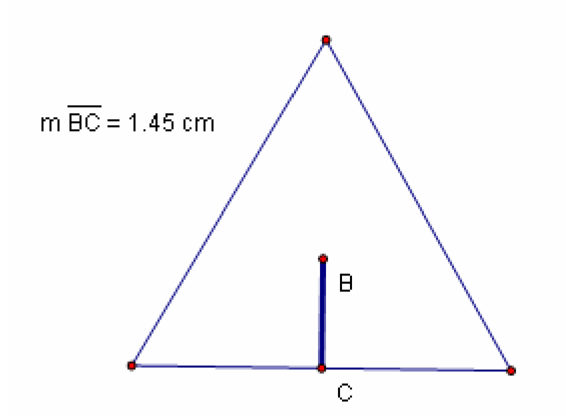
To hide the circle and the unnecessary lines, simply highlight them and use the **Display** and **Hide Path Objects** from the Menu Bar.



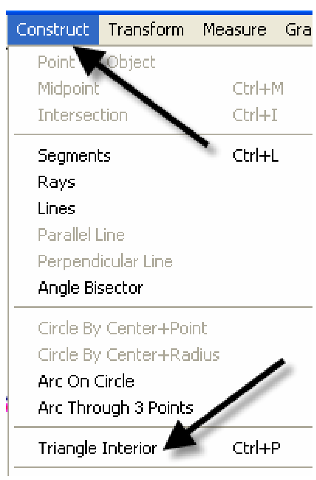



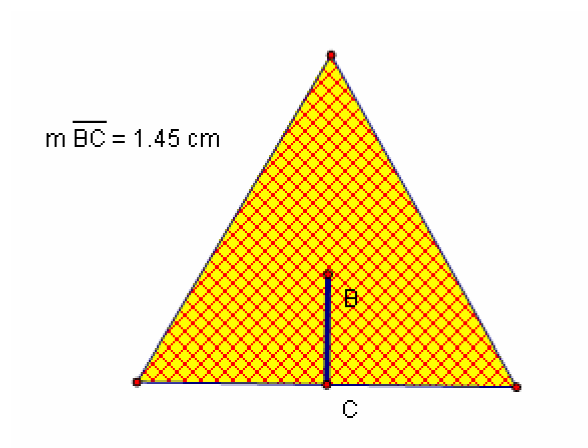
To measure the length of the apothem, highlight it and use the **Measure** and **Length** options from the Menu Bar. The points will automatically be labeled, and the measurement will appear. This measurement will be highlighted, and a click in the blank white area of the screen will deselected it.

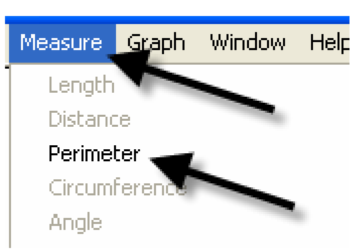


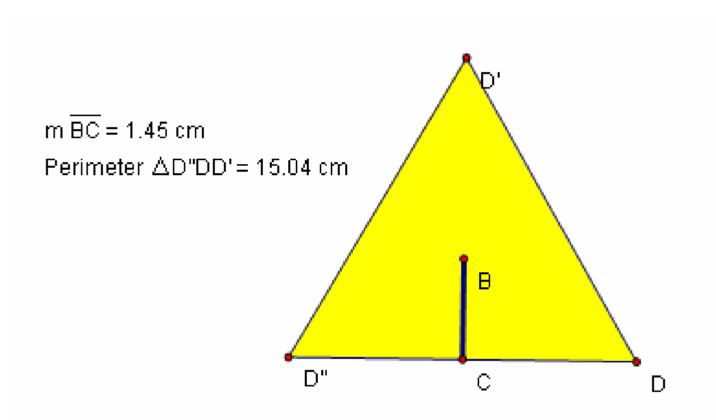




To use the **Measure** and **Perimeter** options from the menu bar, the *interior* of the triangle must first be constructed. Highlight the vertices. Use the **Construct** and **Triangle Interior** options from the Menu Bar. Once constructed, the interior is automatically selected (This is shown by cross hatching.) allowing the **Measure** and **Perimeter** options to become available. Click on them to measure.

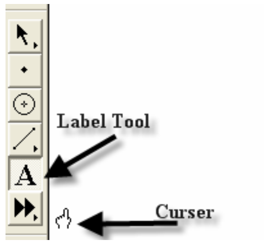


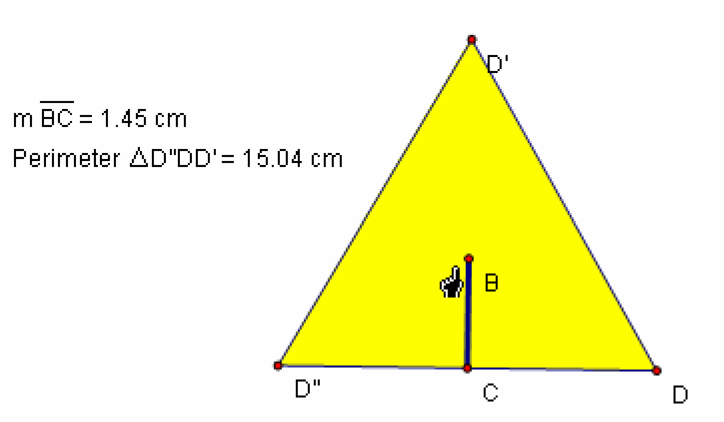



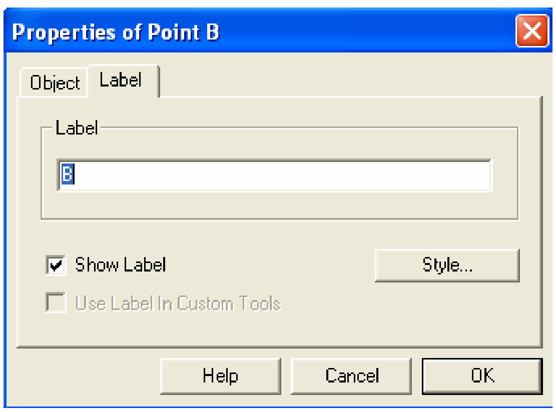



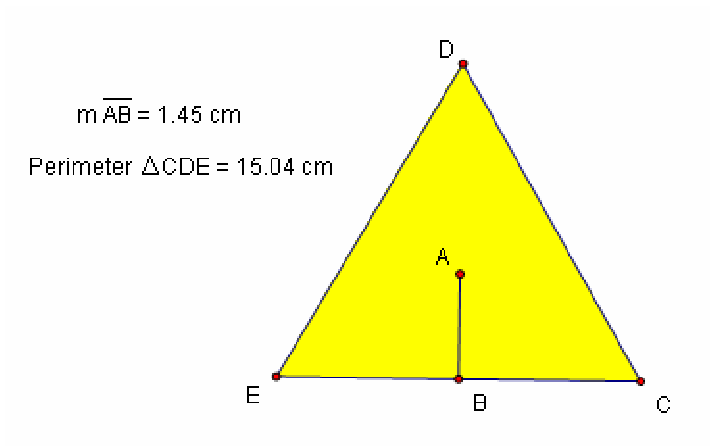



Geometer's Sketchpad labels automatically. In this particular case it used point D as the original point on the circle and D' and D" as the rotated points. If desired, rename the points by selecting the Text tool. A little outline of a hand will appear as the curser. As the curser becomes lined up with a label, it will change. Double click on a point and a window will pop up with a box allowing for the new name to be entered. As points are changed, the label with their respective measurements will also change.

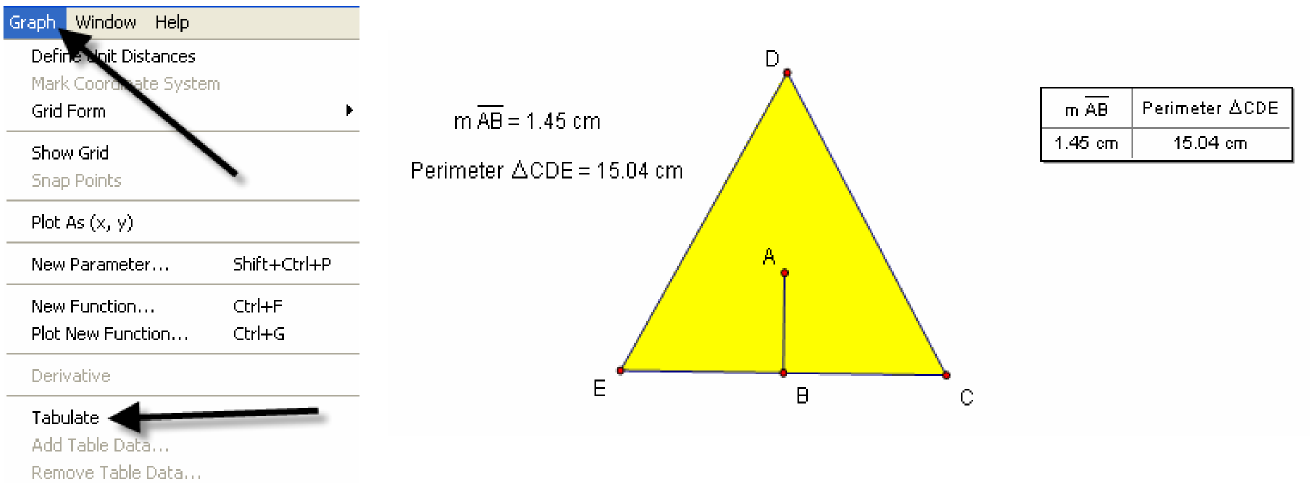


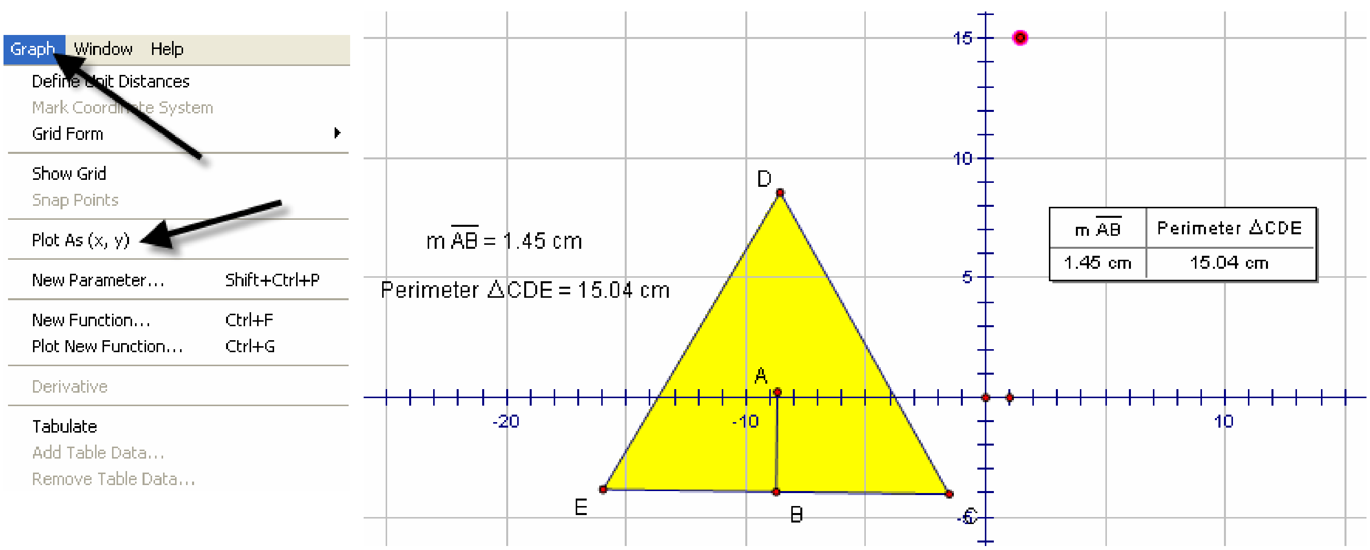



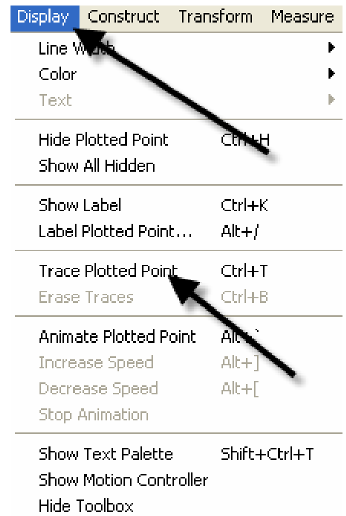
To build a table to explore the relationship between the length of the apothem and the perimeter, highlight in order first the independent variable followed by the dependent variable. In this case, the *measurement* of the apothem is the independent variable and the *perimeter* the dependent variable. Once highlighted, use the **Graph** and **Tabulate** options from the **Menu Bar.** A table with each value will appear. This table can be moved anywhere on the screen that is convenient.

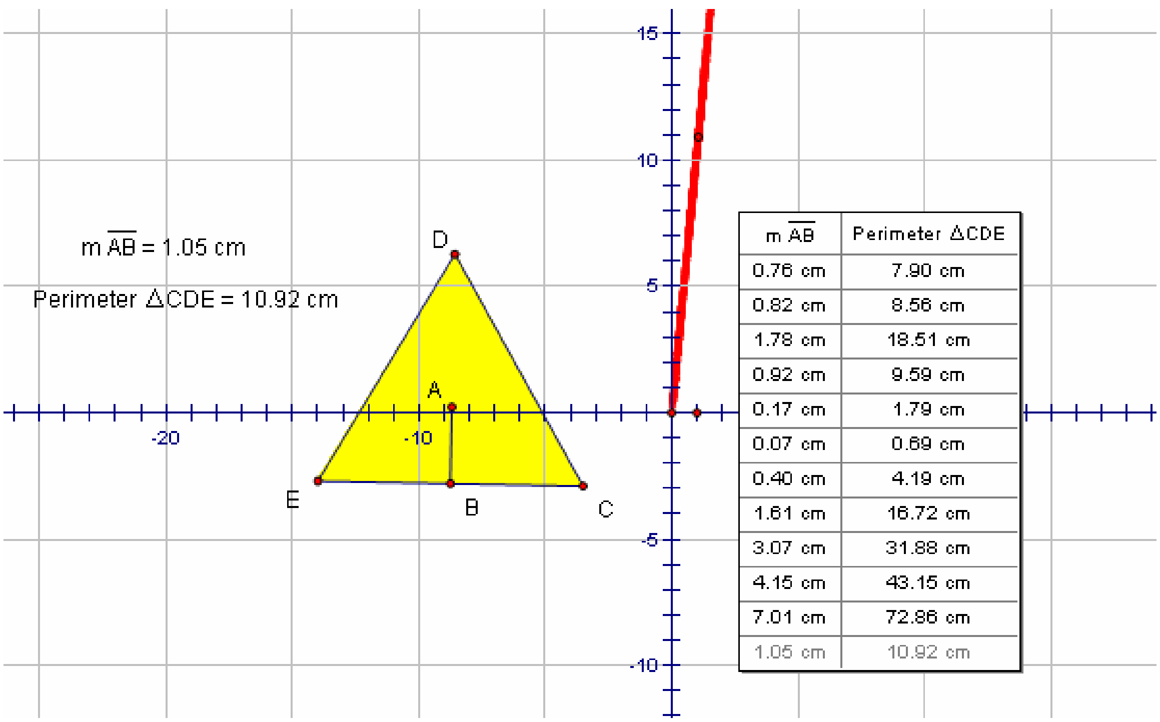



To plot the points, again highlight the independent then the dependent variable. In this case the length of the apothem then the perimeter. Once highlighted use the **Graph** and **Plot as (x, y)** option from the **Menu Bar**. A coordinate grid appears behind the triangle with the point highlighted.



To trace the point, highlight it and use the Display and Trace Plotted Point options from the Menu Bar. The point will trace on the coordinate grid as the triangle is manipulated from any one of its vertices. To add data to the table, double click in the table, adjust the size of the triangle and repeat until the number of data points desired are accumulated.

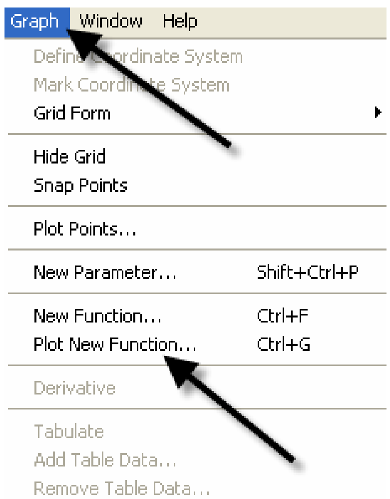


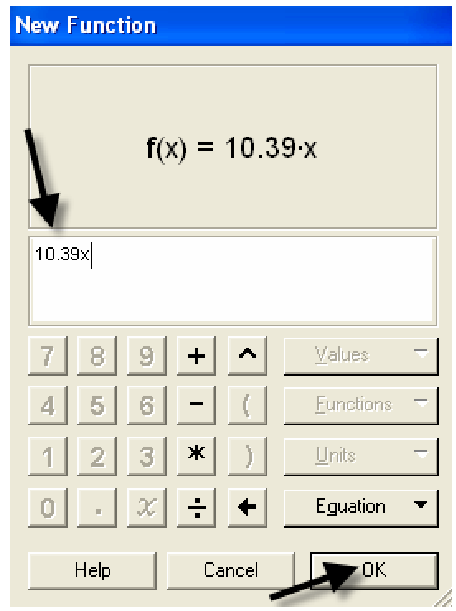



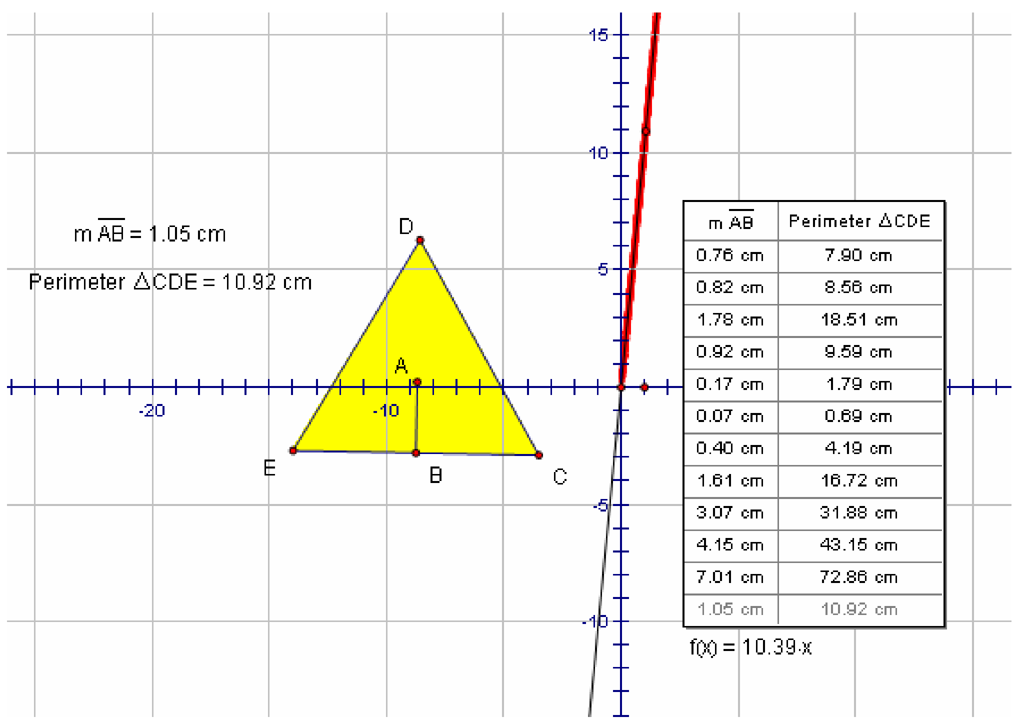

| m $\overline{AB}$ | Perimeter △CDE |
|---|---|
| 0.76 cm | 7.90 cm |
| 0.82 cm | 8.56 cm |
| 1.78 cm | 18.51 cm |
| 0.92 cm | 9.59 cm |
| 0.17 cm | 1.79 cm |
| 0.07 cm | 0.69 cm |
| 0.40 cm | 4.19 cm |
| 1.61 cm | 16.72 cm |
| 3.07 cm | 31.88 cm |
| 4.15 cm | 43.15 cm |
| 7.01 cm | 72.86 cm |
| 1.05 cm | 10.92 cm |

## Verify a Function Rule

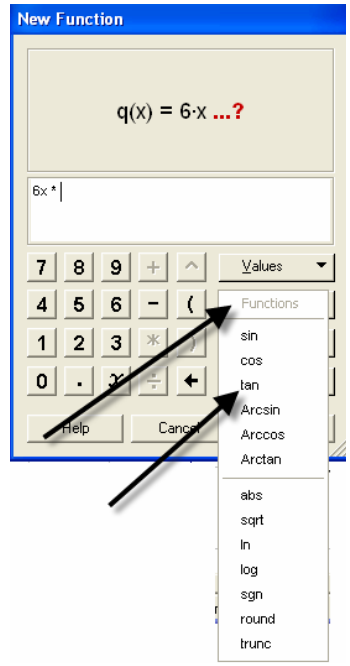
To use Geometer's Sketchpad to verify a function rule, in this case $y = 10.39x$, use the **Graph** and **Plot New Function** options from the Menu Bar. A calculator window will pop up allowing the equation to be entered. After entering the function, click the **OK** button. The function will graph, hopefully over the existing data, thus verifying the function rule.

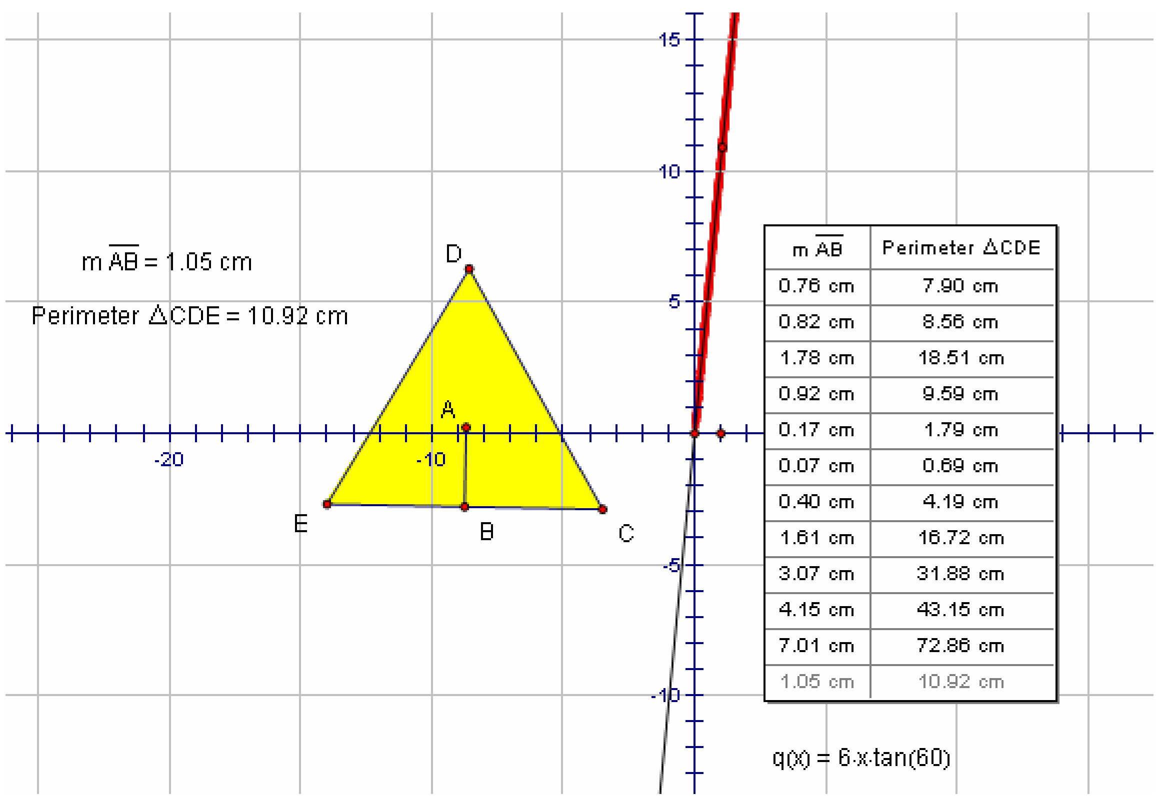


To verify using a trigonometric function, follow the same procedure, but use the calculator to enter the specific trigonometric function desired. For instance, $y = 6x\tan(60^{o})$. The function will then graph verifying the plotted data.




| m $\overline{AB}$ | Perimeter $\triangle$CDE |
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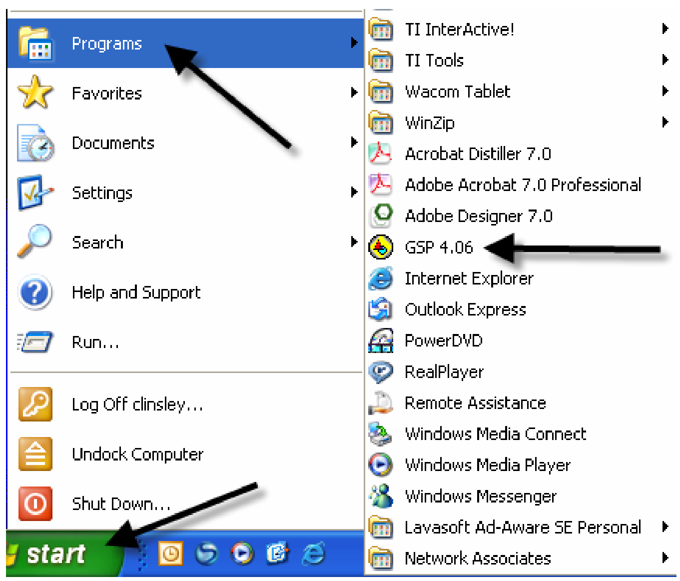
| 0.40 cm | 4.19 cm |
| 1.61 cm | 16.72 cm |
| 3.07 cm | 31.88 cm |
| 4.15 cm | 43.15 cm |
| 7.01 cm | 72.86 cm |
| 1.05 cm | 10.92 cm |

# Sketchpad Skills Investigation

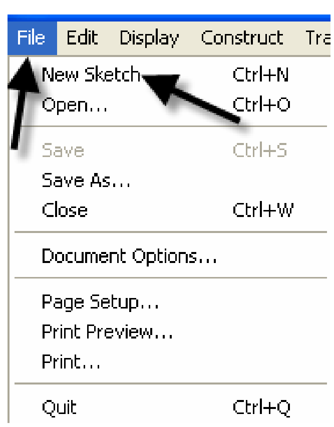
## Opening a New Sketch

1. To **open** the Geometer's Sketchpad ,click on the icon on your desktop


or click on **Start**, **Programs** and find the GSP icon. A new blank sketch will open up.




2. To open a **new sketch** in Geometer's Sketchpad, click on **File**, **New Sketch**.



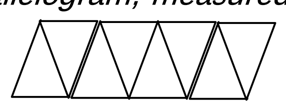
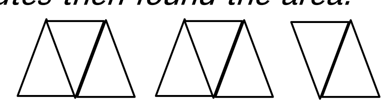
## ENGAGE

*The Engage portion of the lesson is designed to create student interest in the relationships between the area of regular polygons and the area of the triangles that compose them. This part of the lesson is designed for groups of three to four students.*

1. Distribute a sheet of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on the sheet of patty paper. Distribute the **Area of Regular Polygons** activity sheet. Students should follow the directions on the sheet.
3. Students will use paper folding to construct an octagon then cut it into 8 congruent triangles.
4. Students should use the available measuring tools to measure critical attributes then calculate the area of the octagon.
5. Students will share their method of calculating the area with their group, and then with the whole class.

*Facilitation Questions – Engage Phase*

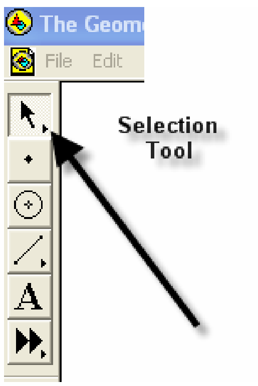
1. When you folded your circle into 8 equal sectors, what was the measure of each central angle?
   *45°, 360° divided by 8.*
2. What do you observe about the 8 triangles that you cut out?
   *Answers may vary. Students may observe that the triangles are congruent.*
3. How do you know the triangles are congruent?
   *Answers may vary. Students may stack the triangles or offer an informal proof using SAS since all radii and all central angles of the octagon are congruent.*
4. How did you use your triangles to determine the area of your octagon?
   *Answers may vary. Students should be able to explain their method. Students may have measured the base and height of one triangle, calculated the area of the triangle then multiplied it by 8. The may have arranged the triangles into the shape of a parallelogram or two trapezoids and a parallelogram, measured the critical attributes then found the area.*

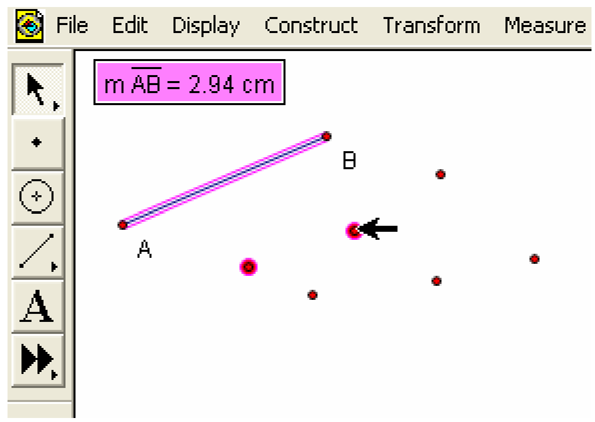
   
   

   *This activity sets the stage for exploring the relationship between the apothem of a regular polygon (height of the triangle) and the area of the polygon.*
5. How can you determine a method that could be used to calculate the area of any regular polygon?
   *Answers may vary. Students should realize that data for several polygons could be collected and analyzed to verify conjectures.*

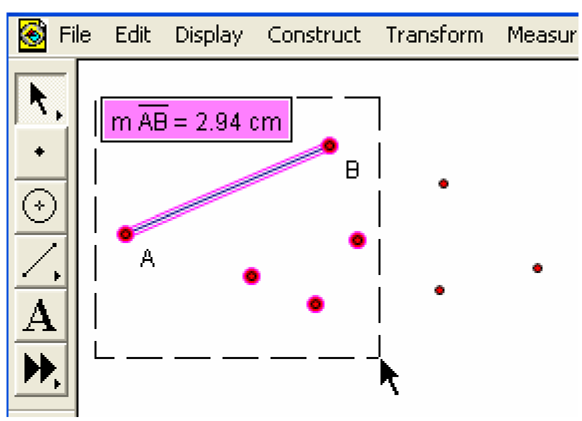
## Selection Tool

The selection tool allows for selection/deselection of items in two different ways. First, simply click on the item to be selected/deselected. An item that is highlighted is pink.




The second way is to click and drag. An outline box will appear that will select/deselect everything it touches.

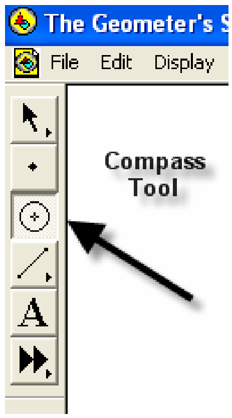


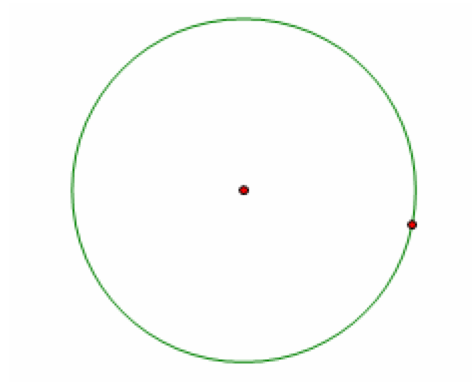

## Deleting

To delete items, simply select them, so they are highlighted, and then hit the **Delete** key on the keyboard.

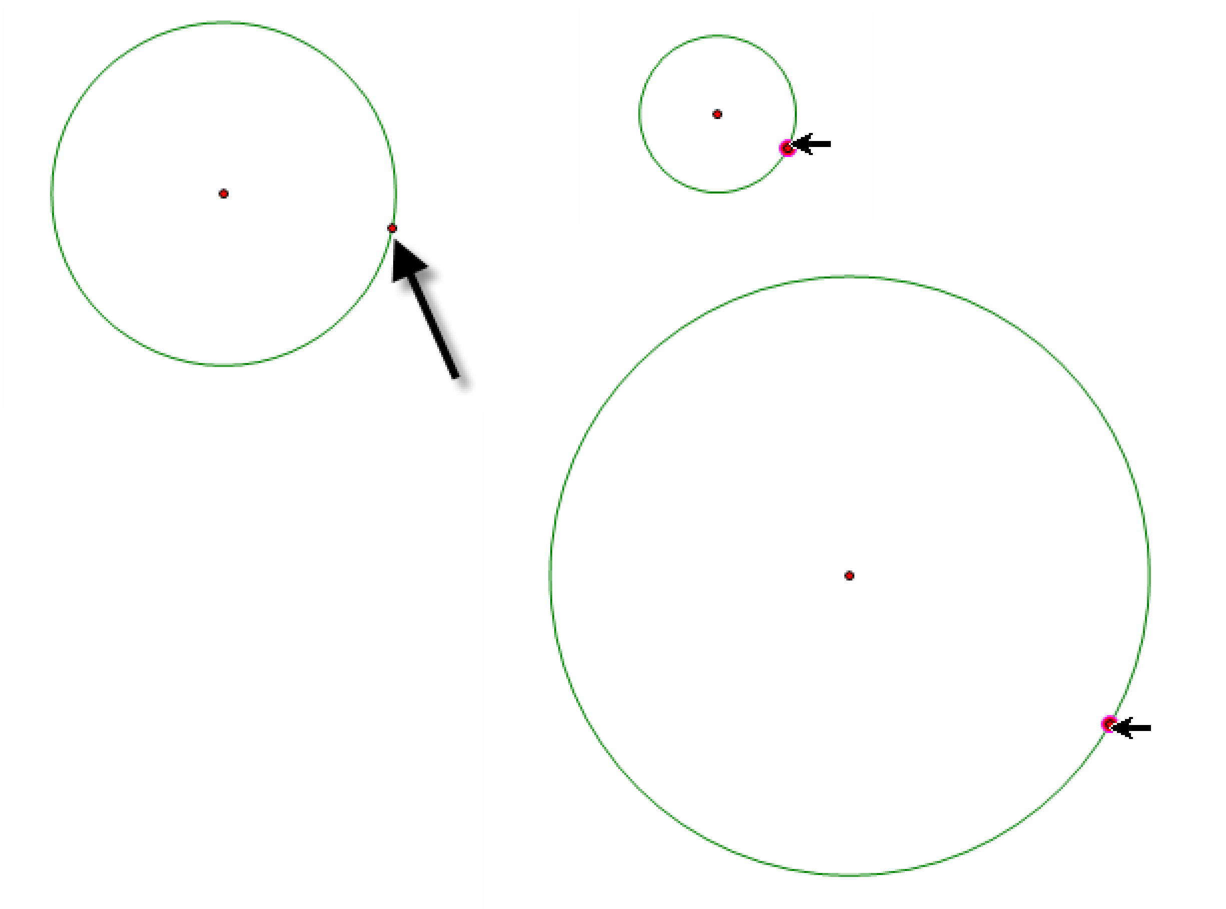
## Circles

To construct a circle use the **Compass Tool**.  Notice that the circle forms from the inside out.



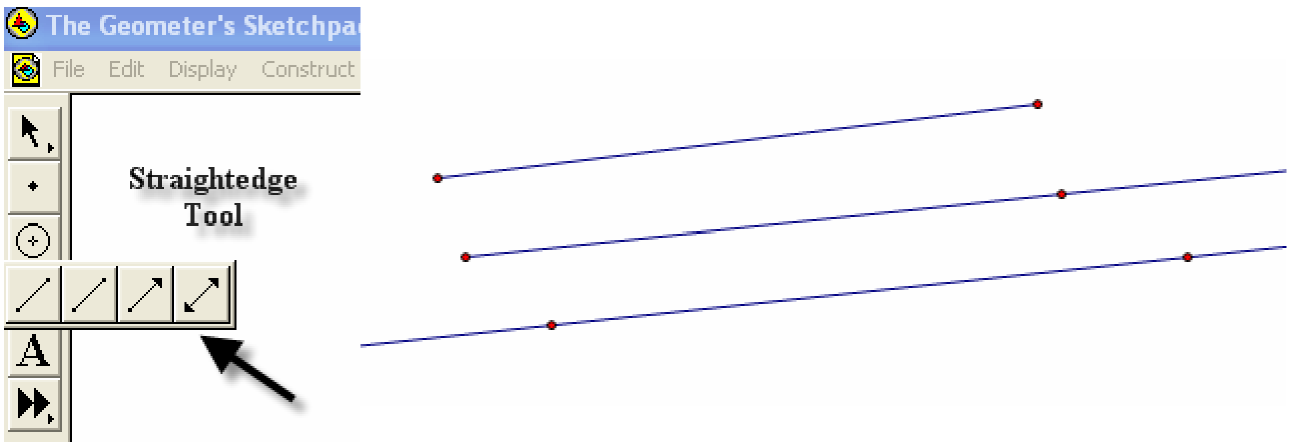



The point on the side of the circle is a control point that will allow the size of the circle to get larger and smaller by clicking and dragging.

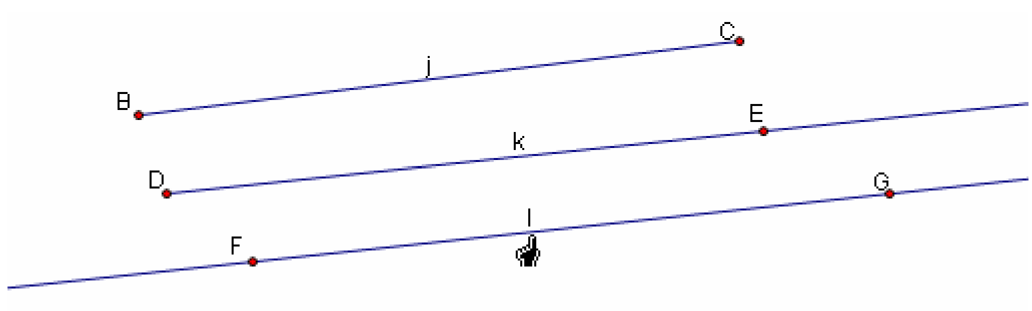


## Lines, Rays and Segments

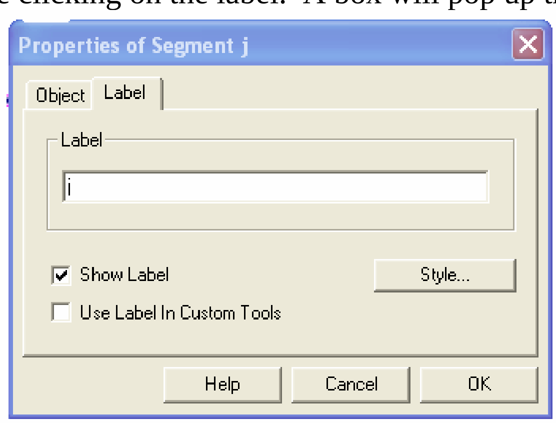
To create lines, rays or segments, click on the **Straightedge** tool, then slide the cursor to the right to choose the desired tool. Each figure is formed from two points. The segment has two distinct endpoints; the ray has one endpoint and then travels off the screen, and the line has both ends traveling off the screen.




Label the figures by first selecting the **Text** tool and either clicking on two points on the figure or by clicking on the figure between two points.

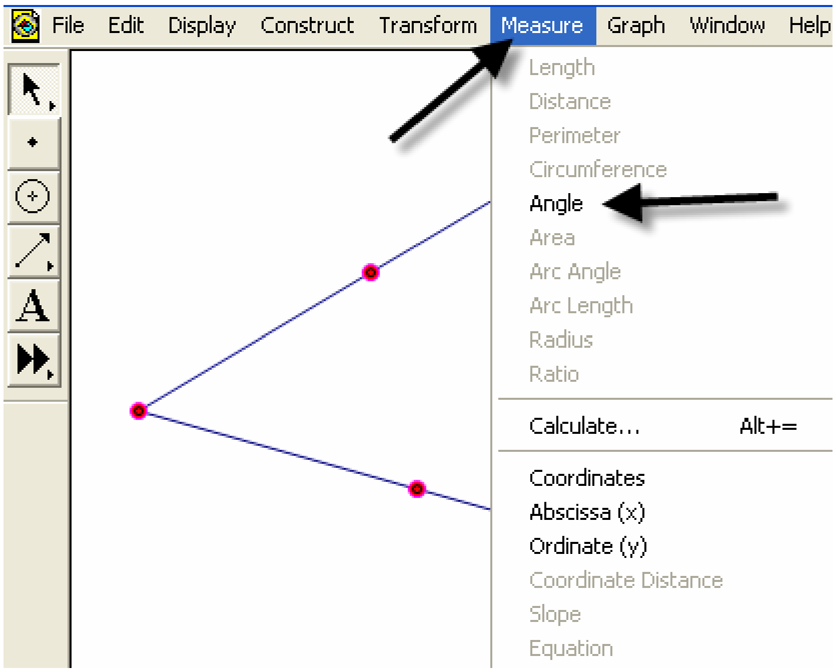



Labels can be changed by double clicking on the label. A box will pop up that provides a place to edit or delete a label.

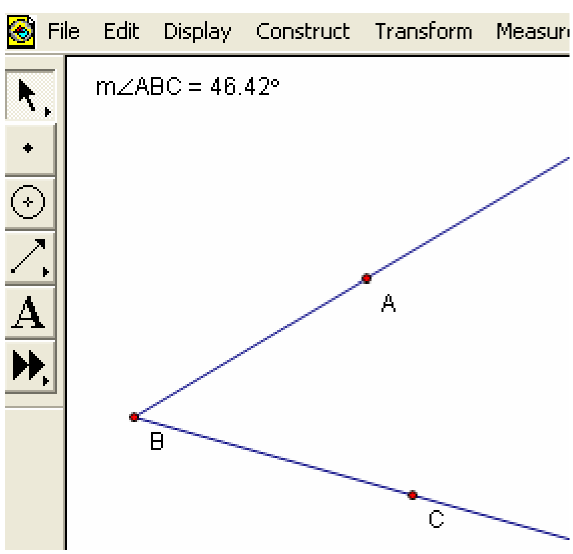


Measuring an Angle

To measure an angle, first highlight it by clicking on three points that could be used to name it, one on a side, then the vertex, and then one on the other side. Use the **Measure** option on the menu bar and select **Angle**.

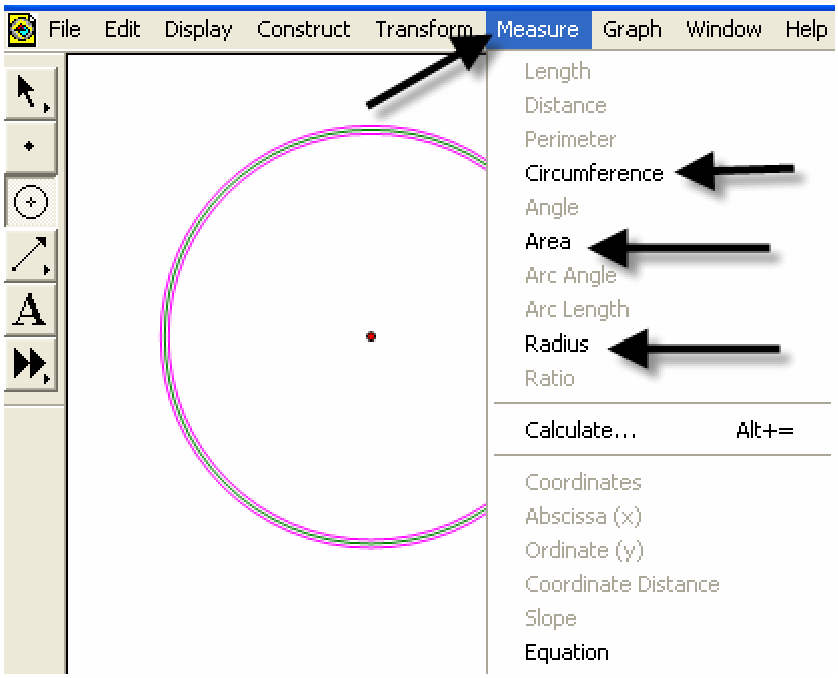



The measurement will appear and the program will automatically label points if they weren't labeled prior to measurement.

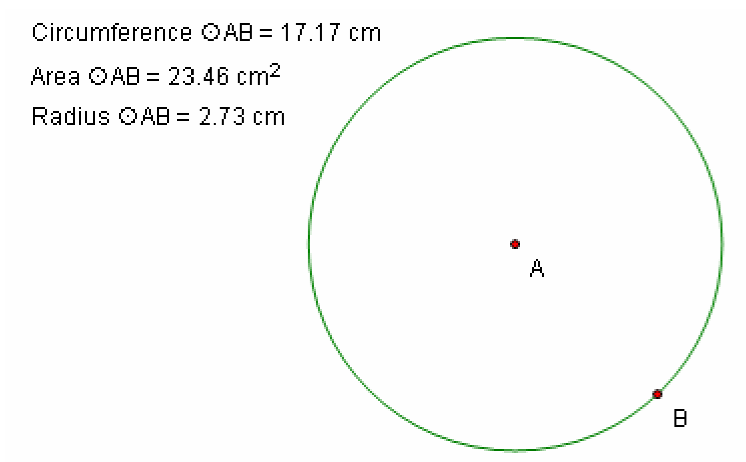


## Measuring a Circle

To measure a circle, first highlight it by clicking on it. Use the **Measure** option on the menu bar and select the measurement desired.

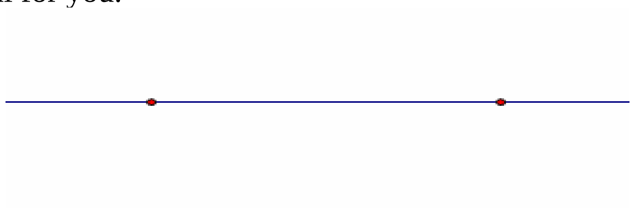



The measurement will appear, and the program will automatically label points if they weren't labeled prior to measurement.

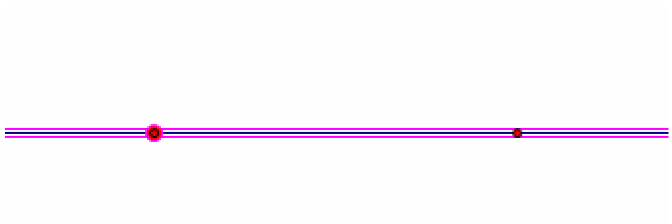


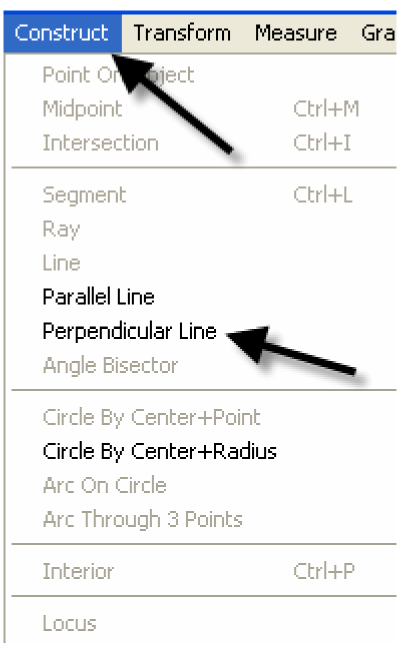
## 30-60-90 Triangle

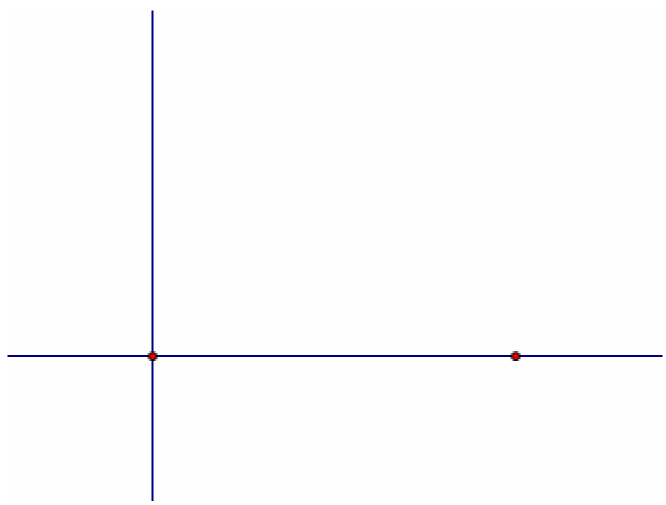
1. Draw a horizontal line. If you hold the shift key before letting of the line, it will make it horizontal for you.



2. Construct a perpendicular line by first highlighting the line and one of the points, then clicking on the Construct menu.

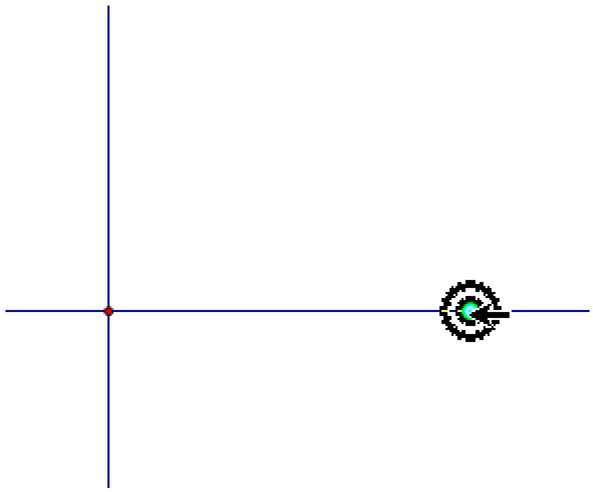




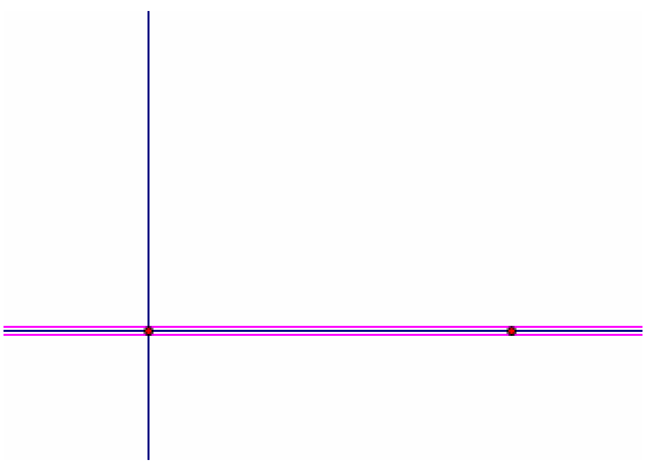



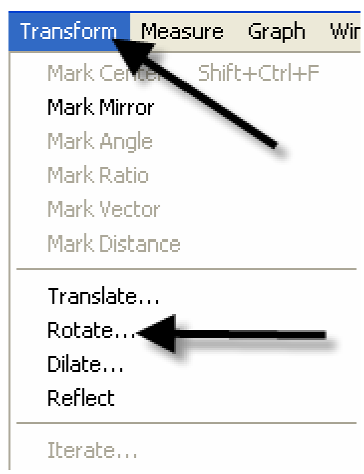
3. Create the third side by rotating the original line to form either a 30 or 60 degree angle.

   a) Mark the point of rotation by double clicking on it. There will be a quick flash of concentric circles around the point as it is marked.

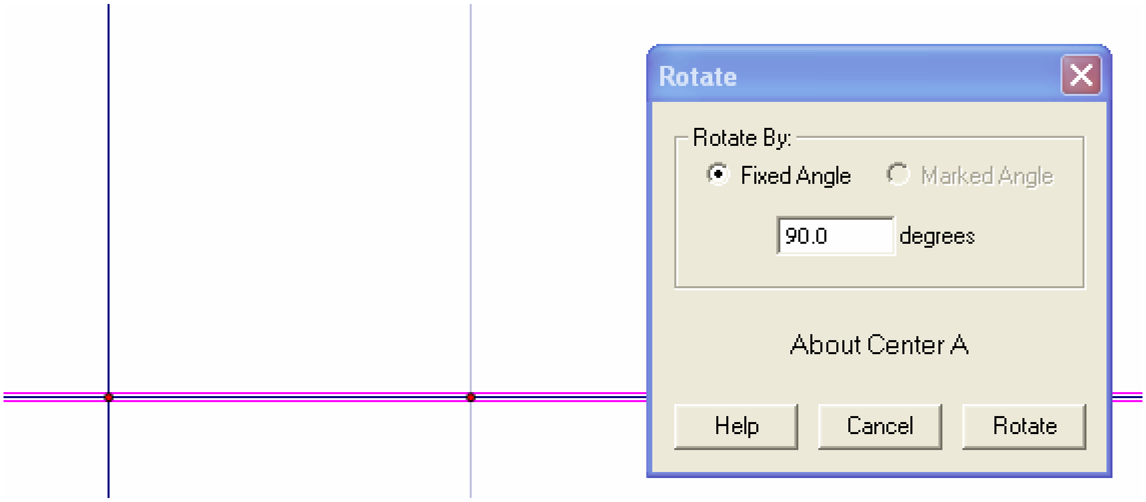


   b) Highlight the original line and use the **Transform** menu with the **Rotate** option.

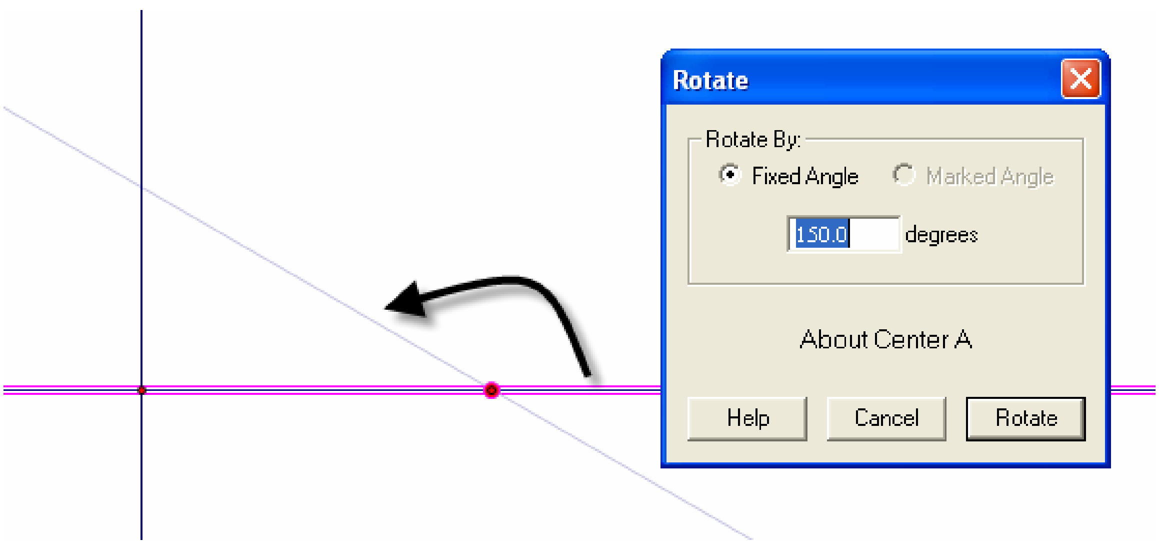




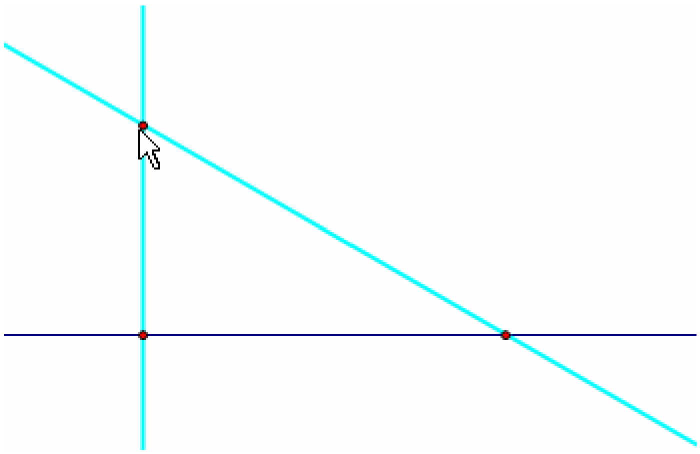
A box will pop up that allows the number of degrees of rotation to be entered. Notice that a shadow of the rotated line appears. This shadow line is a preview of where the rotated line will go. Geometer's Sketchpad has a default of 90 degrees.

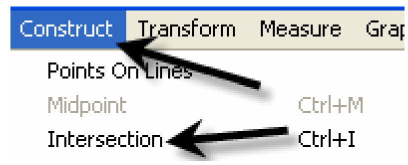



Change the number to a multiple of 30 to get the desired effect. Geometer's Sketchpad treats the point of rotation as the origin and rotates from the side of standard position.



4. Construct a point of intersection where the perpendicular line meets the rotated line either by using the **Point** tool and placing a point or by highlighting both lines and using the **Construct** menu with the **Intersection** option.

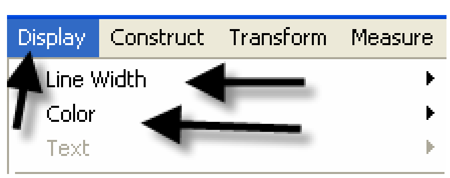


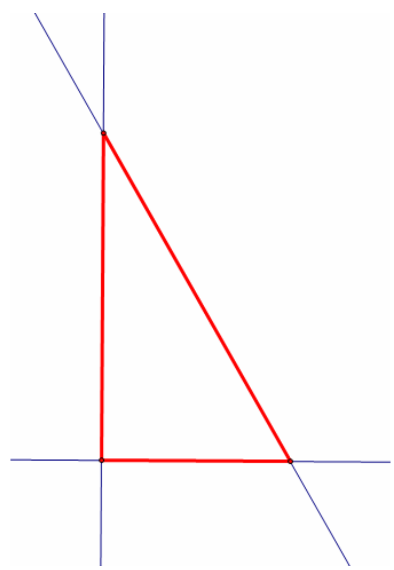



## Construction Clean Up

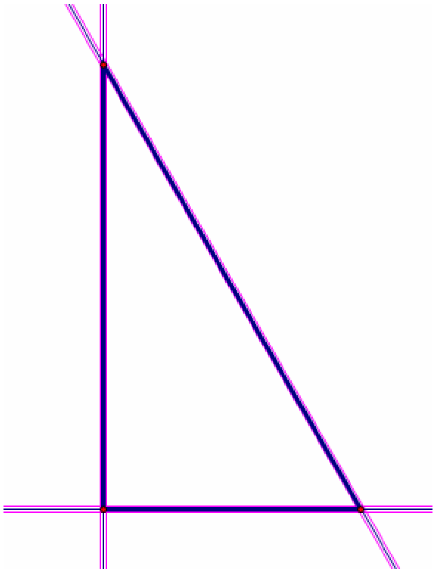
1. To “clean up” a construction, it is often necessary to construct segments, arcs, etc. over the parts of the final product. Follows is an example of a 30-60-90 triangle.

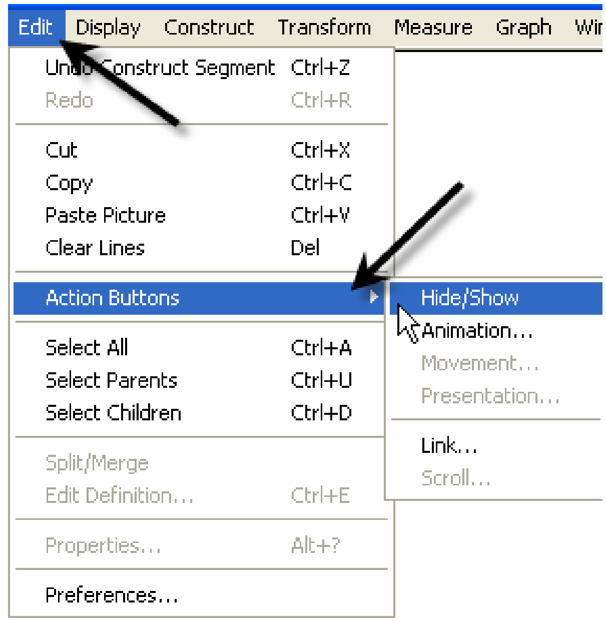
1. Use the **Straightedge tool** to draw segments on top of the sides of the triangle. After drawing the first one, use the **Display** menu to change the **Line Width** and **Color** of the segment. Subsequent segments will then be drawn with this color and thickness.



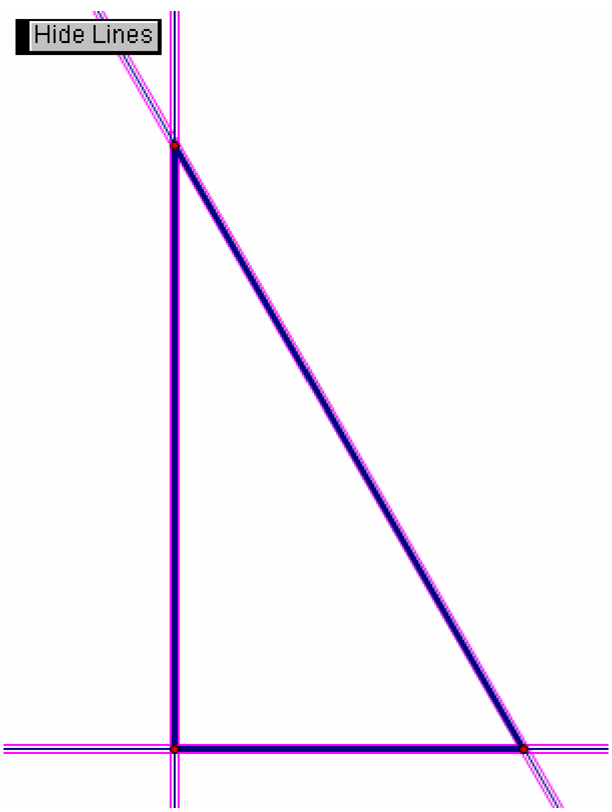



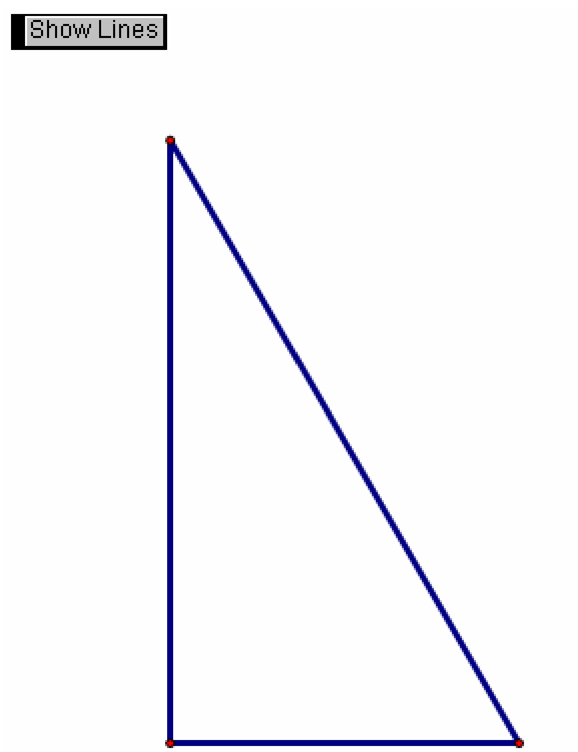
2. To hide construction lines ,create a **Hide/Show** button by highlighting the lines then using the **Edit** menu and the **Action Buttons**⟶**Hide/Show** option**.**





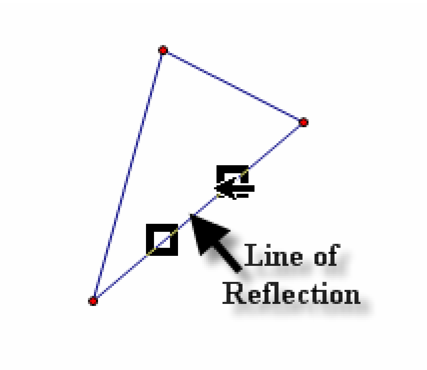

The **Hide Lines** button appears which works as a toggle switch between **Hide** and **Show** when clicked on.

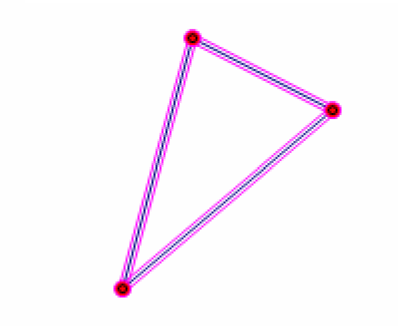


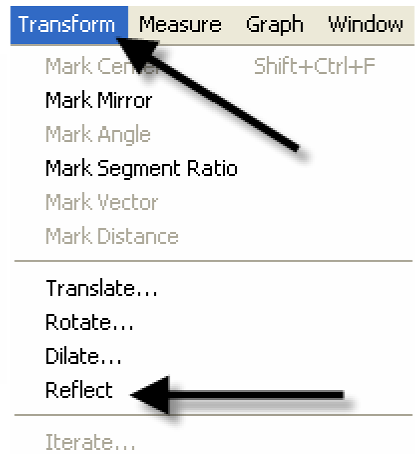



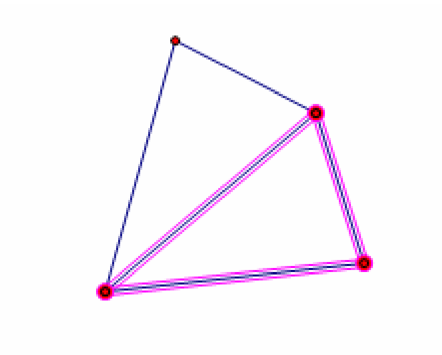
## Reflecting

To transform a figure by reflecting, first mark the line of reflection by double clicking on it. A quick flash of two sets of concentric squares will appear on the line as the marking process is taking place. Next, use the Selection tool to highlight the figure to be reflected. Use the Transform menu and the Reflect option to complete the reflection.

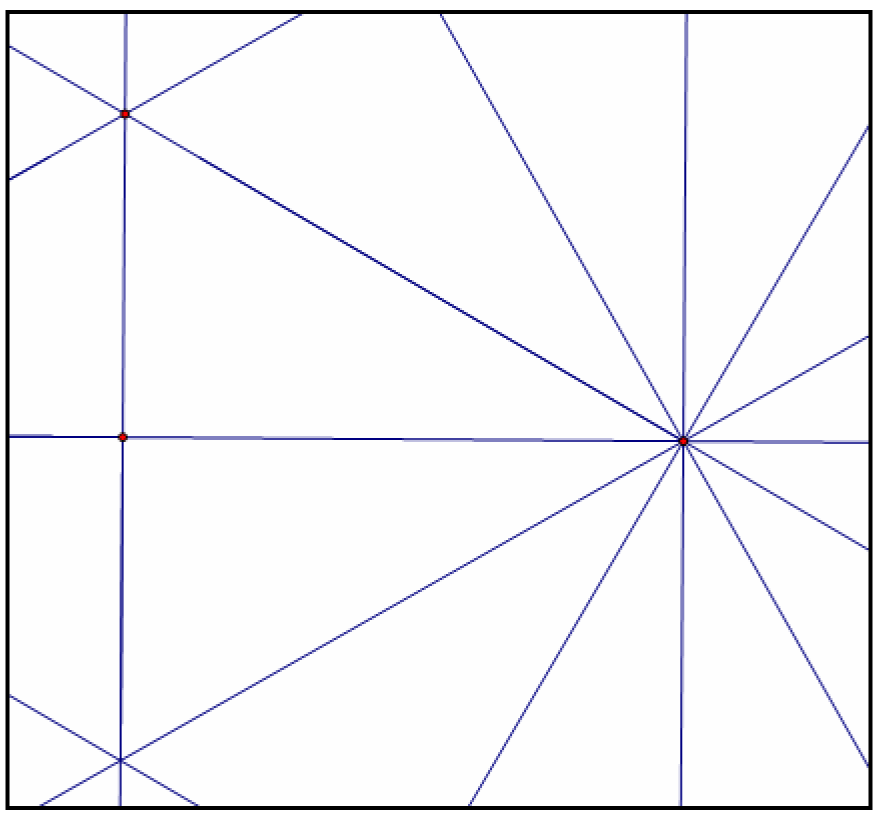




30-60-90 triangle tessellation by reflection

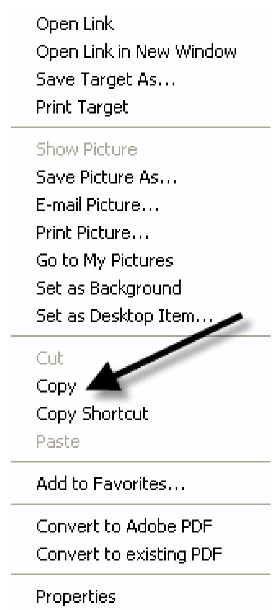


# Explore Geometric Properties in the World

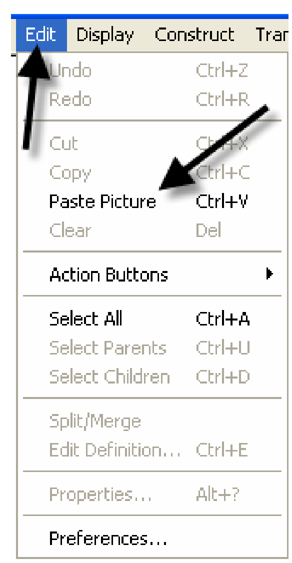
## Importing Pictures from the Internet

1. Position the cursor on the picture.

2. RIGHT click on your mouse and select COPY.



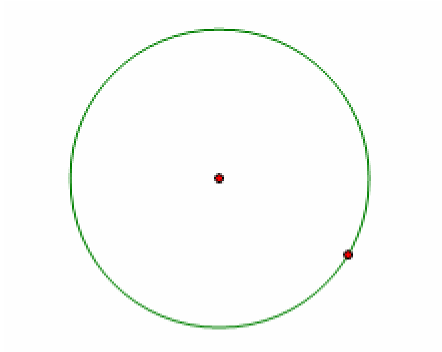

3. Return to your sketch in Geometer's Sketchpad. Use the **Edit** and **Paste Picture** options from the **Menu** bar.



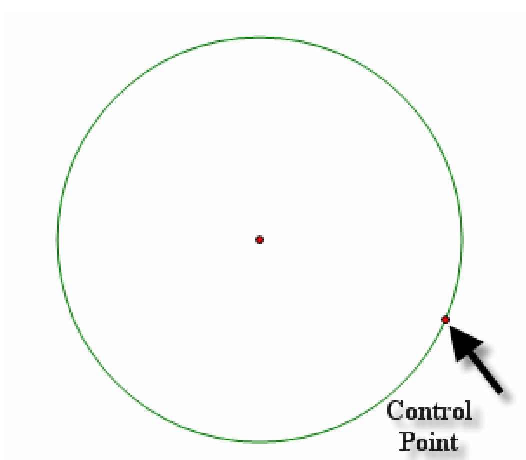
# Dome Floor Dilemma

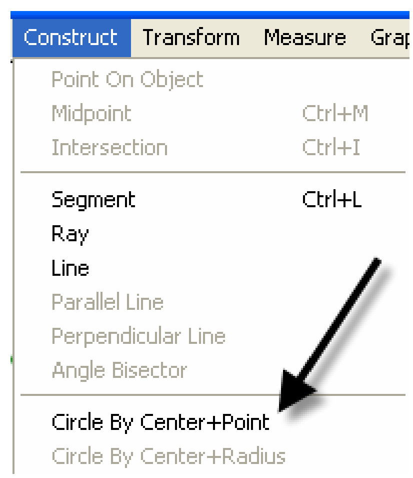
## Sector Construction

1. Circle Construction
   a) Use the **Compass** tool to construct a circle.

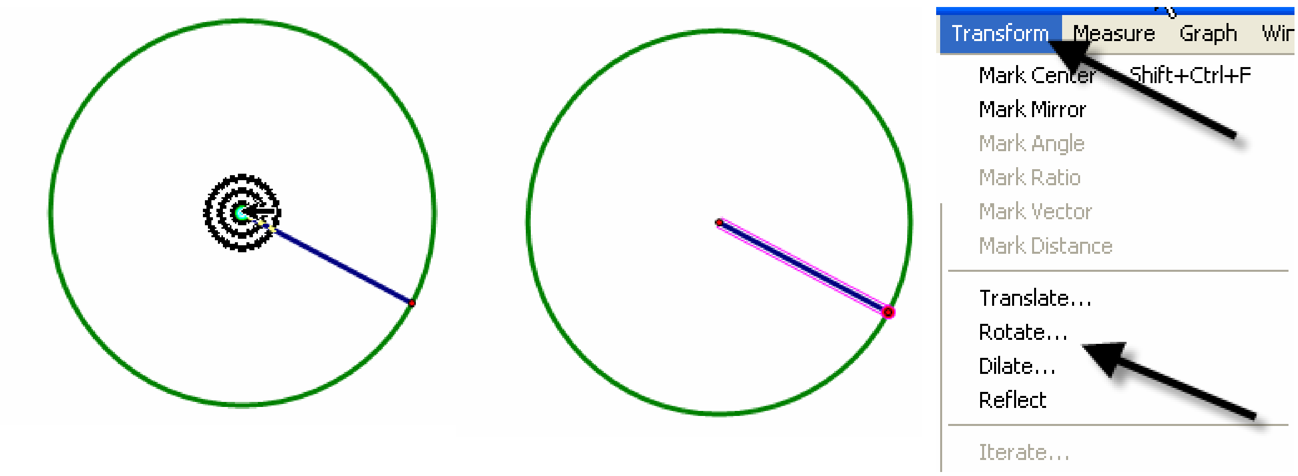


   b) Use the **Segment** tool or the **Construct** menu to construct a radius of the circle. Connect the radius from the center to the "control" point on the circle. To use the **Construct** menu, first select the center and the point on the circle, then use **Construct** with the **Circle By Center+Point** option.

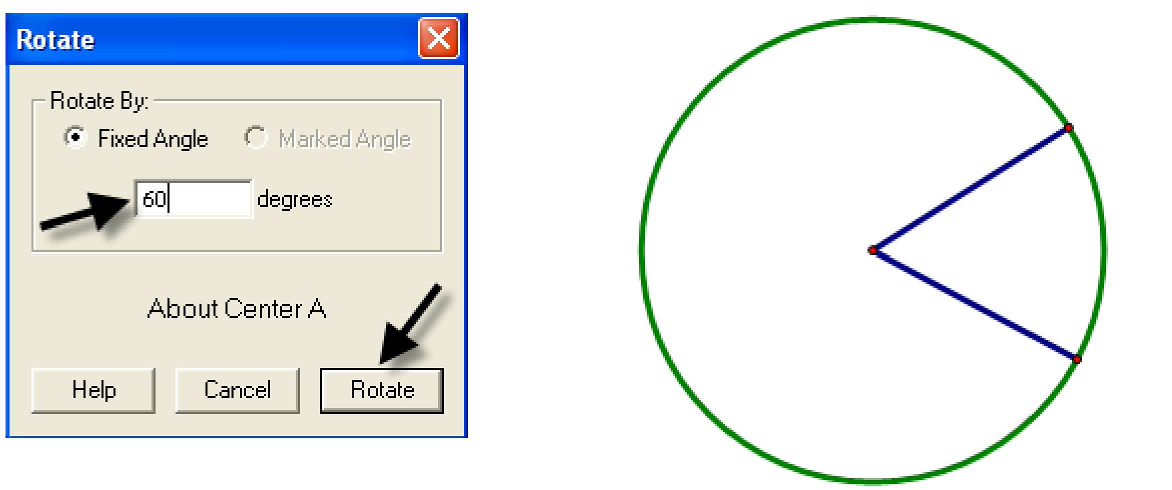


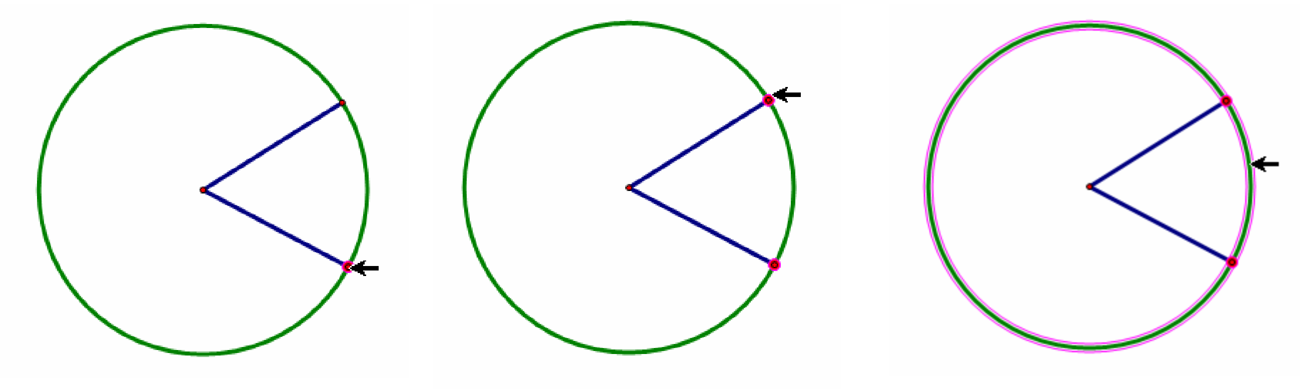
2. Rotate Radius
   a) To rotate the radius and its endpoint that lies on the circle, first mark the point of rotation by double clicking on the center of the circle. You will see a quick flash of concentric circles as the "marking" takes place, then highlight the radius and the endpoint that lies on the circle. Use the **Transform** menu and choose the **Rotation** option.

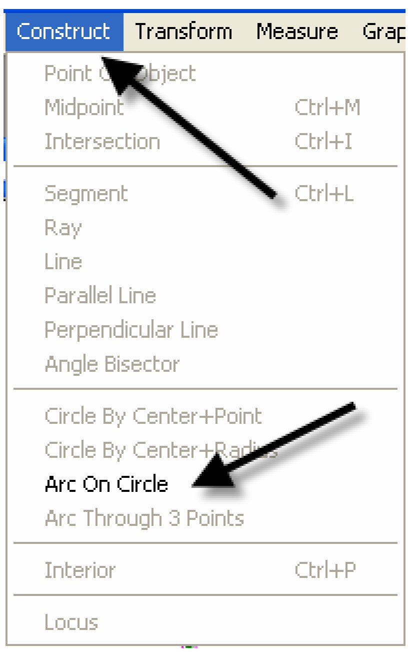


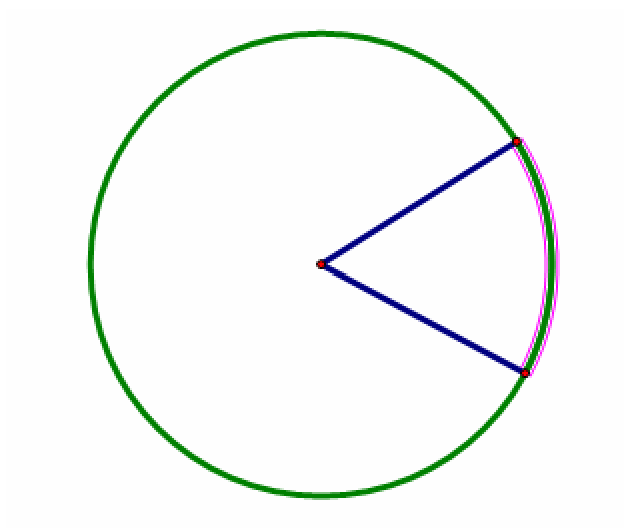

   b) A box will pop up that allows the desired degrees of rotation to be entered. For this construction, enter 60°, then click on **Rotate**.



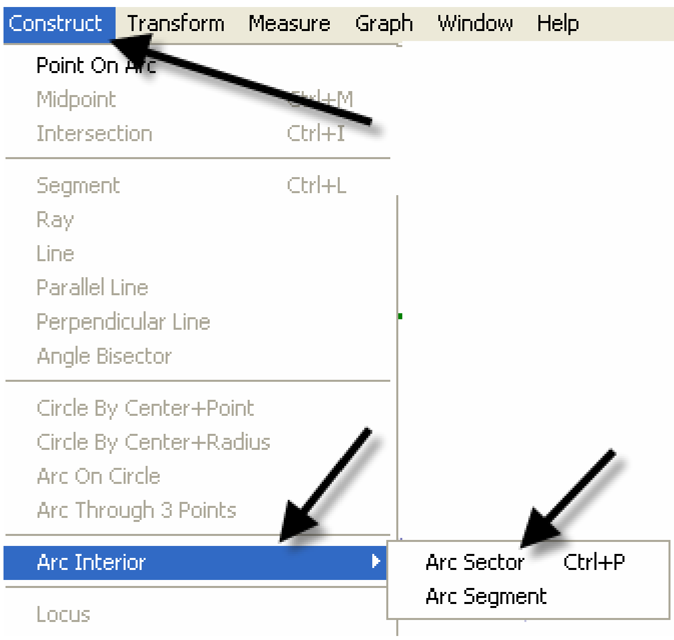
3. Construct Intercepted Arc
   To construct the intercepted arc of the sector, select the endpoints of the radii in a counter clockwise direction. Then select the circle and use the **Construct** menu to construct **Arc on Circle**.

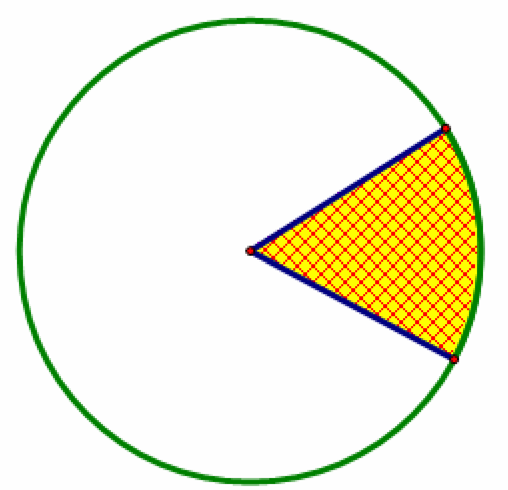




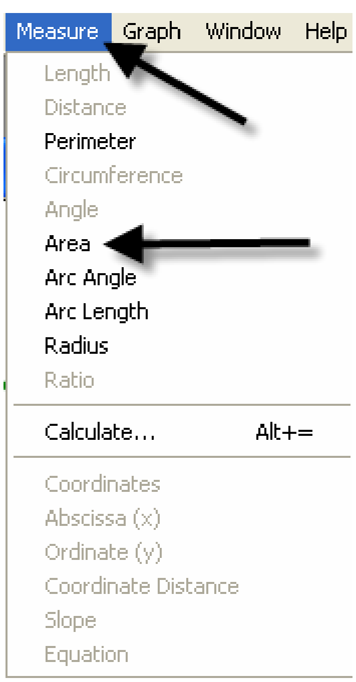



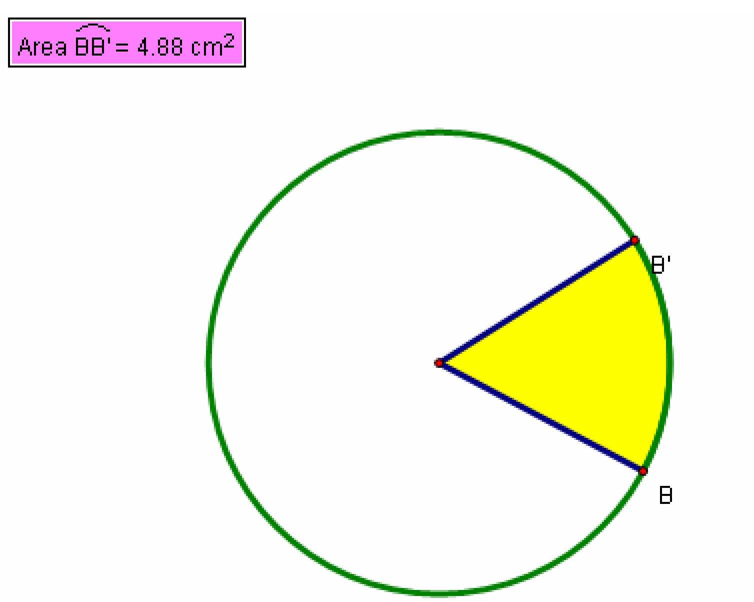
4. Construct Arc Sector
   While the newly constructed arc is still highlighted, create the arc sector interior by using the **Construct** menu with the options, **Arc Interior** then **Arc Sector**.



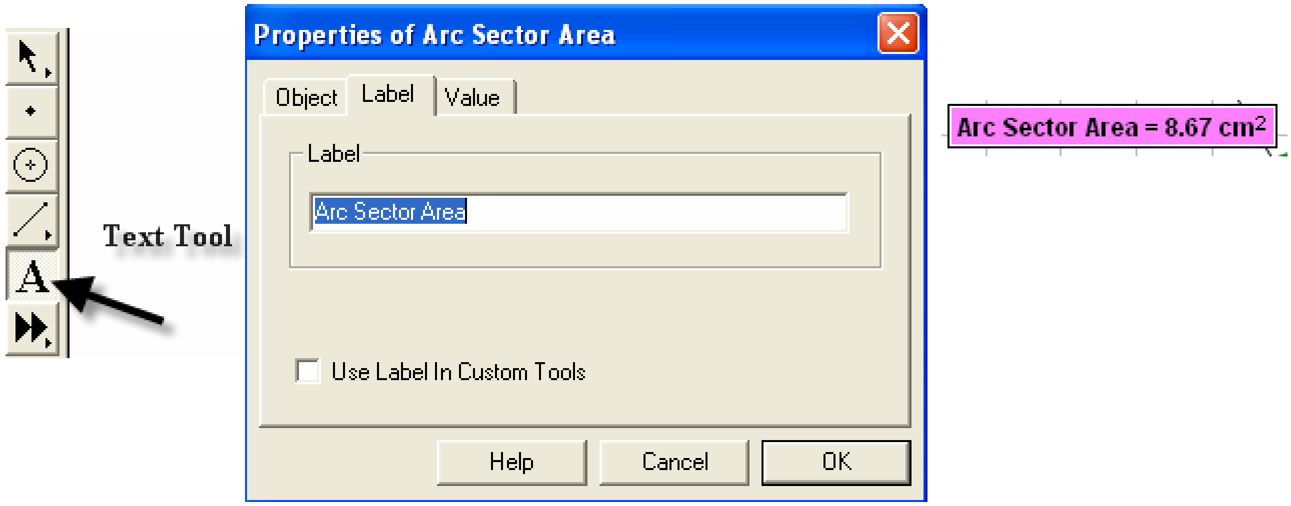



5. Measure Area and Length
   a) To measure the area of the sector, highlight the sector by clicking in it, then use **Measure** from the menu bar with the **Area** option. A highlighted labeled box will appear. Be sure to un-highlight the box by clicking in any white space on the sketch.

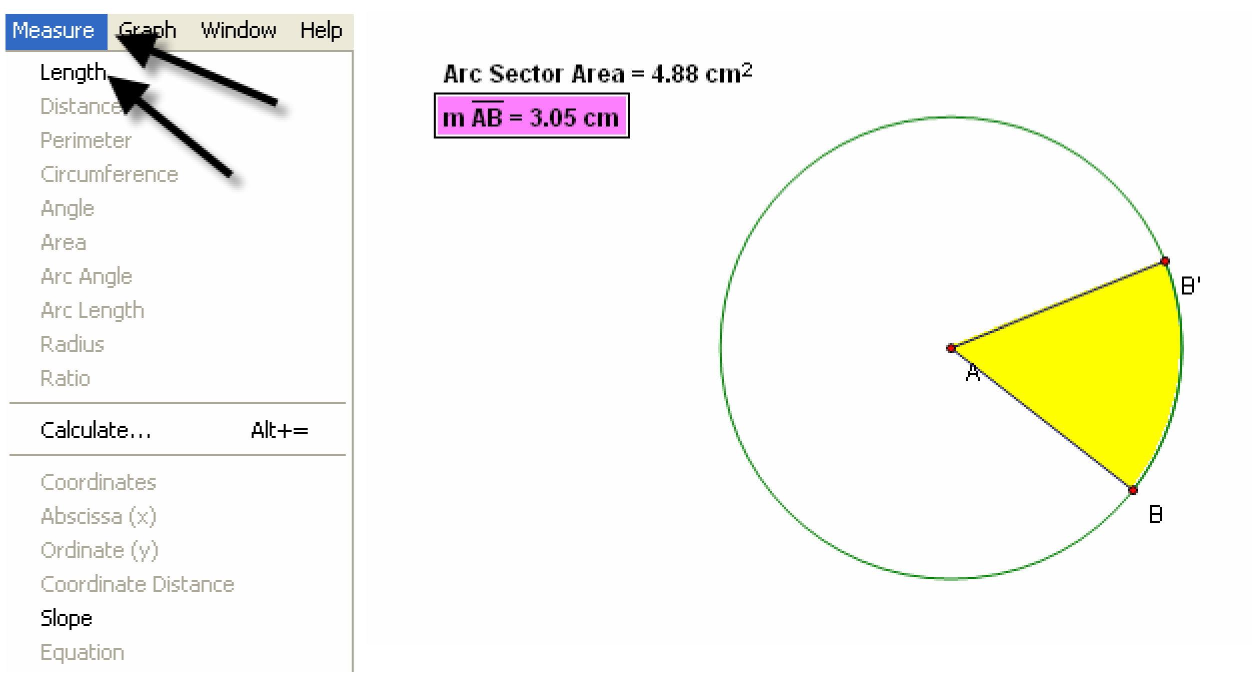


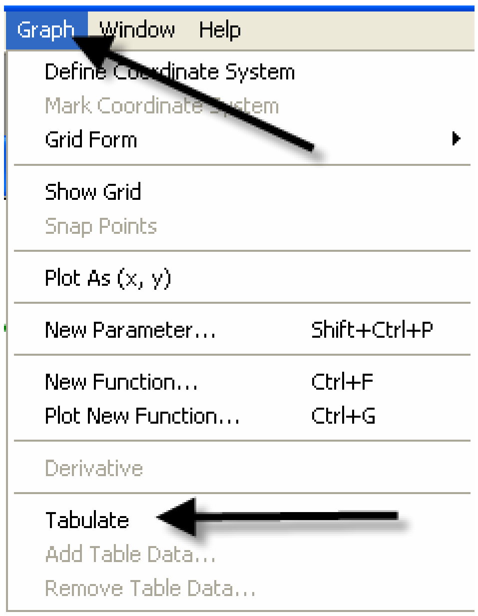
b) Change the label of the Area to read **Arc Sector Area** by first selecting the Text tool, then double clicking on the Area label and typing in the new label in the pop-up window.

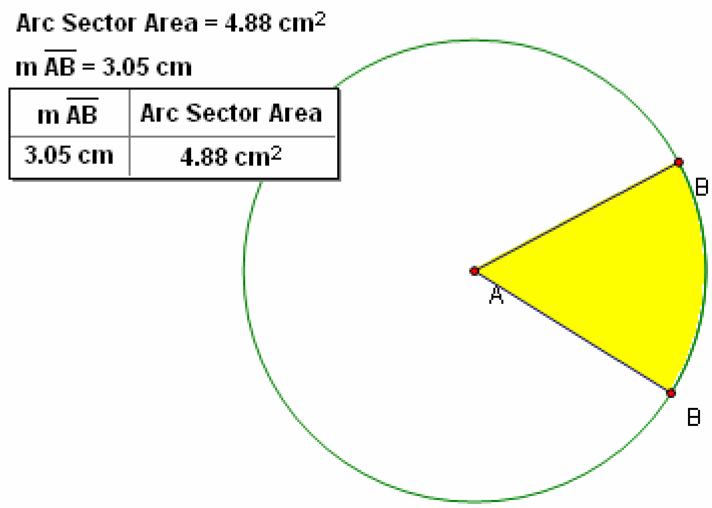



c) To measure the length of the radius, first highlight any radii, then use the Measure menu with the Length option.  Again a labeled highlighted box will appear.



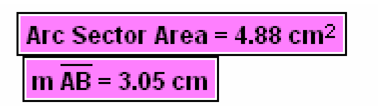
6. Create a Table
   To create a table to explore the relationship between the length of the radius and the area of the sector, highlight their measures respectively. Then use **Graph** from the menu bar with the **Tabulate** option. A labeled highlighted table will pop up on the sketch.

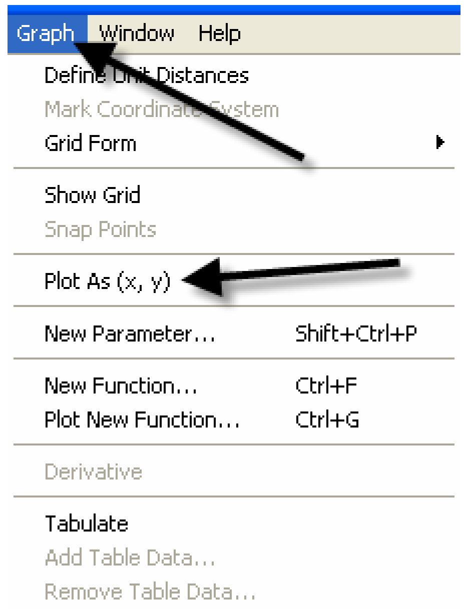


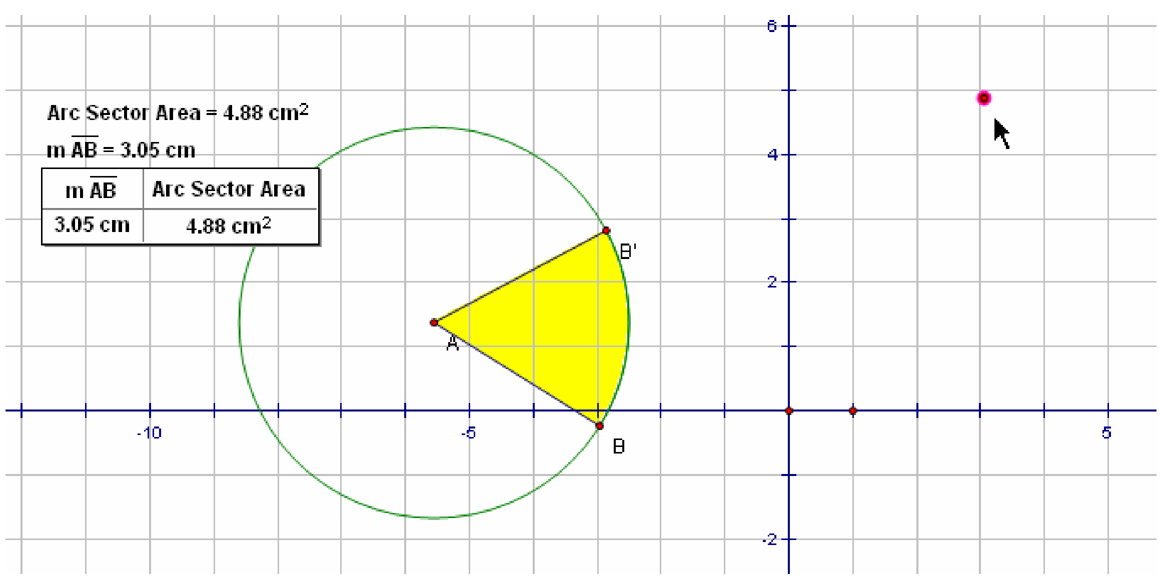
7. Plot Point

a) To plot the point represented in the table, again highlight the measure values in the respective order: length of radius then area of sector. Use **Graph** from the menu bar with the **Plot as (x,y)** option.

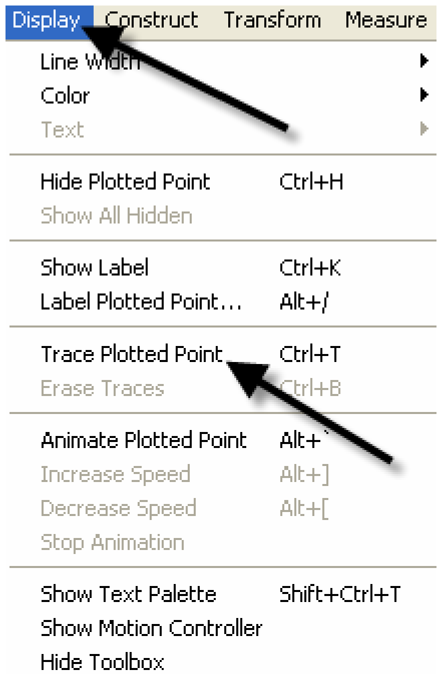



b) The coordinate grid appears with the highlighted point on the grid.

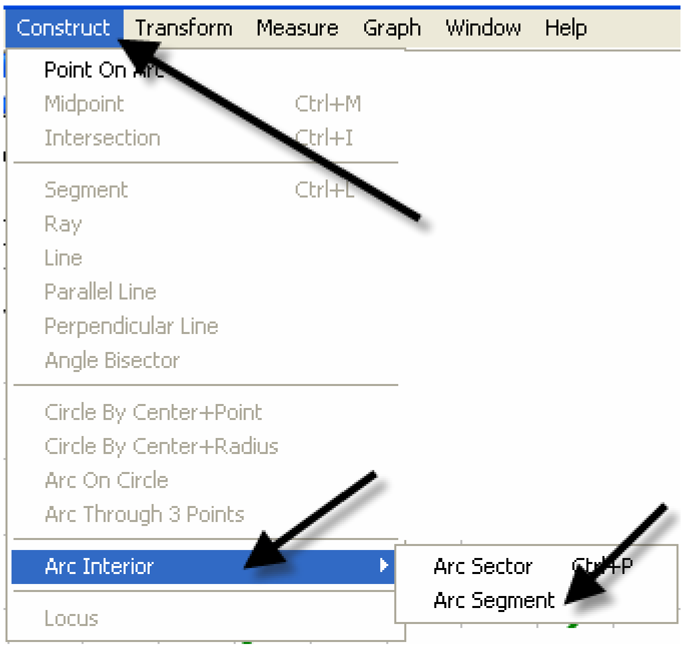


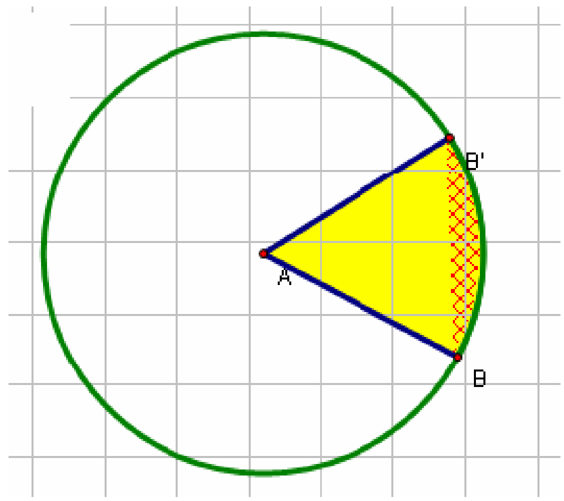
c) To turn on the trace option, highlight the plotted point and use **Display** from the menu bar with the **Trace Plotted Point** option. This will allow any new points added to the table to be plotted automatically.



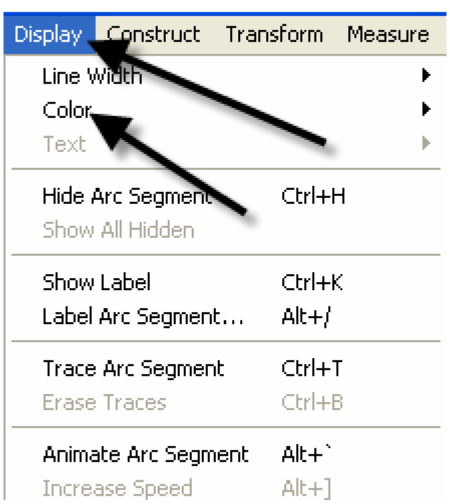
# The Arc Segment Construction

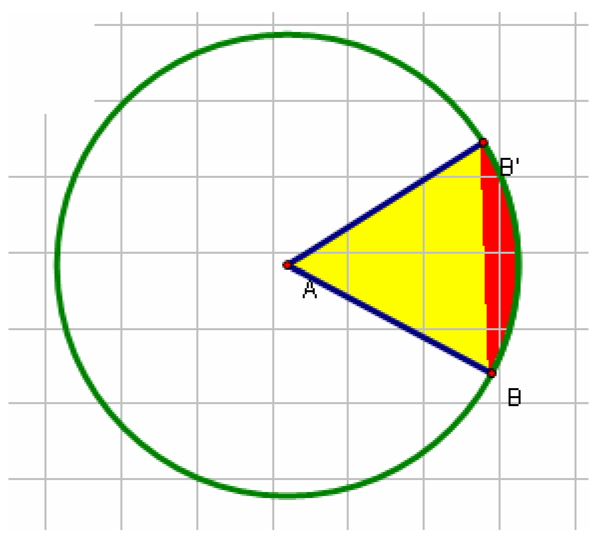
1. Construct Arc Segment
   To construct the arc segment, first select the arc by double clicking on the arc. Then use **Construct** from the menu bar with the **Arc Interior,** then **Arc Segment** options.



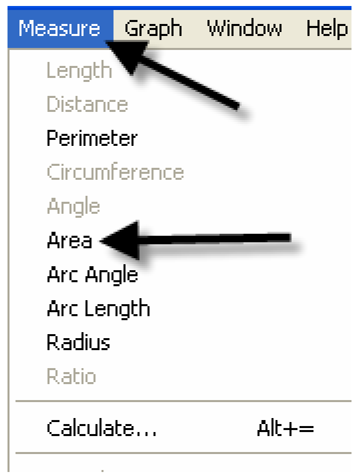



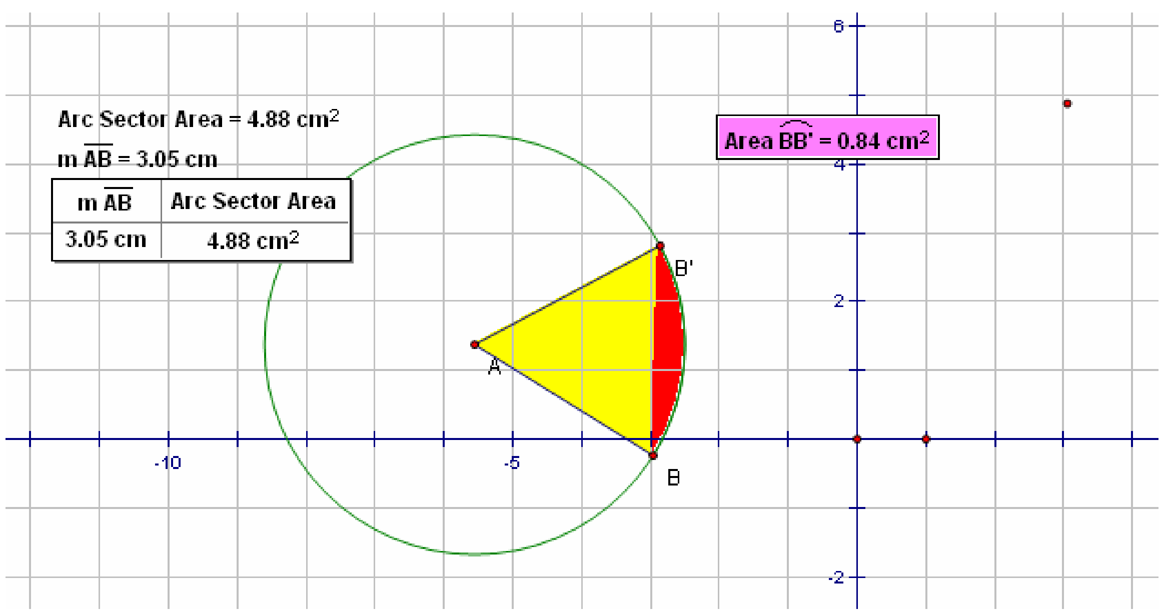

2. Change the color of the segment by using Display with the Color option.



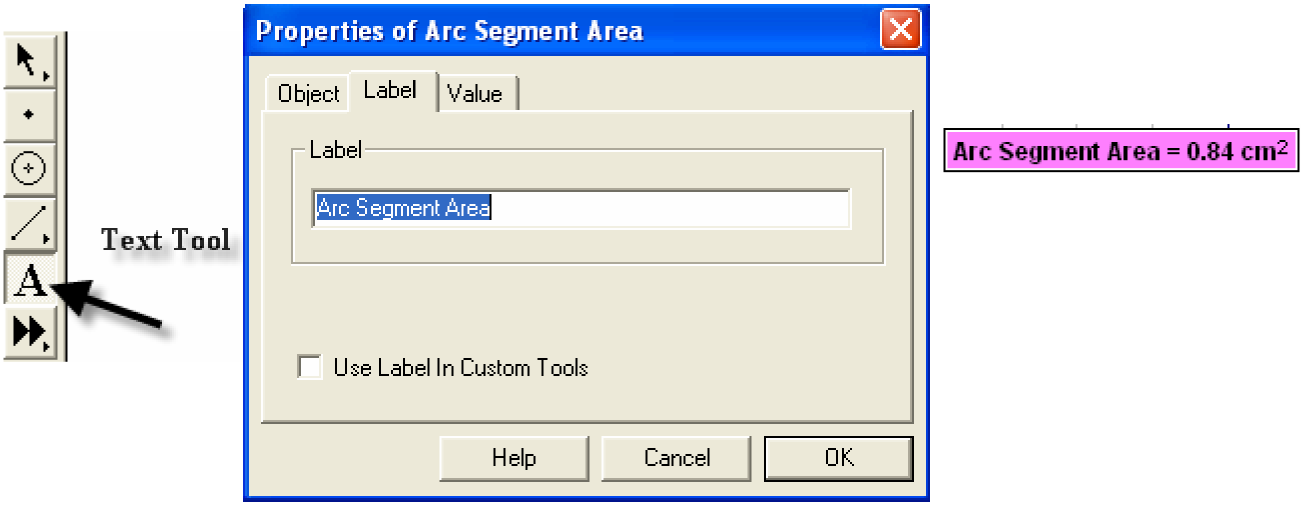



3. Measure Arc Segment Area
   a) To measure the area of the arc segment, highlight it by clicking in the interior of the arc sector, then use **Measure** from the menu bar with the **Area** option. With the measurement still highlighted, you may move it to a new location on the sketch for easier viewing. Remember to click in any blank space to deselect the measurement.

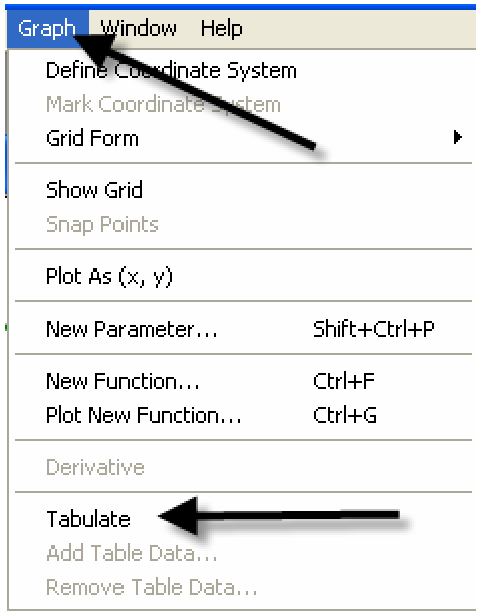


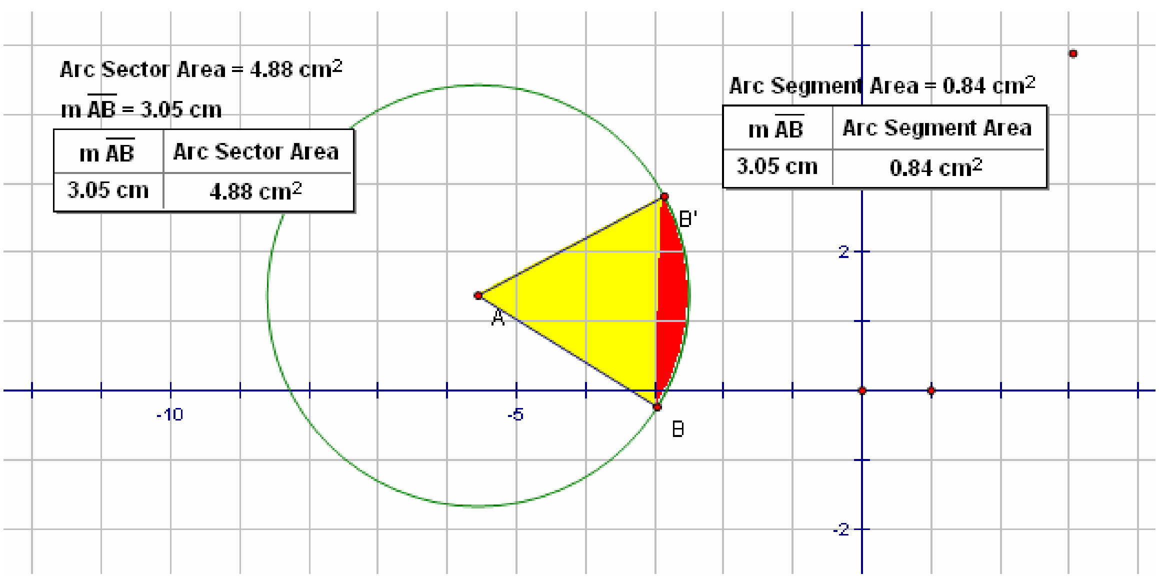



b) Change the label of the Area to read **Arc Segment Area** by first selecting the Text tool, then double clicking on the Area label and typing in the new label in the pop-up window.

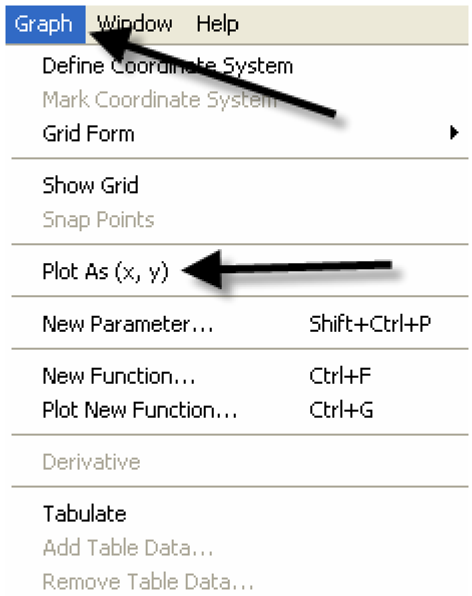


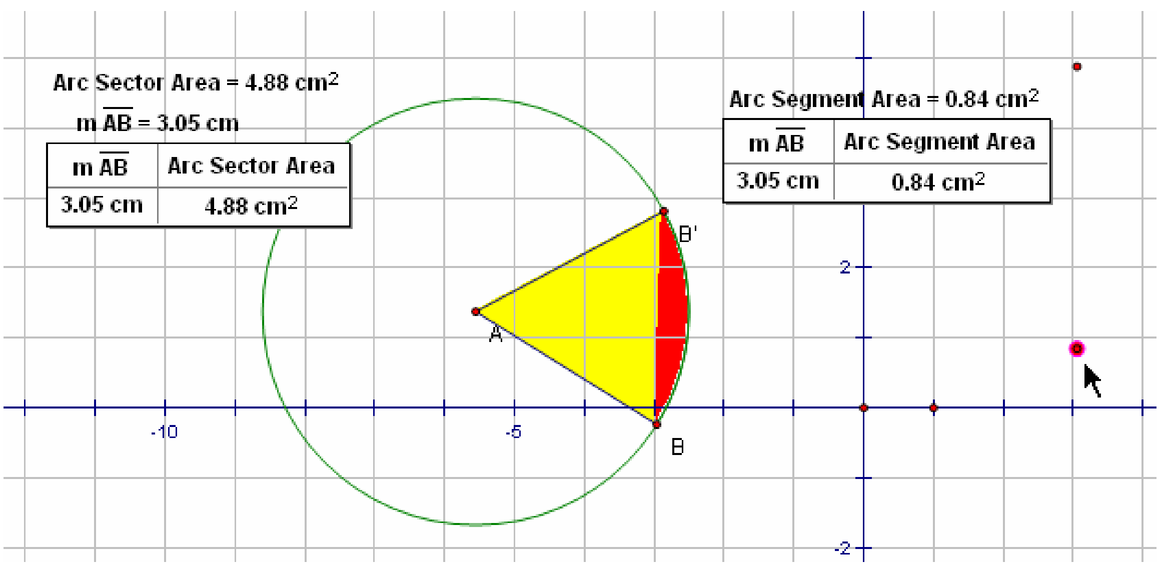
4. Create Table
   To create a table to explore the relationship between the length of the radius and the arc segment area, highlight their measures respectively. Then use Graph from the menu bar with the Tabulate option. A labeled highlighted table will pop up on the sketch.



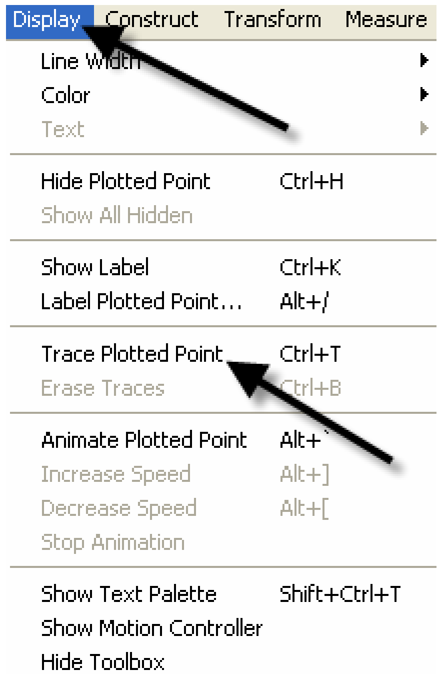



5. Plot and Trace Point
   a) To plot the point represented in the table, highlight the measure values in the respective order: length of radius, then area of the arc segment. Use **Graph** from the menu bar with the **Plot as (x, y)** option.



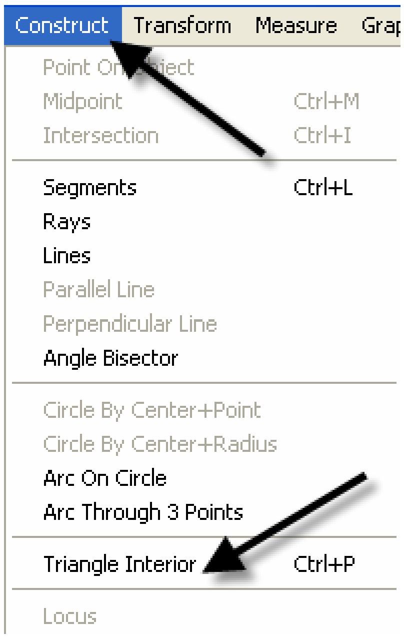
b) To turn on the trace option, highlight the plotted point and use Display from the menu bar with the Trace Plotted Point option. This will allow any new points added to the table to be plotted automatically.

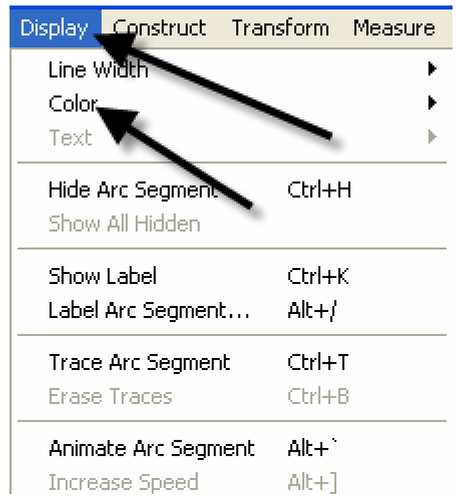


## The Triangle Construction

1. Construct Triangle Interior

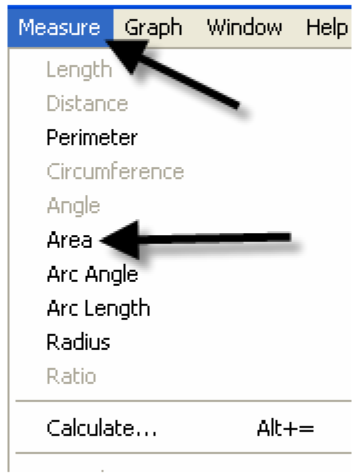
   To construct the triangle interior, first select the vertices of the triangle, then use **Construct** from the menu bar with the **Construct Triangle Interior** option. The color will change to the last color selected, so use **Display** from the menu bar with the **Color** option to make the triangle a different color than the arc segment.

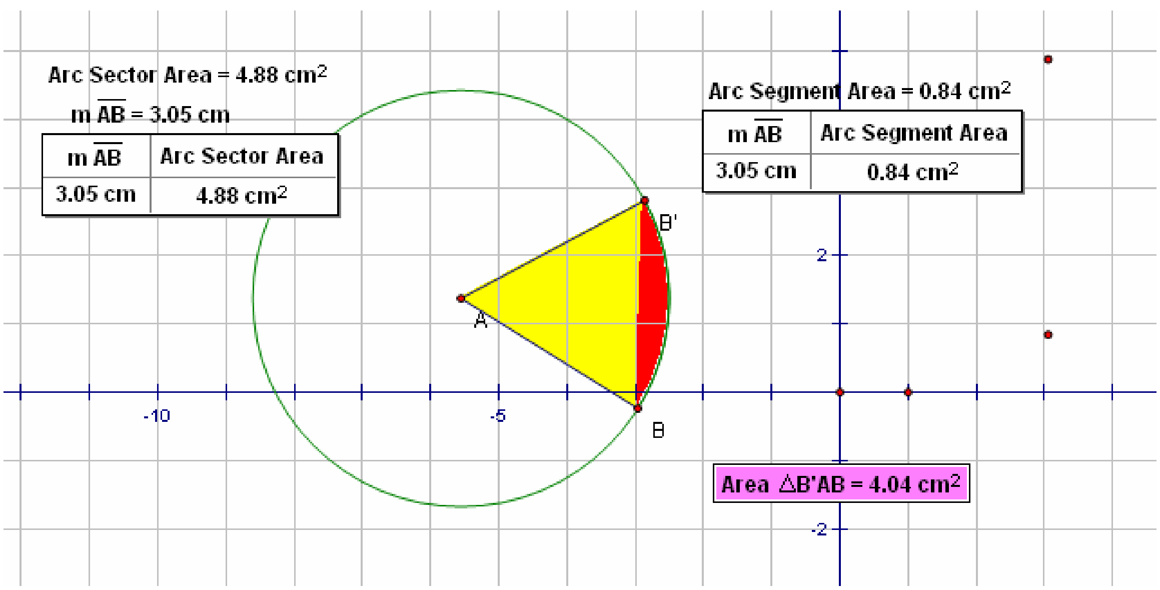


2. Measure Triangle Area

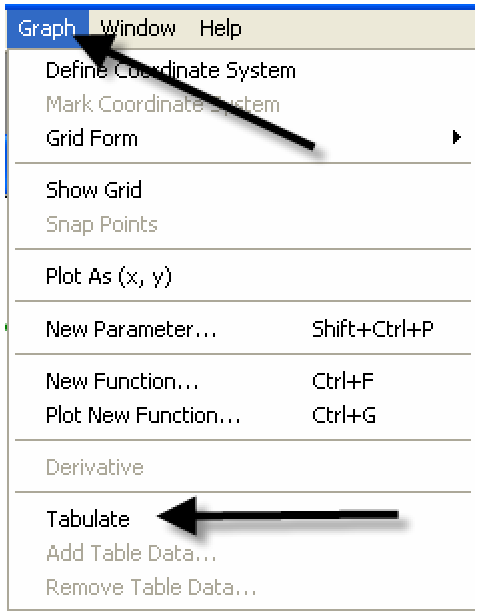
To measure the area of the triangle, click the triangle interior (may require double clicking to keep from selecting the entire sector) and use **Measure** from the menu bar with the **Area** option. With the measurement still highlighted, you may move it to a new location for easier viewing.

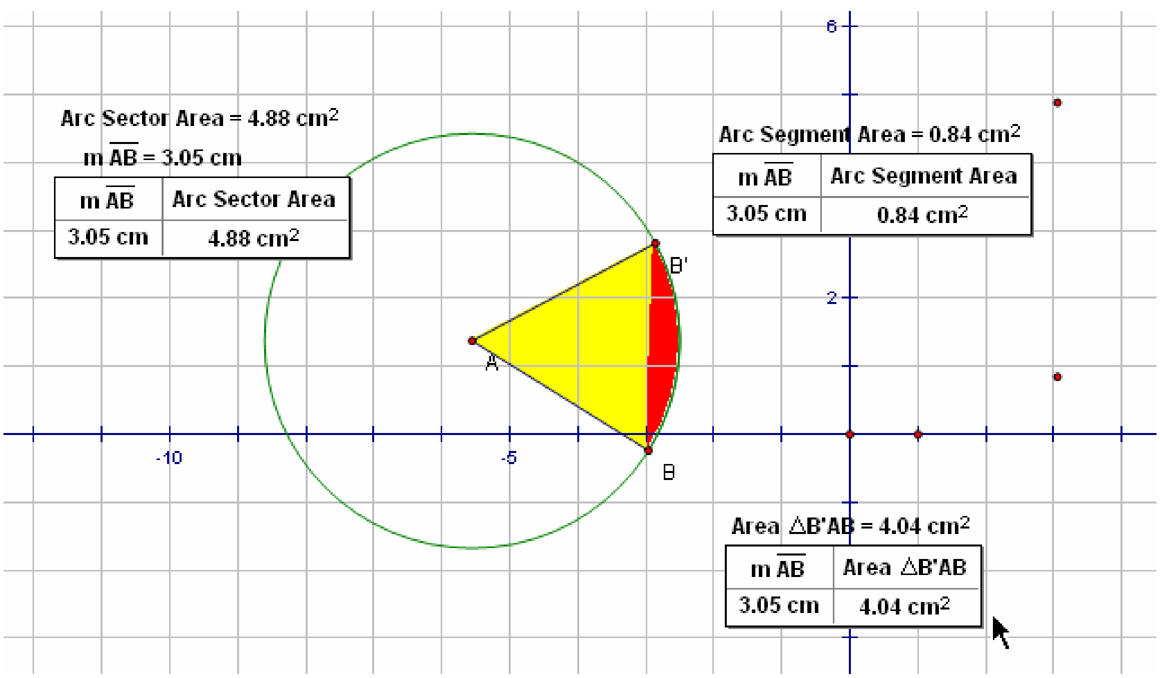


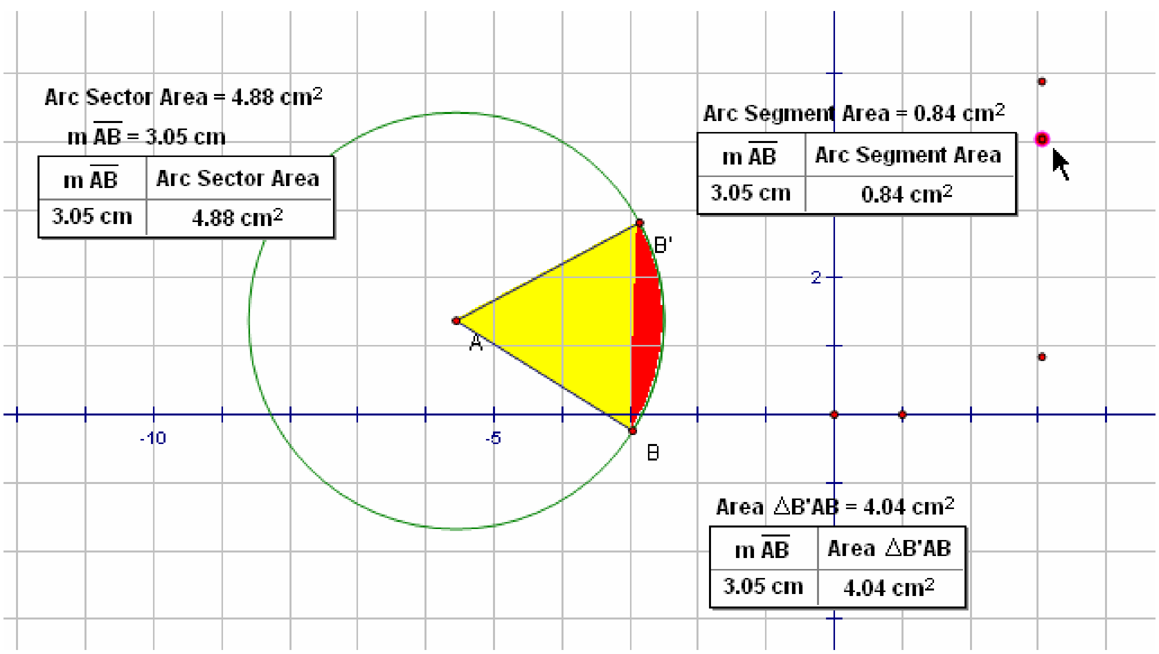
3. Create Table

To create the table to explore the relationship between the length of the radius and the area of the triangle, highlight both measures respectfully. Use **Graph** from the menu bar with the **Tabulate** option.

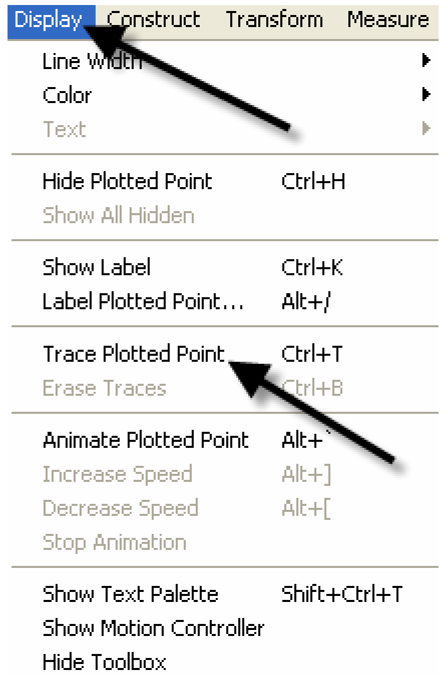


4. Plot and Trace Point
   a) To plot the point in the table, highlight the measures again: length of the radius and area of the triangle. Use **Graph** from the menu bar with the **Plot as (x, y)** option.

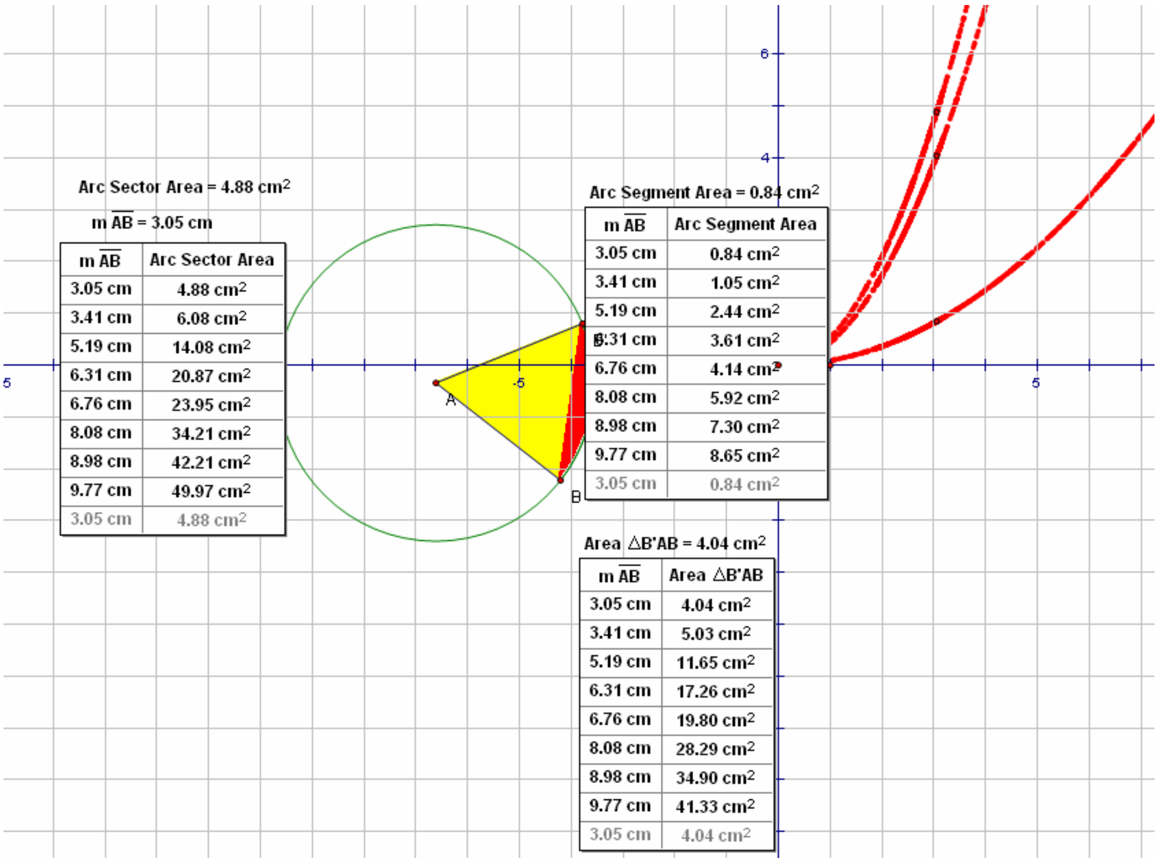
b) To trace the plotted point, highlight the plotted point and use **Display** from the menu bar with the **Trace Plotted Point** option. This will allow any new points added to the table to be plotted automatically.

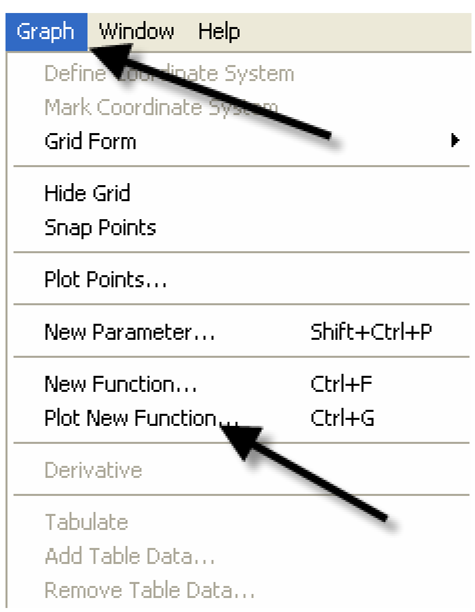


# Dome Floor Dilemma—Function Rule Verification

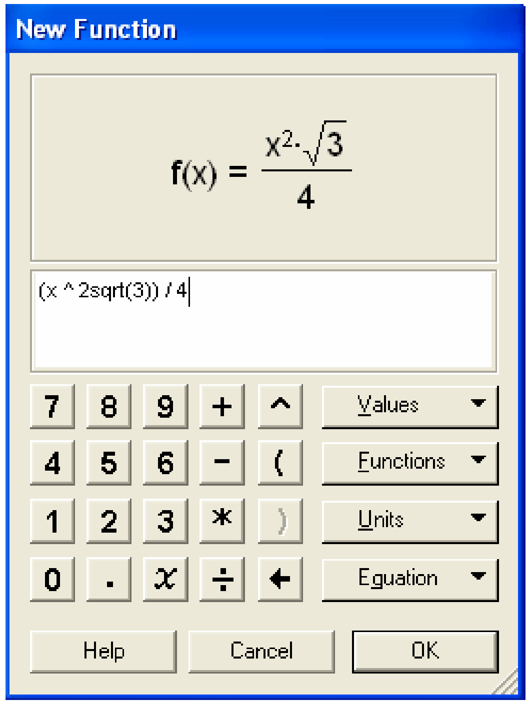
Function Rule Verification—Geometer's Sketchpad.

1. Using the existing sketch from the Dome Floor Dilemma Exploration, use **Graph** with the **Plot New Function** option.

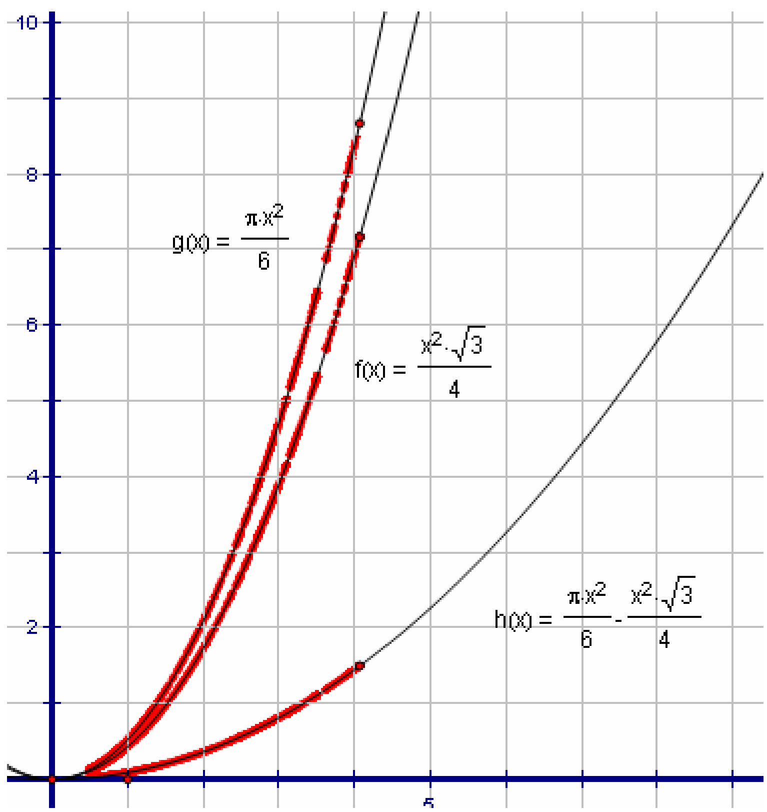


A **New Function** box will pop up, allowing the function rule to be entered. Then click on the **OK** button.



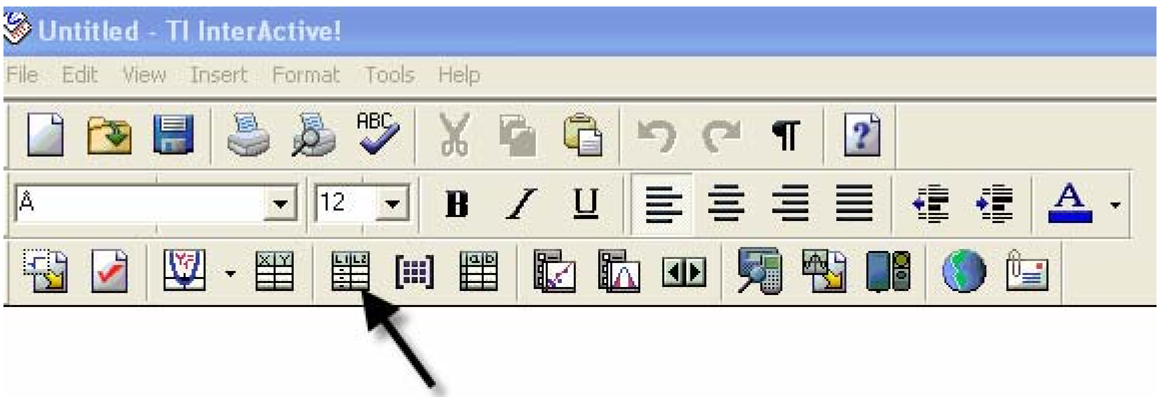

The function will then graph on the coordinate grid. If it is right, it will graph directly on top of its corresponding points, thus verifying the rule. Repeat this process for all function rules.

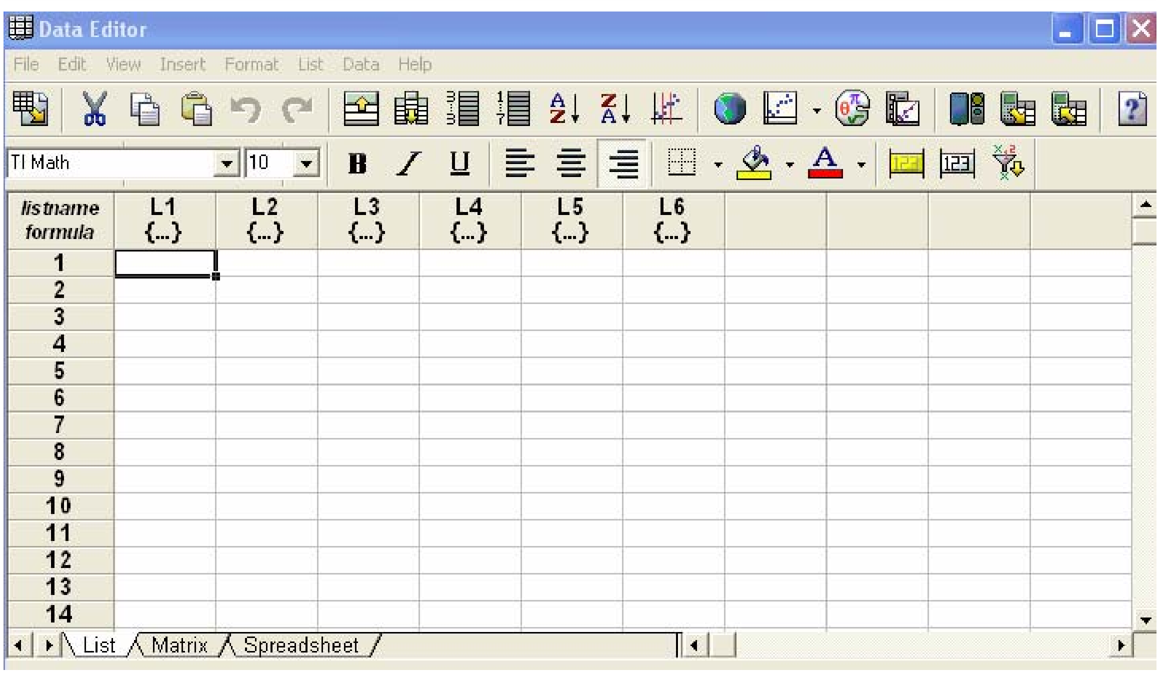


Function Rule Verification—TI-Interactive.

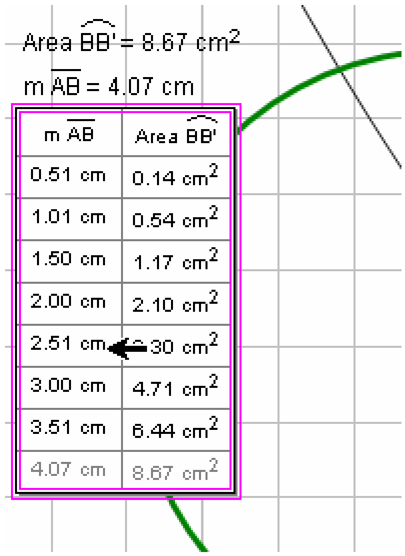
1. With your sketch in Geometer's Sketchpad still open, open TI-Interactive by pressing on the TI-Interactive icon.

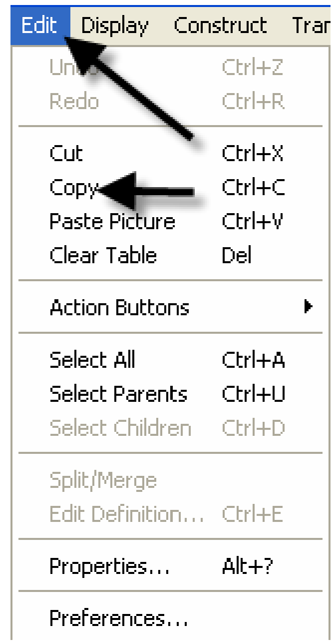
2. Click on the **List Icon** to get the **Data Editor** screen.



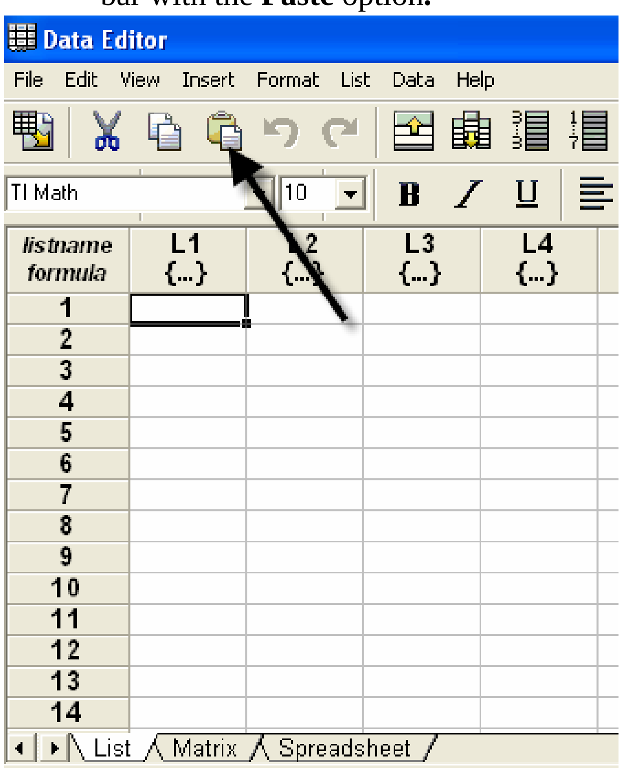



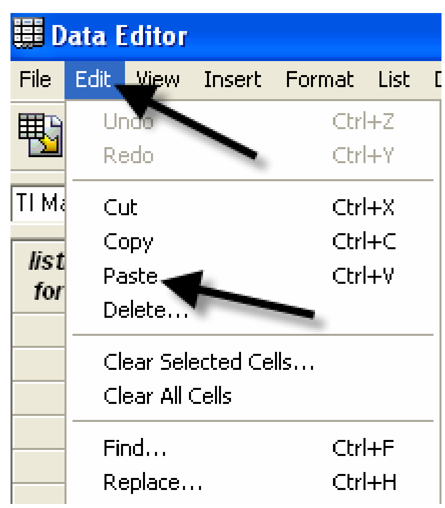
3. Select one of the tables from your sketch in **Geometer's Sketchpad** by clicking on it. Use **Edit** from the menu bar with the **Copy** option.



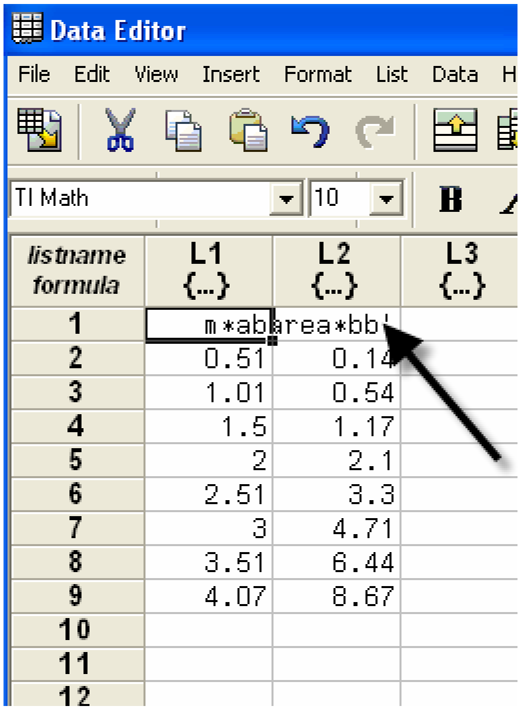



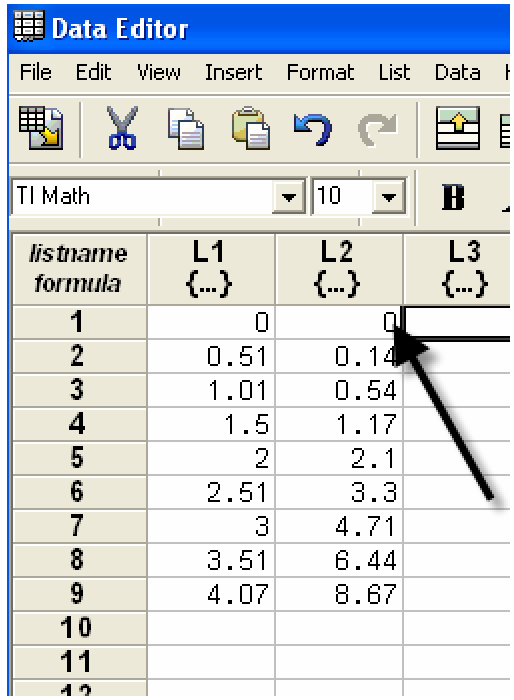

4. Return to the **Data Editor** and click on the **Paste** icon or use **Edit** from the menu bar with the **Paste** option**.**



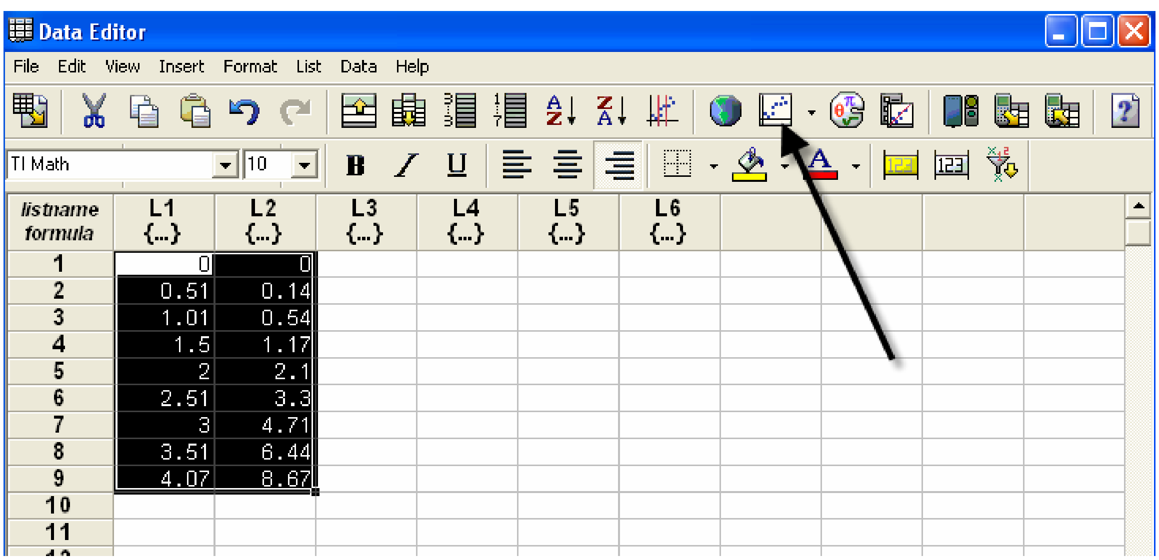



5. Notice that the table headings also transfer. Delete the non-numerical data; and if you like, enter the point of origin in its place.

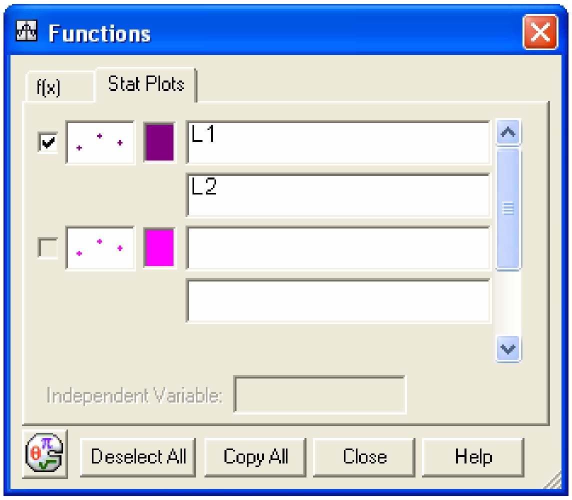


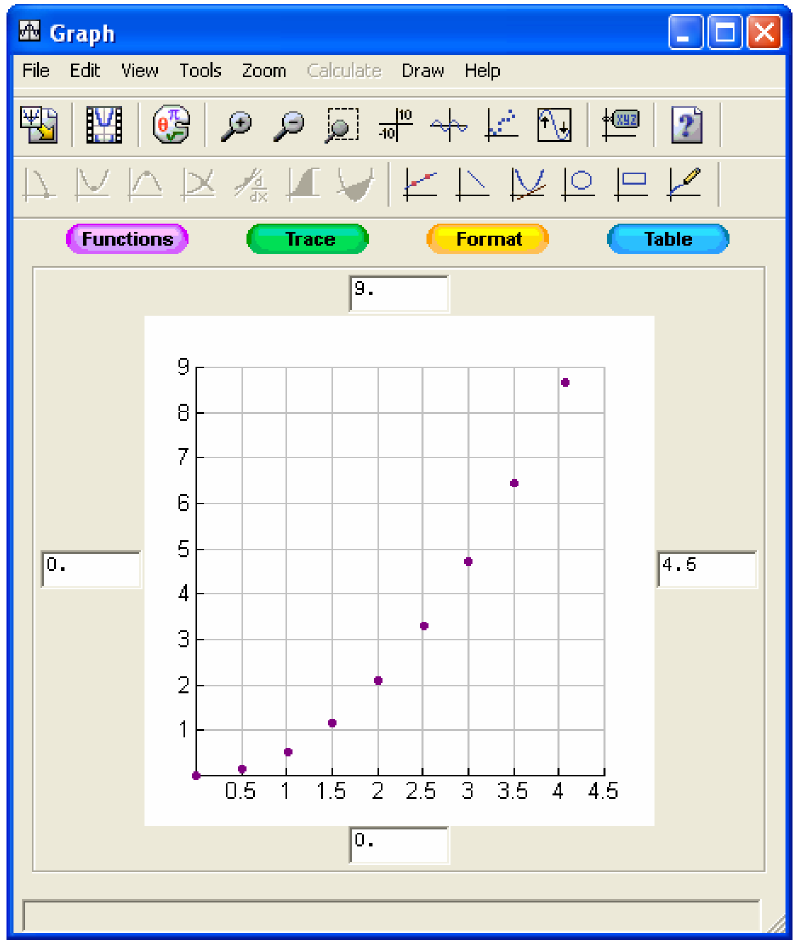




6. Highlight the data you want to graph and click the **Scatter Plot** icon.

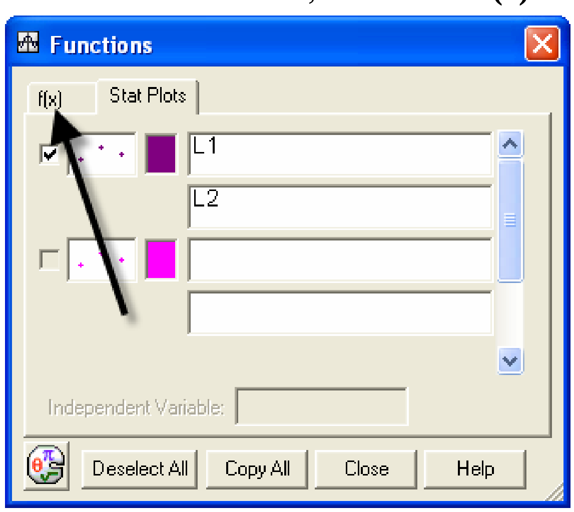


The **Functions** window and the **Graph** window will pop up with **L1** and **L2** listed in the **Stat Plots** windows and the points plotted on the **Graph**.

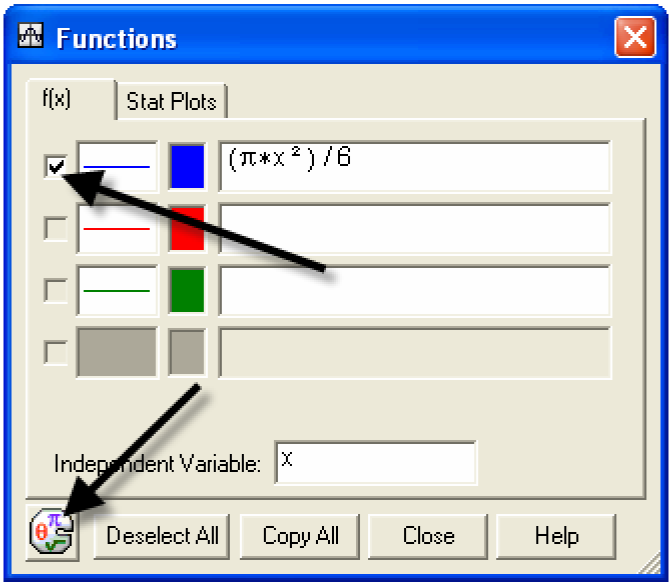


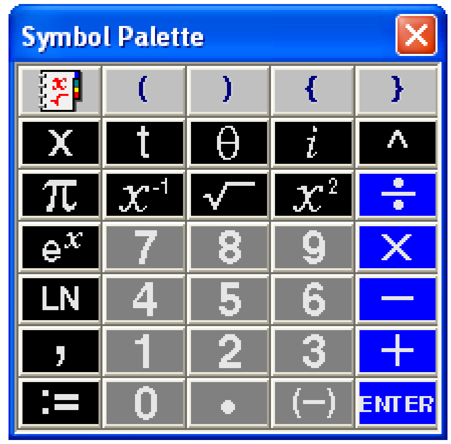



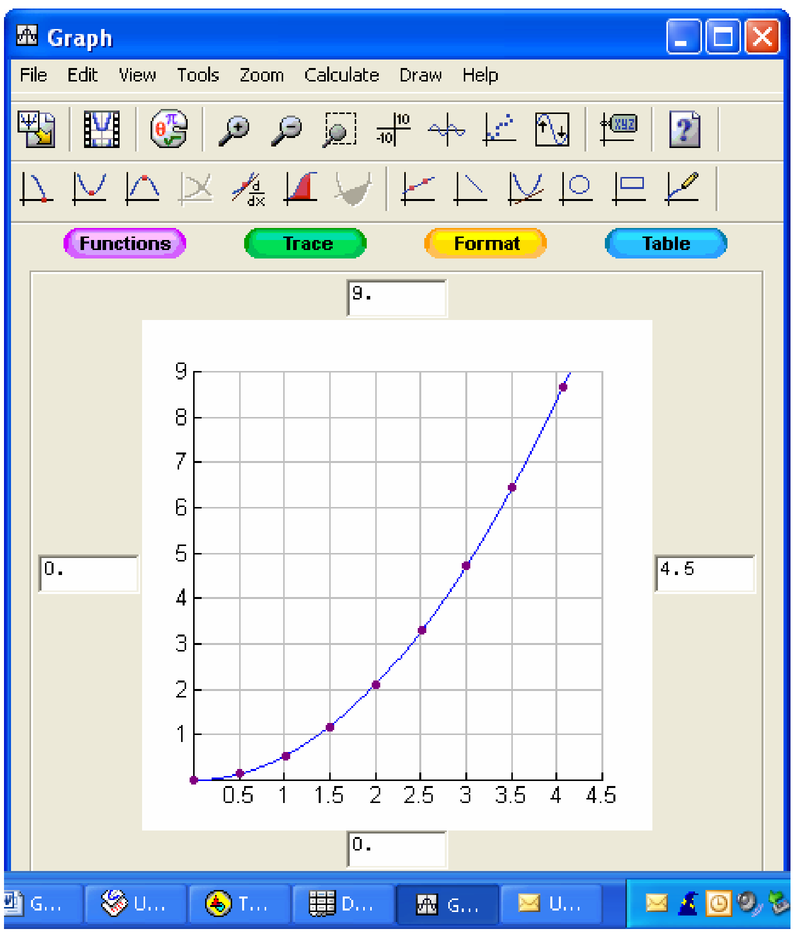

**7.** To enter the function for verification, click on the **f(x)** tab.



**8.** To enter the function, press the Symbol Pallet icon. This lets the Symbol Palette pop up. Enter the function and check the box to graph the function.

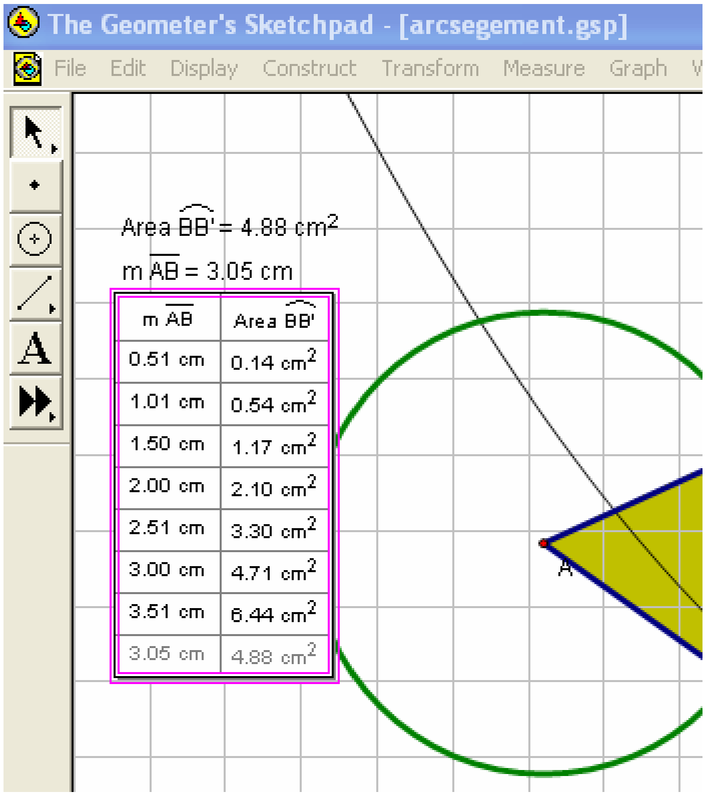


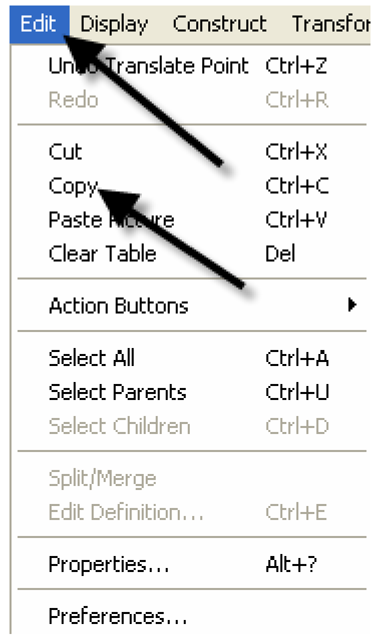



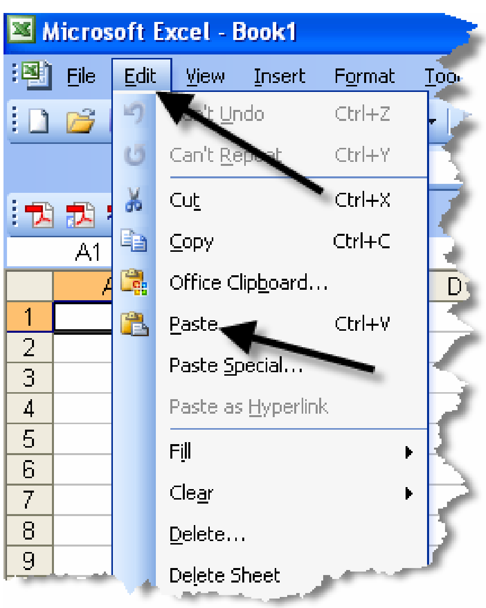
# Function Rule Verification—Spreadsheet

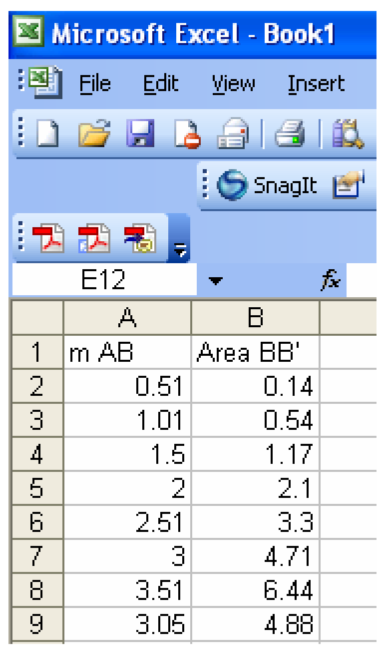
**1.** Copy the table from **Geometer's Sketchpad** by first selecting it, then use **Edit** from the menu bar with the **Copy** option.



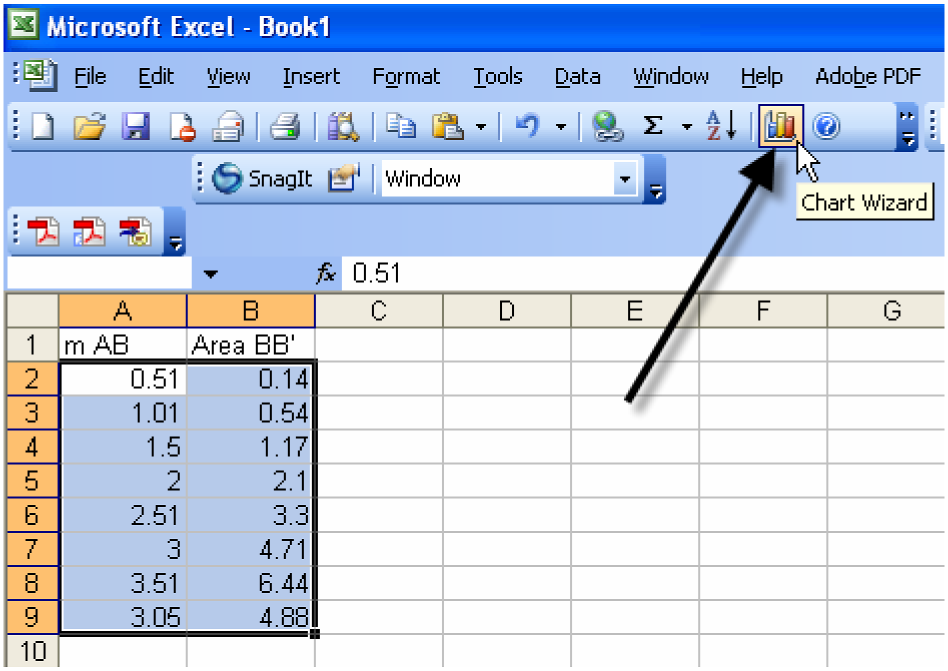



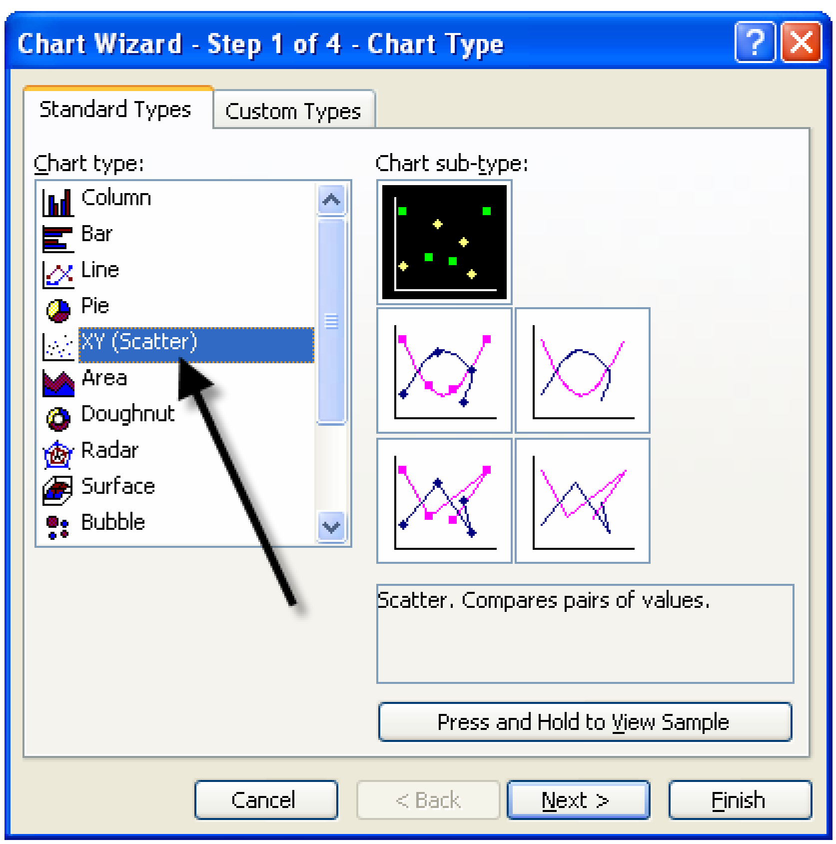

**2.** Open a blank Spreadsheet and paste into the spreadsheet by using the **Edit** from the menu bar with the **Paste** option.



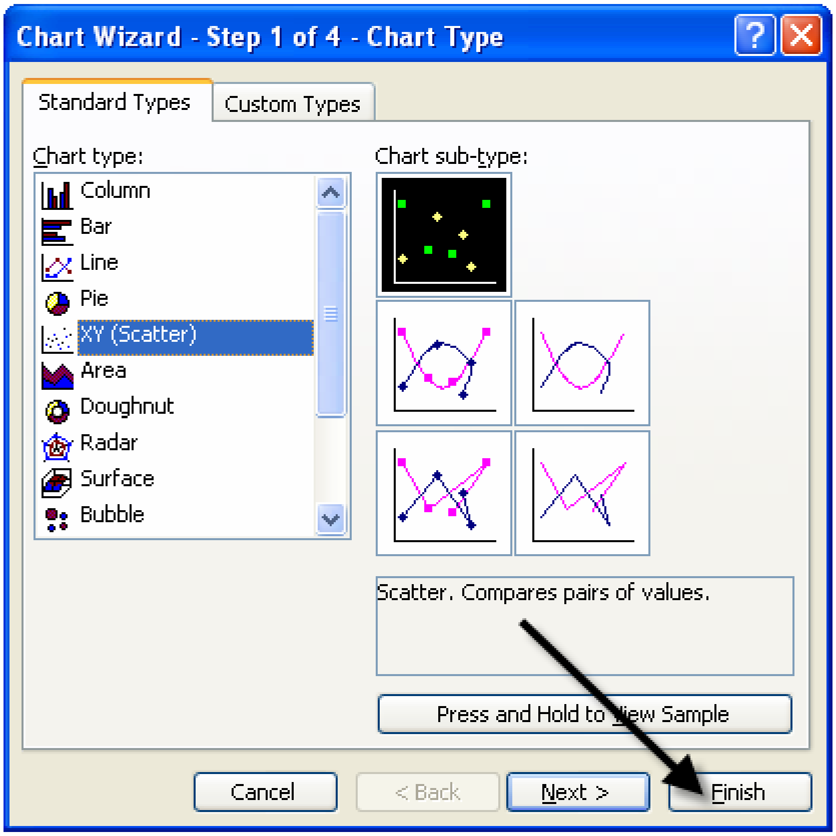



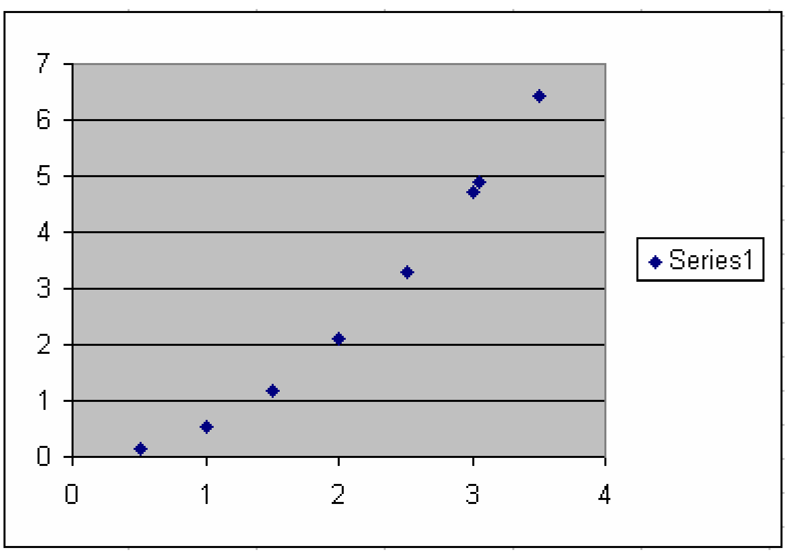
**3.** Highlight the data you want to graph, then click on the **Chart Wizard** icon. The Chart Wizard box will pop up on the screen. Select **XY (Scatter)**.



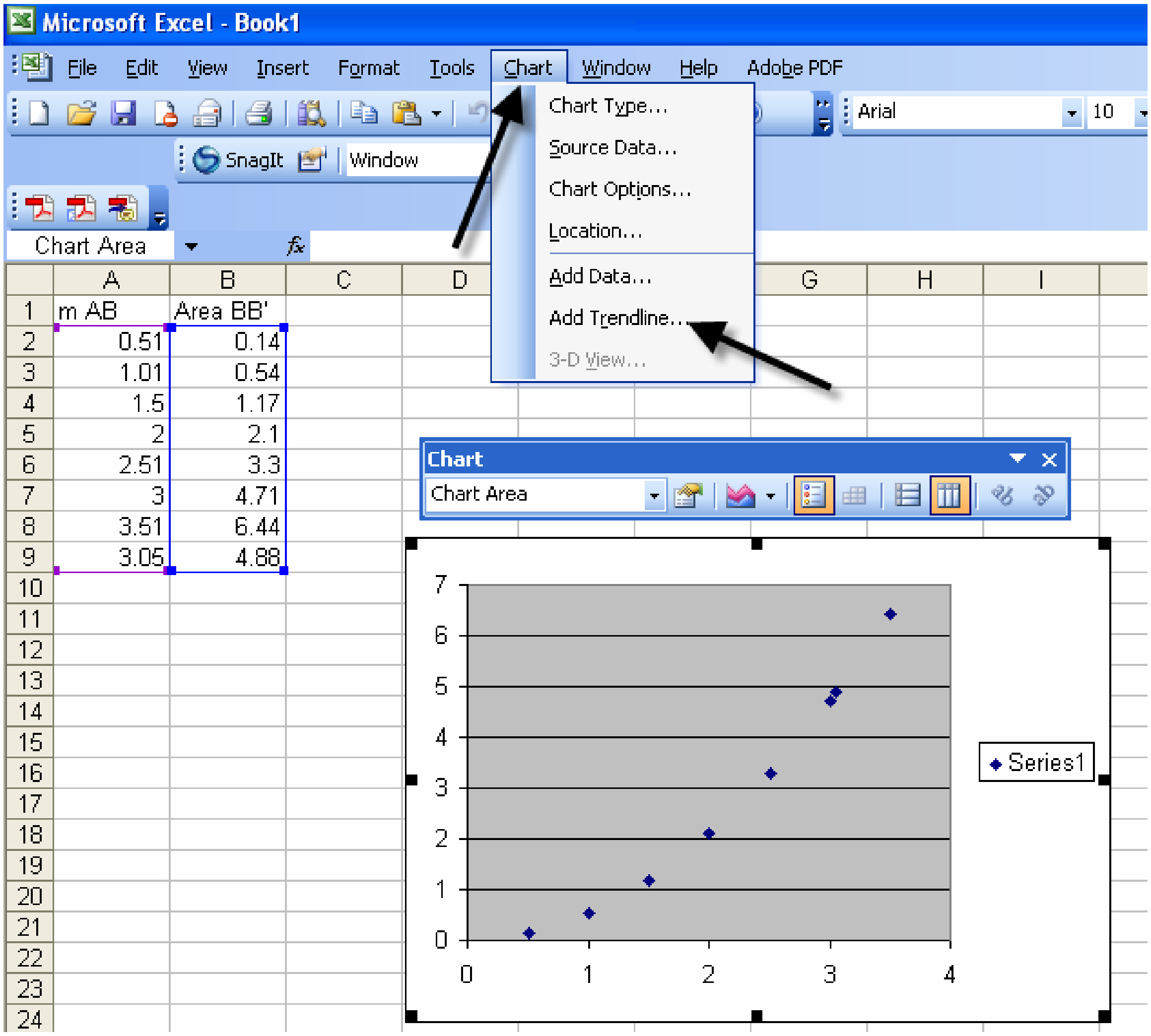



**4.** Click **Finish** to view the graph.

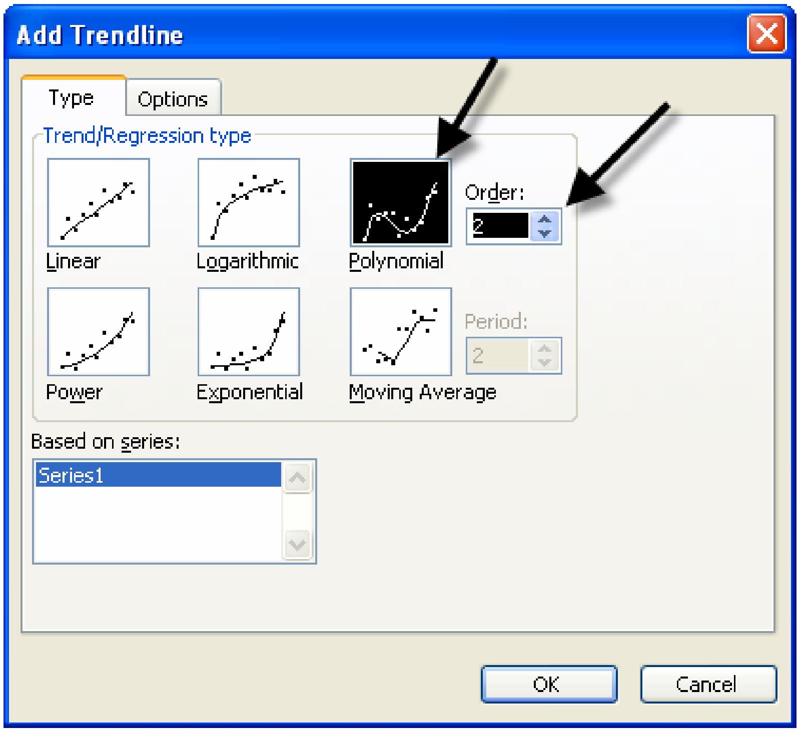


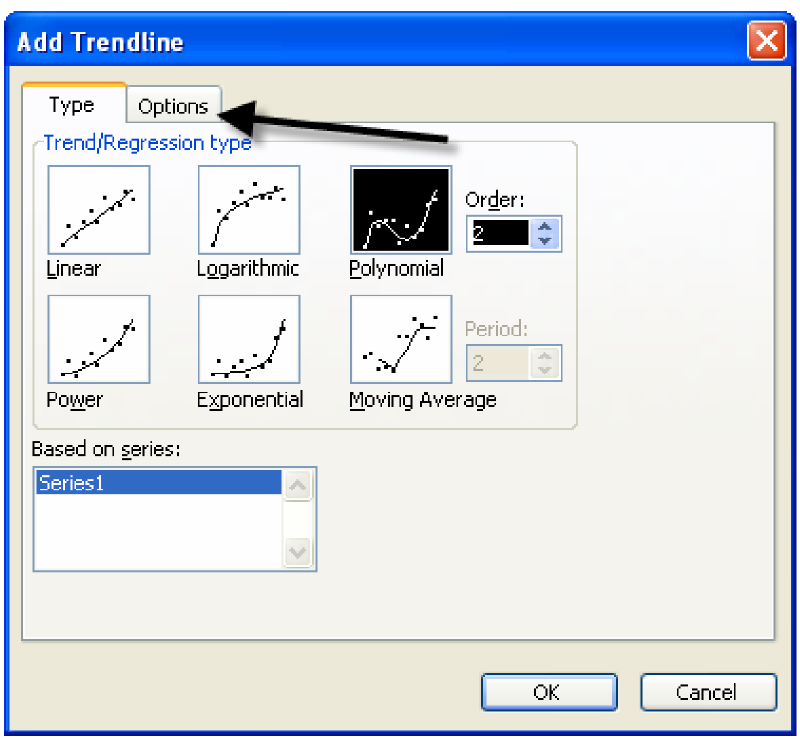
**5.** Select the graph, then use the Chart menu with the **Add Trendline** option.



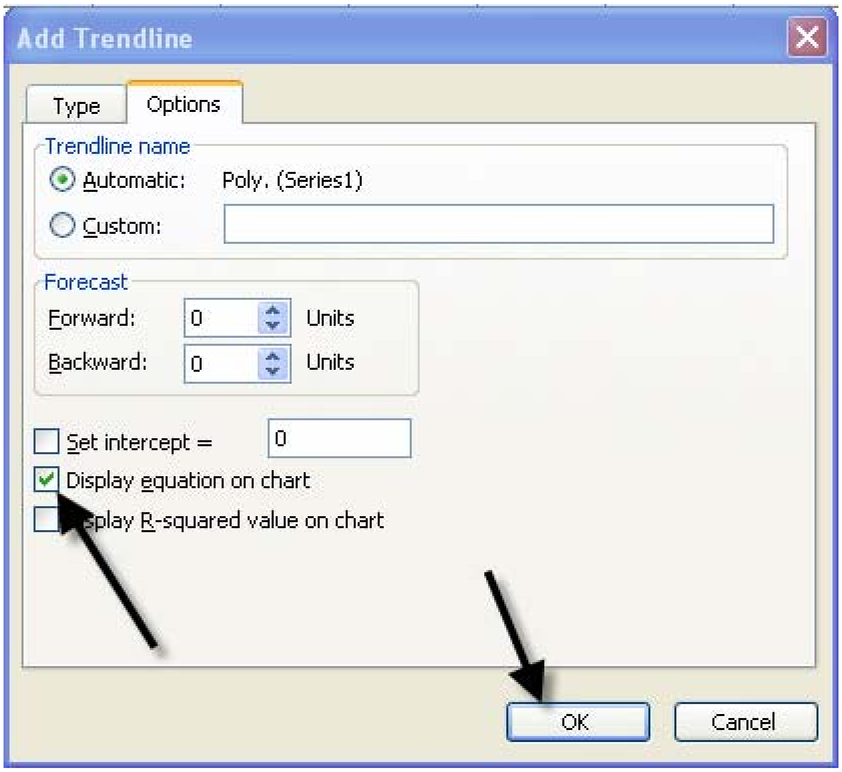
**6.** Since the scatterplot appears to be quadratic, select **Polynomial** order 2.

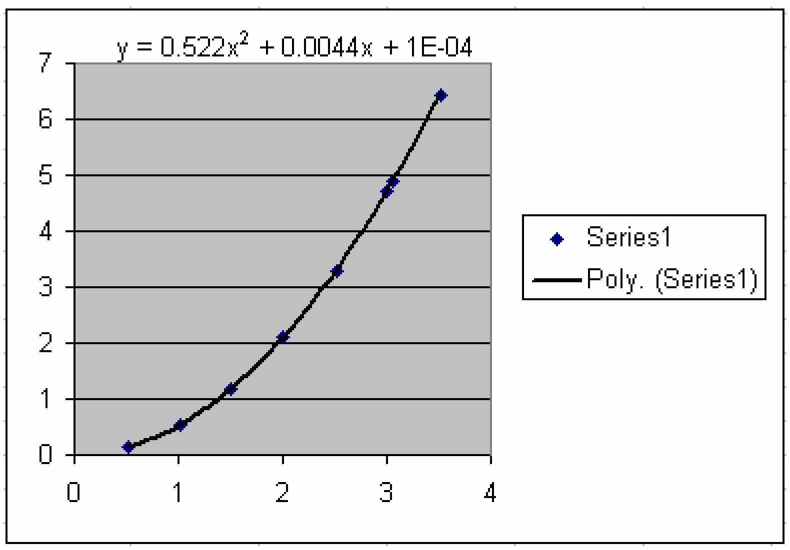



**7.** Click the **Options** tab.



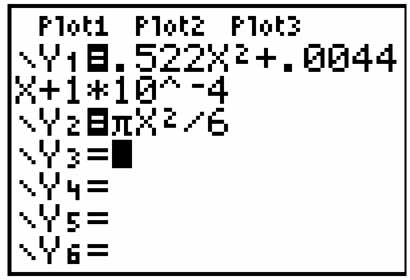
**8.** Check **Display Equation on Chart**, then click **OK**.



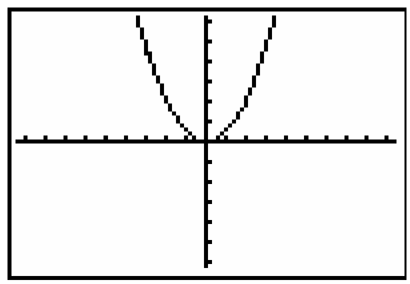



**9.** Use the graphing calculator to verify that the function that was developed by the spreadsheet is equivalent to $A_{sec} = \dfrac{\pi r^2}{6}$ .

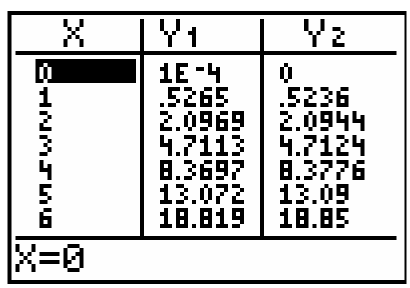
Press [ON].
Enter both functions into [Y=] .



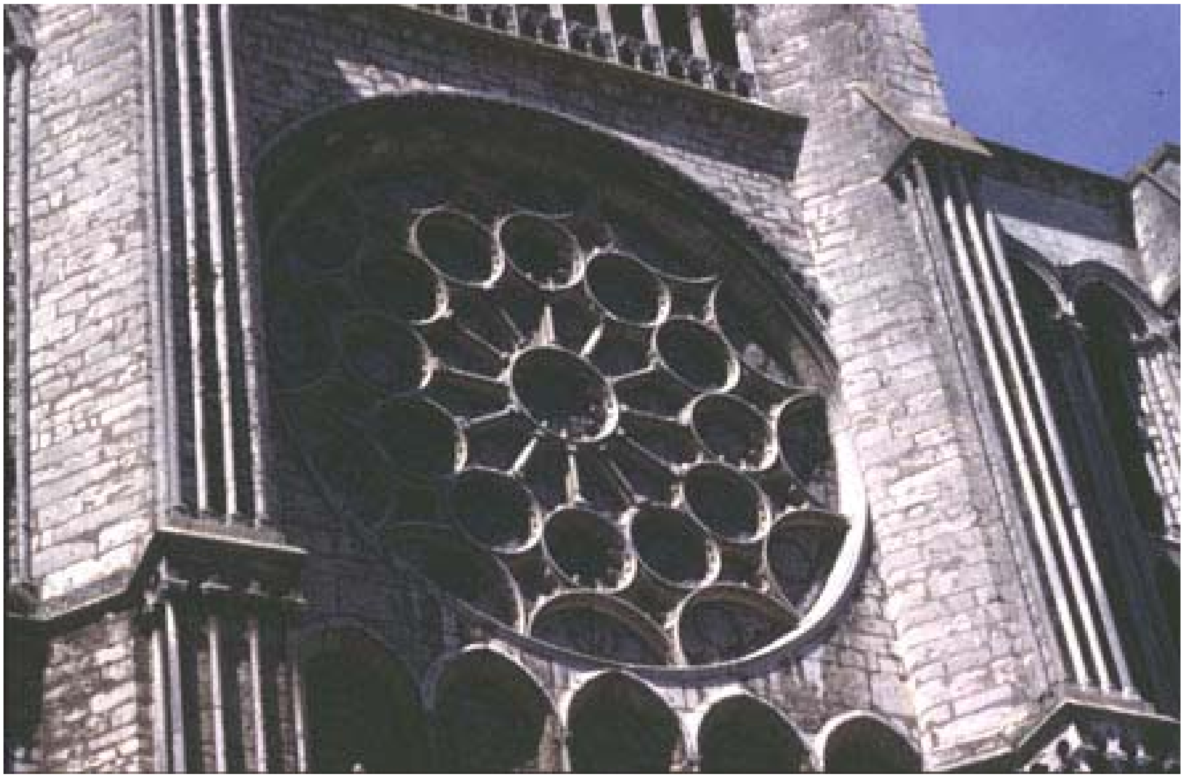

Press the [GRAPH] key. If the functions are equivalent, they will graph on top of each other and the graphing window will show what appears to be only one graph.

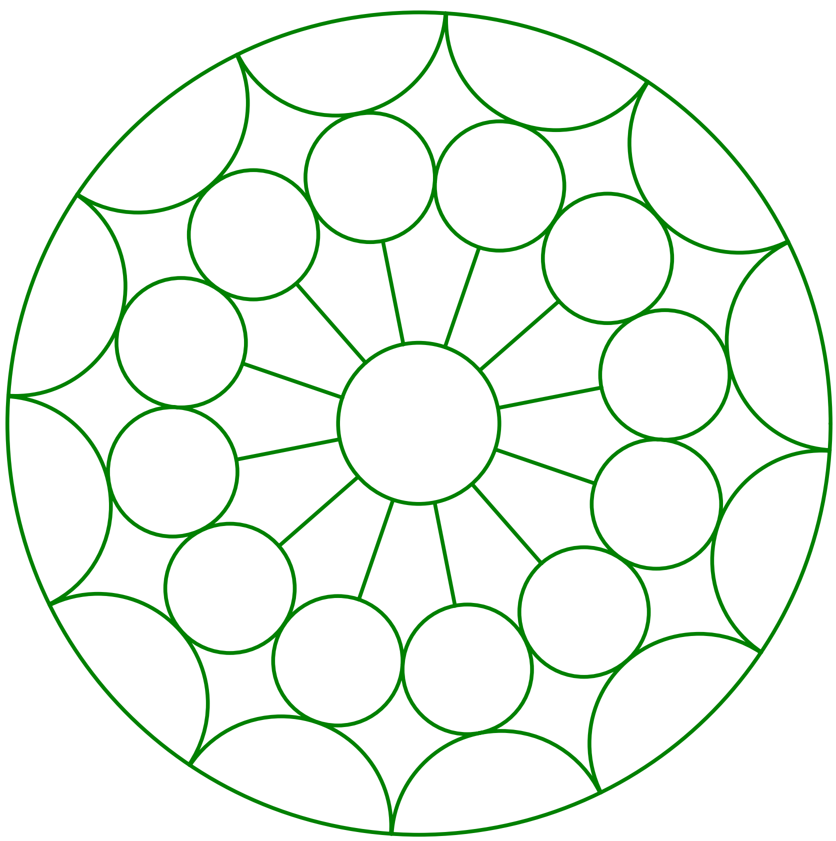


For further verification, press [2nd] [GRAPH] to examine the table values.



# Geometer's Sketchpad—Rose Construction

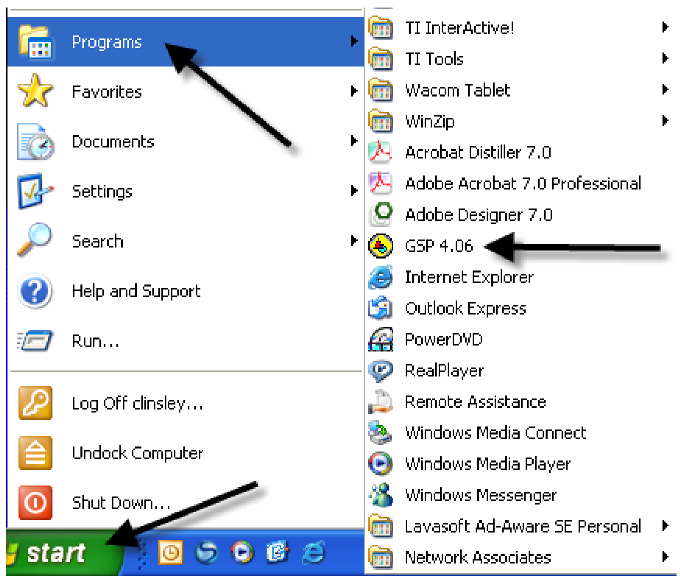




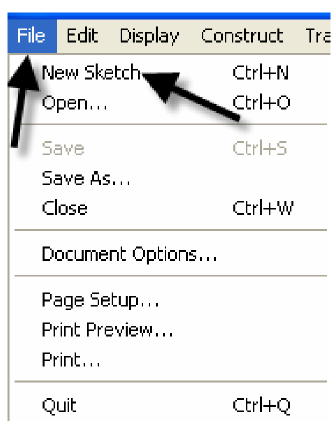
## Opening a New Sketch

To **open** the Geometer's Sketchpad, click on the icon on your desktop


or click on **Start**, **Programs** and find the GSP icon. A new blank sketch will open up.

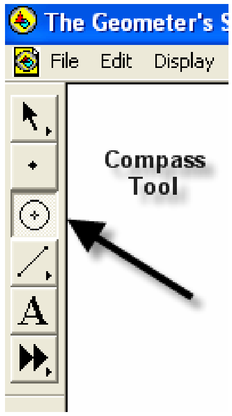


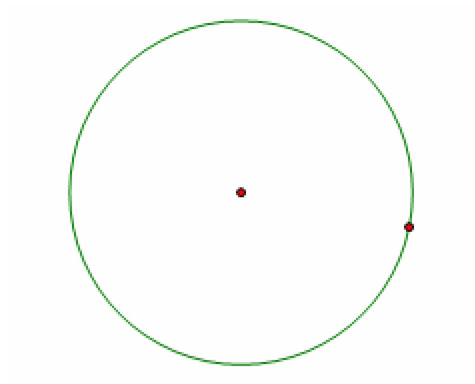

To open a **new sketch** in Geometer's Sketchpad, click on **File**, **New Sketch**.

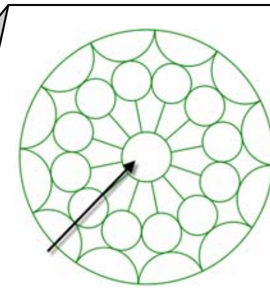


## Circle Construction

Construct a circle with the **Compass Tool**.





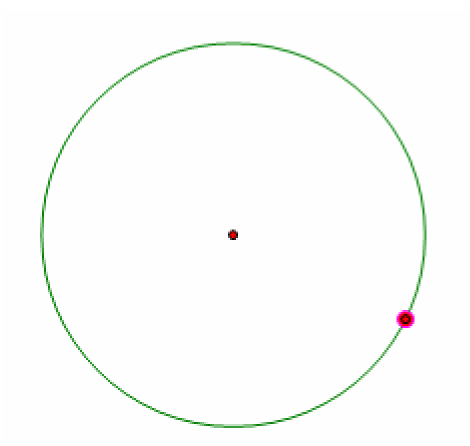
Construct the inner circle.

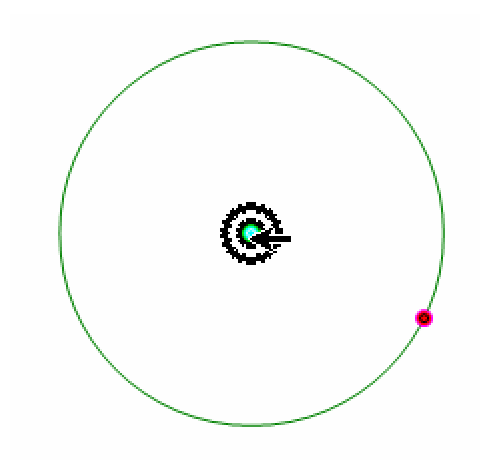
Since there are 12 congruent spokes in the rose, we can use 30 degree rotations in our construction.

## Angle Construction

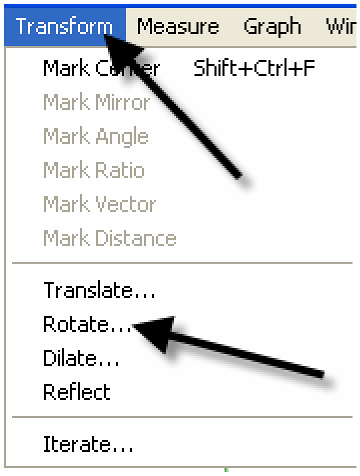
Construct a $30^{o}$ angle by rotating a point on the circle.

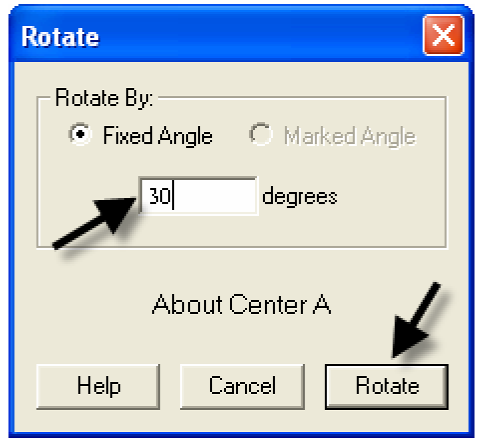
Highlight the point on the circle, then double click the center of the circle to mark the angle of rotation. You will see concentric circles radiating from the center as it is marked.



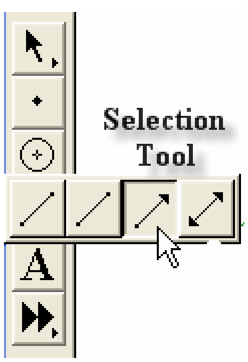


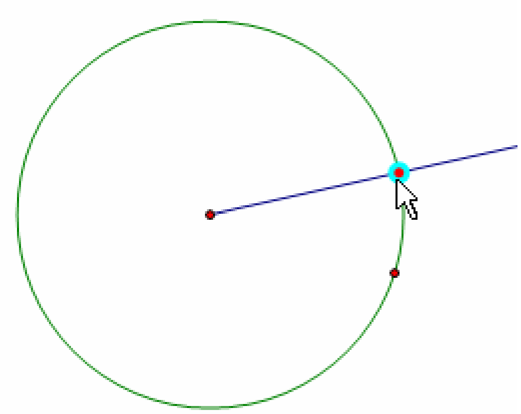
Use **Transform** from the menu bar with the **Rotate** option to rotate.  Enter 30° in the window when the box pops up and click on **Rotate**.

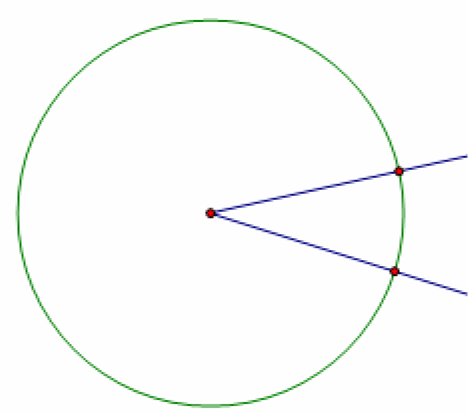


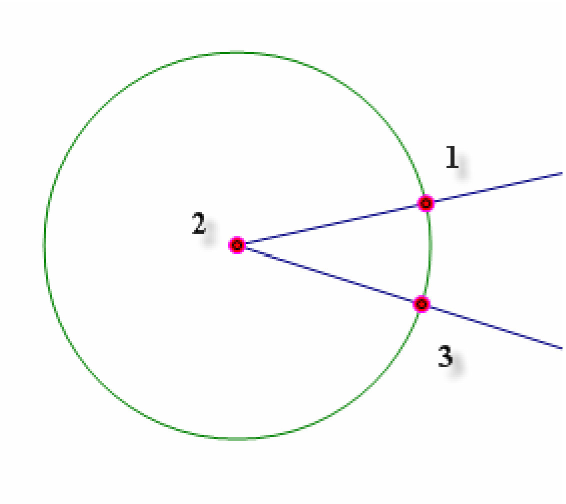

Construct an angle from the center of the circle through each point on the circle.  Using the **Straightedge** tool, select the **Ray** option.  Click on the center of the circle to attach the endpoint, and then line up the point of the ray on top of the point on the circle. Repeat for the second ray.

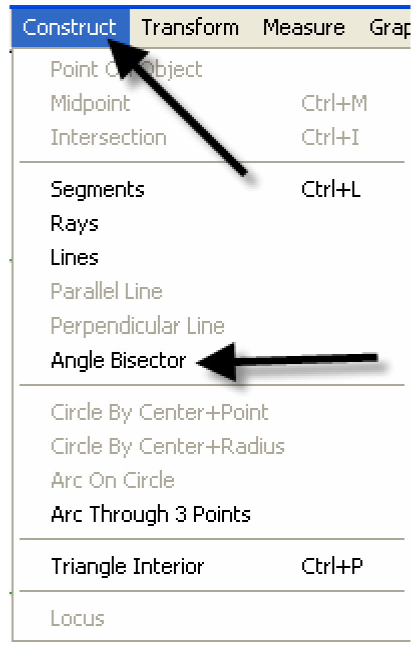


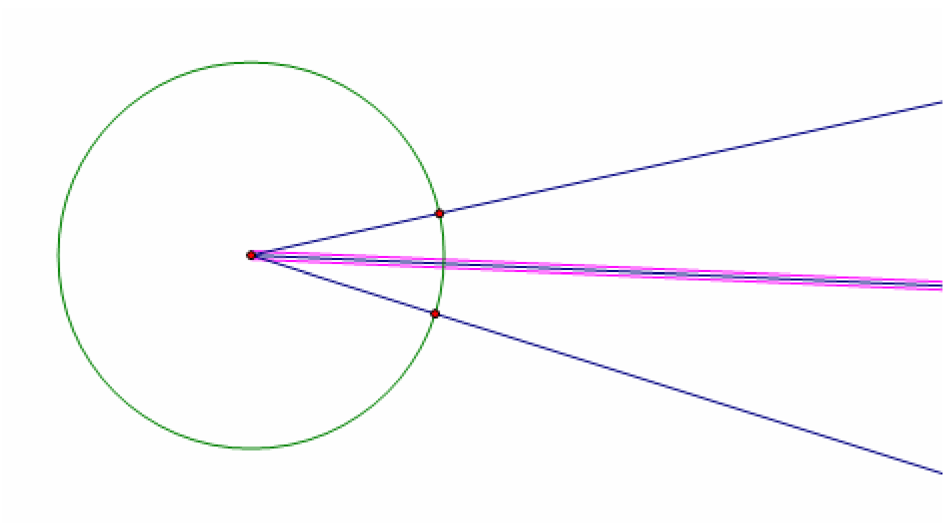



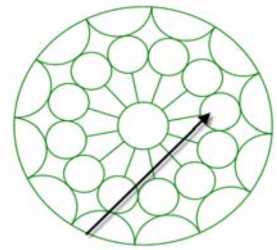


Construct the angle bisector by first selecting the three points of the angle, then using **Construct** from the menu bar with the **Angle Bisector** option.



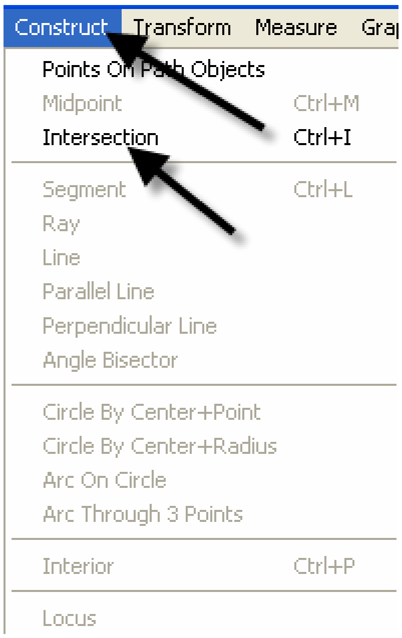



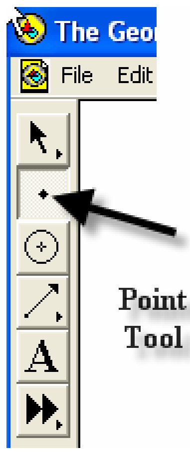



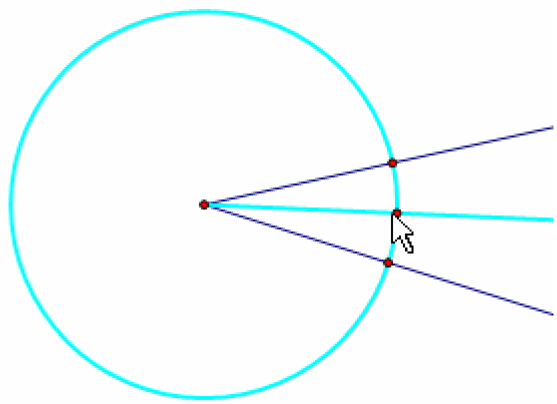


We can find the center of the small circle by reflecting the original circle over a line perpendicular to the angle bisector.

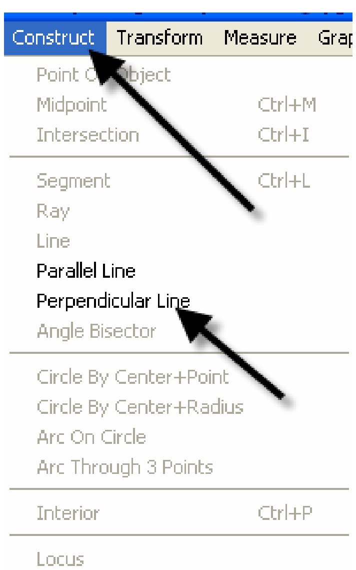
Construct a point where the angle bisector intersects the circle, either by selecting the angle bisector and the circle, then using **Construct** from the menu bar with **Intersection** option or by using the **Point** tool and placing the point on the intersection (You will know you are on the intersection when both the circle and the angle bisector change color.)

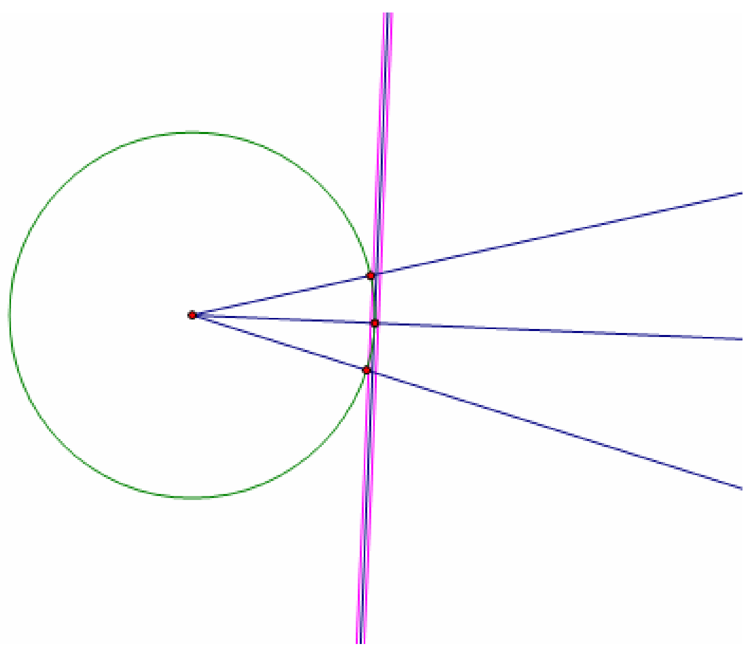


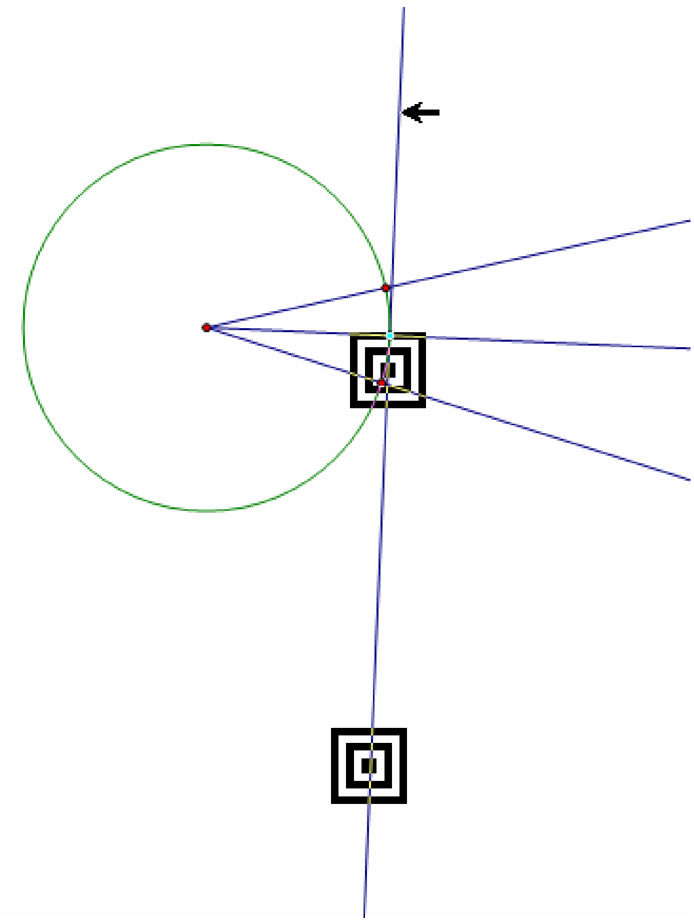



Construct a tangent to the circle through the point of intersection of the circle and the angle bisector by first selecting the point of intersection and the angle bisector, then using **Construct** from the menu bar with the **Perpendicular Line** option.

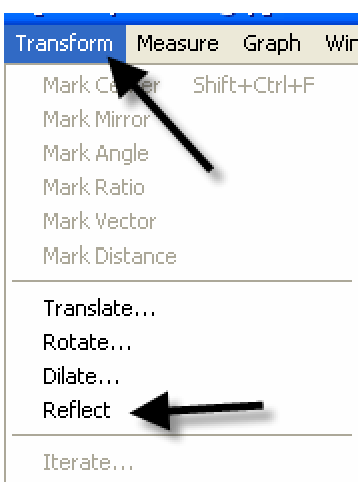


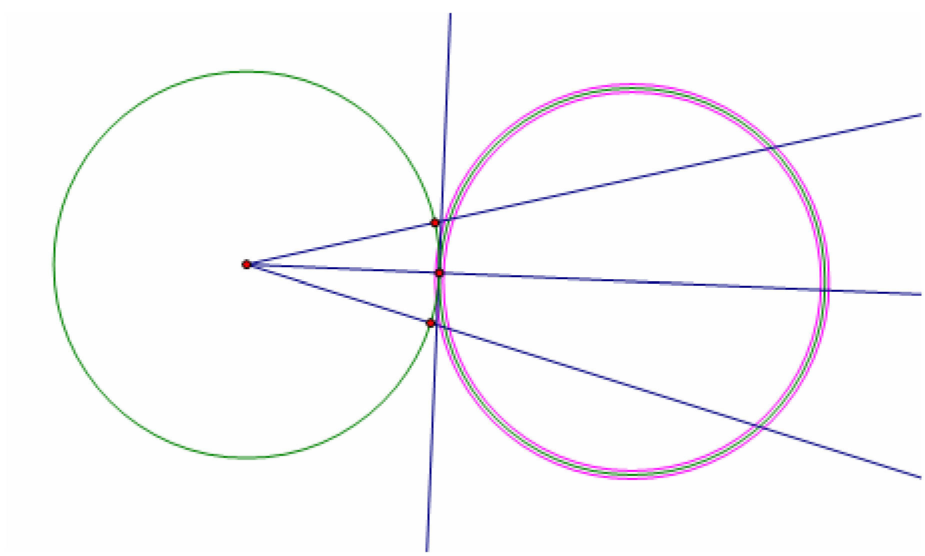



Mark the tangent line as a line of reflection by double clicking on the line. You will see a double set of concentric boxes flash as the line is being marked.

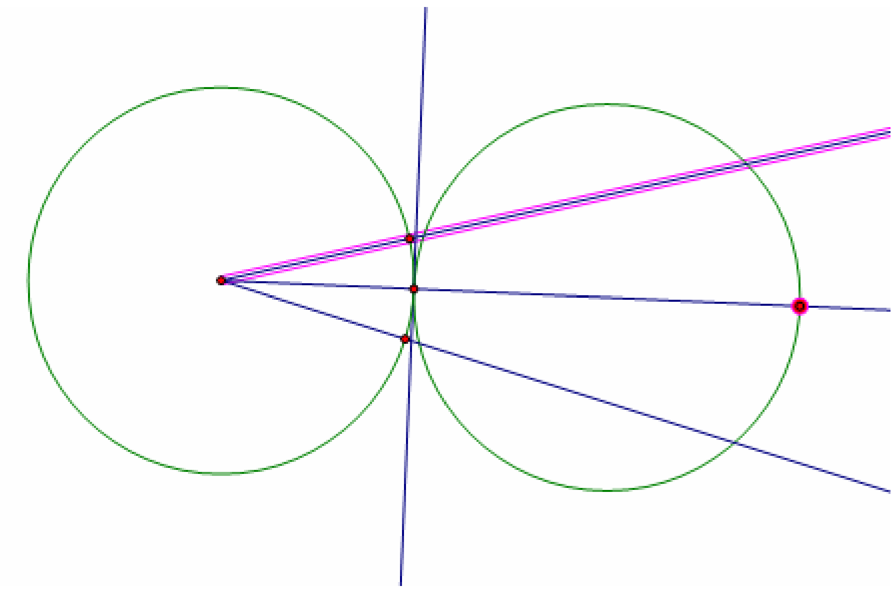


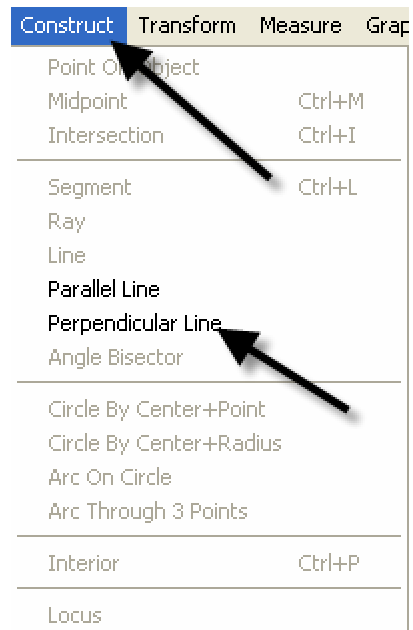
Reflect the circle across the line of reflection by selecting the circle and using **Transform** from the menu bar with the **Reflect** option.

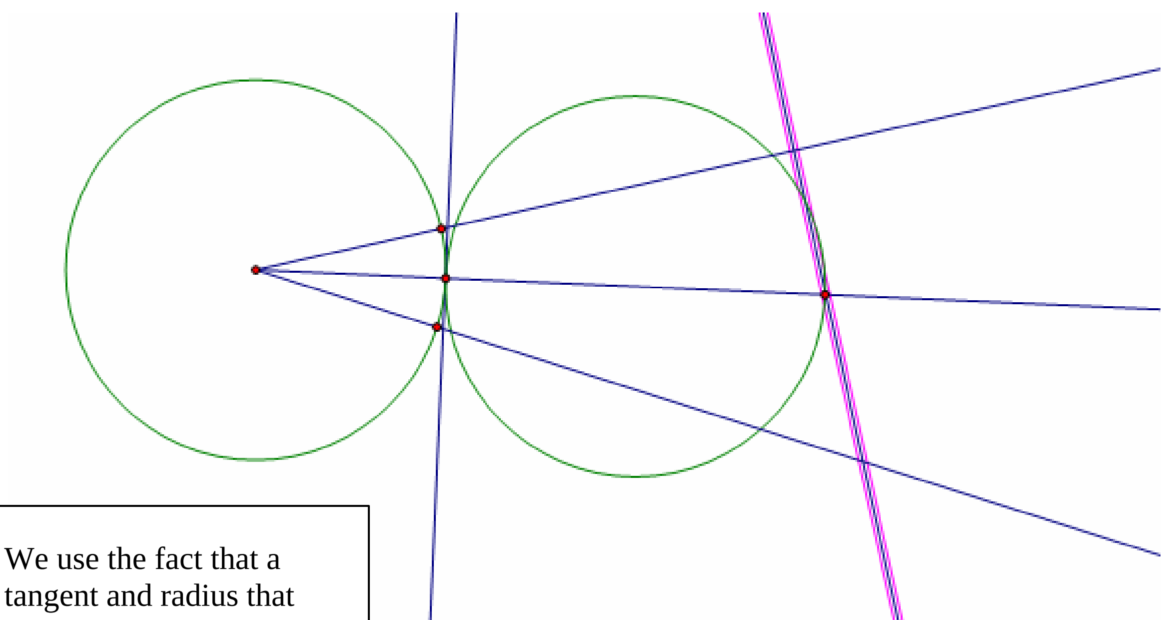


Construct the intersection of the angle bisector and the circle, then construct a line through it and perpendicular to one side of the angle. Select the point of intersection and the ray that forms one side of the angle, then use **Construct** from the menu bar with the **Perpendicular Line** option.

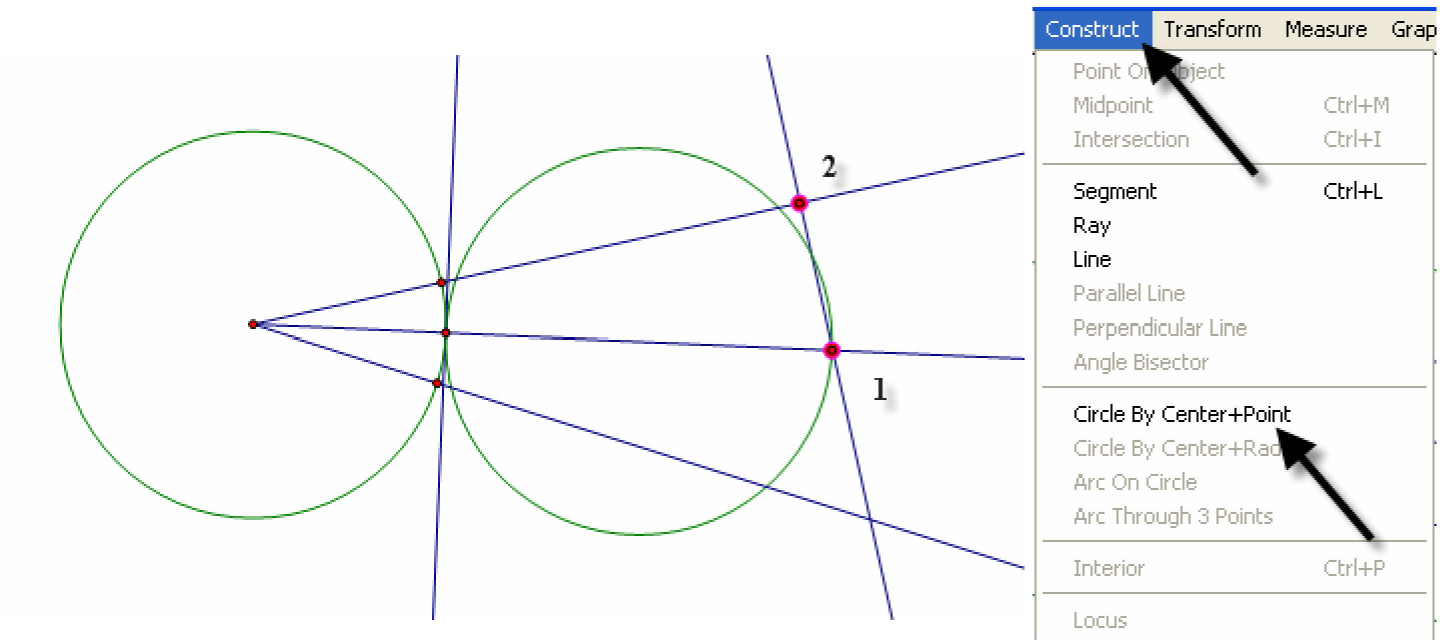


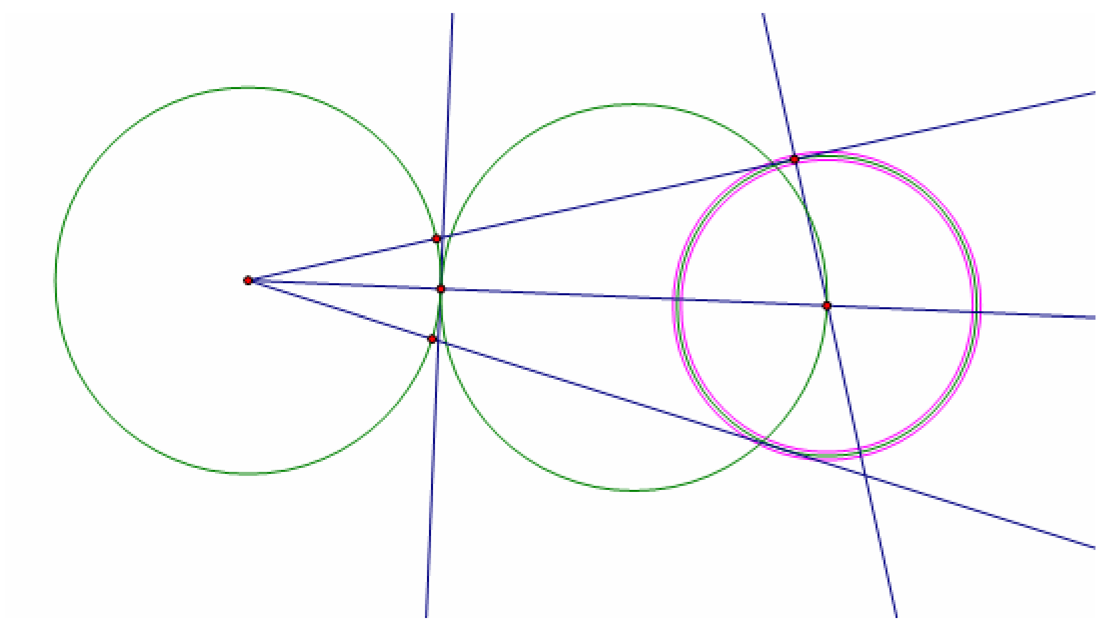


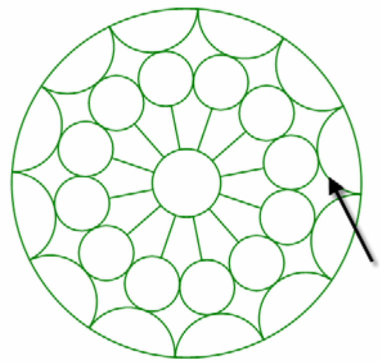
We use the fact that a tangent and radius that intersects it are perpendicular to construct a circle tangent to the rays.

Construct the point of intersection with the new perpendicular line and the side of the angle. Select the points in the order shown below. Use **Construct** from the menu bar with the **Circle By Center+Point**.

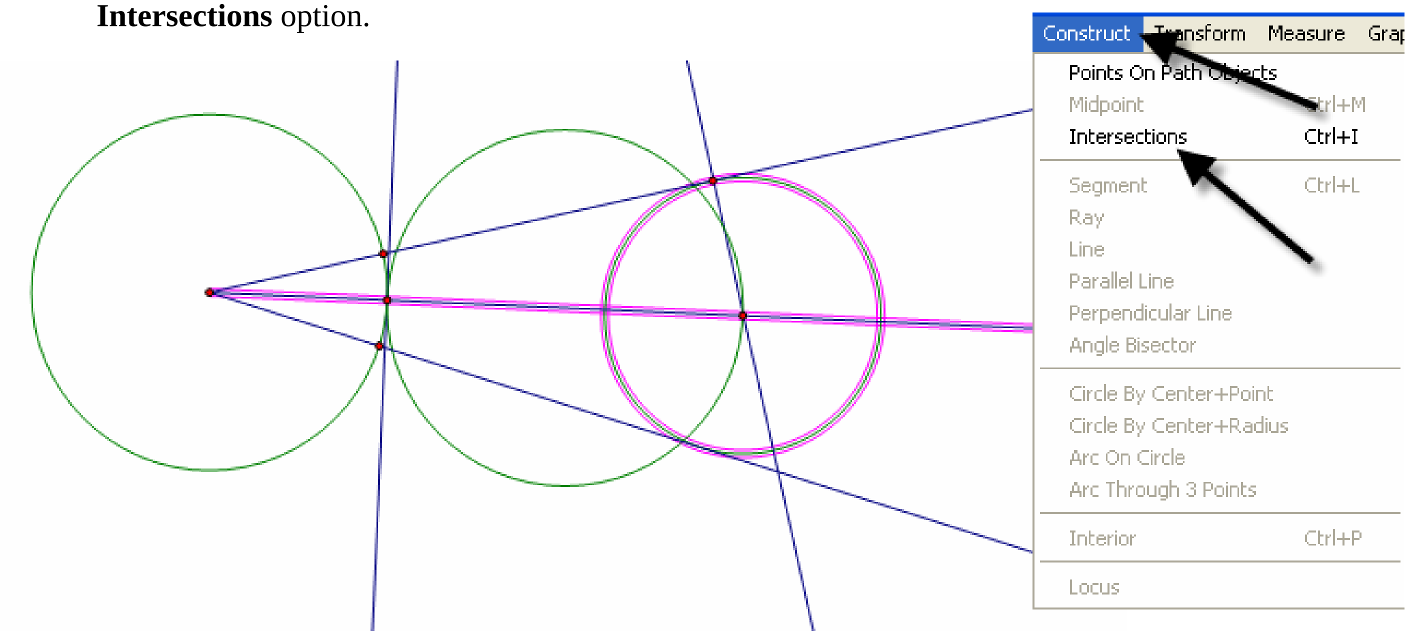


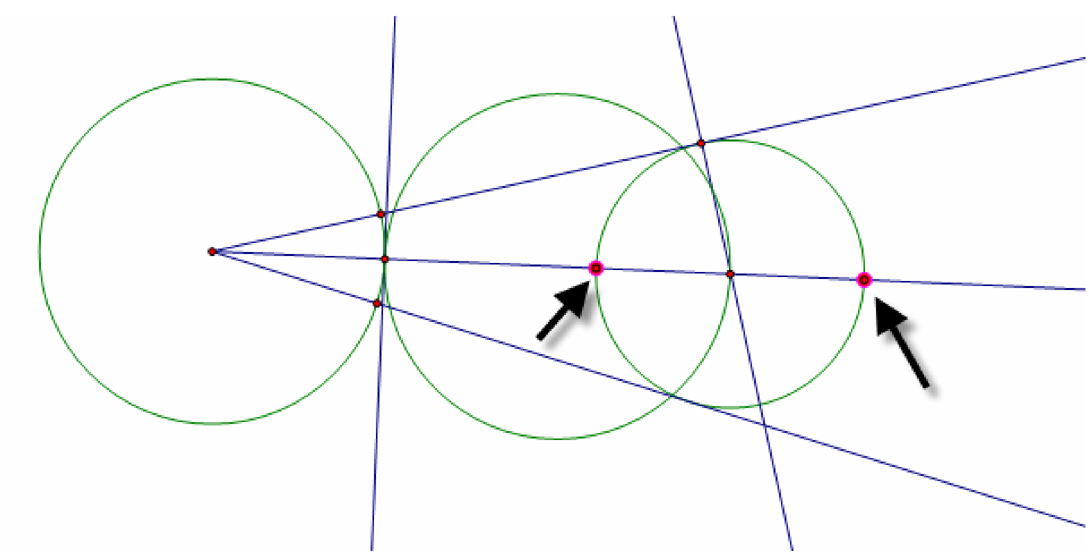



We need to construct an arc that is tangent to the rays that form the angle as well as tangent to the small circle. We will use proportional reasoning to determine the center of the circle the arc lies on.

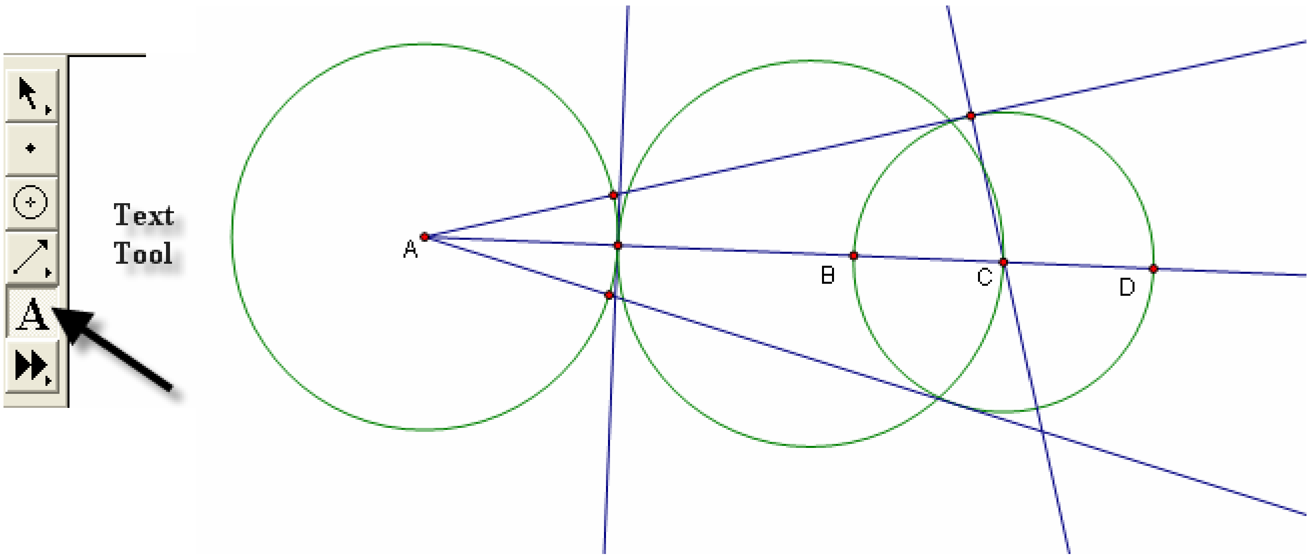


Construct the intersection of the small circle with the angle bisector by first selecting the circle and the angle bisector, then using **Construct** from the menu bar with the **Intersections** option.

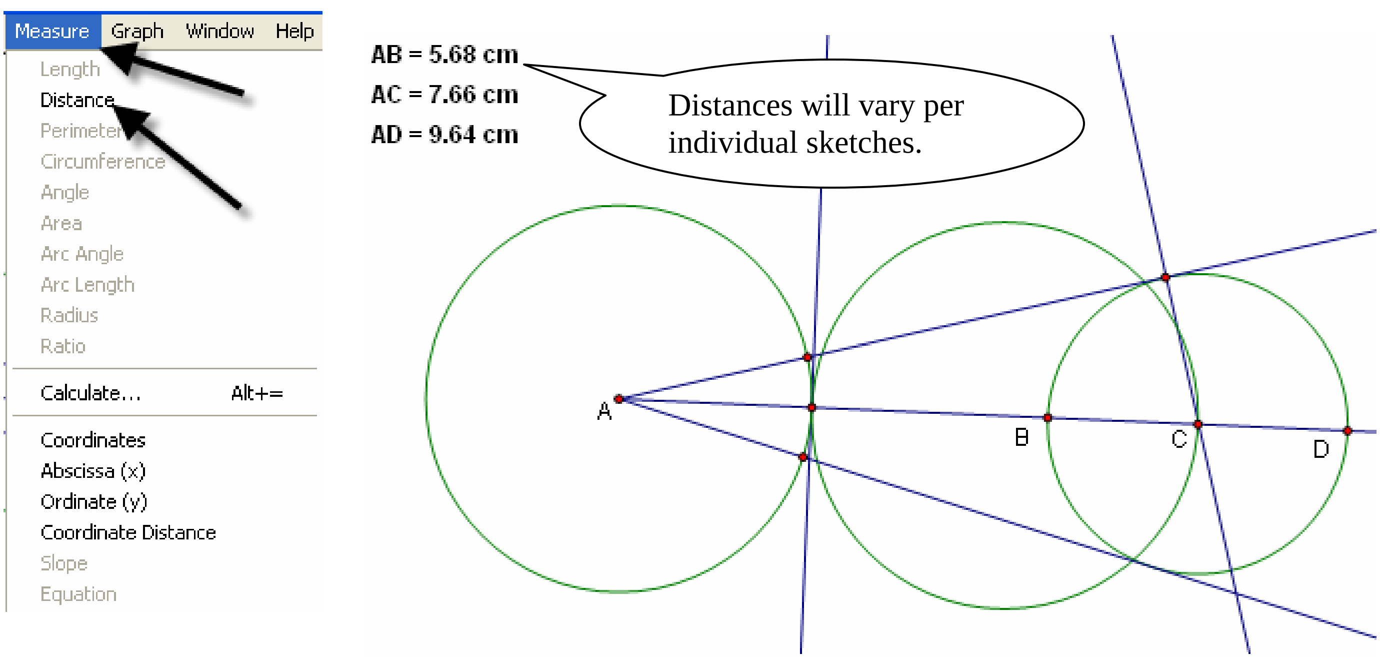


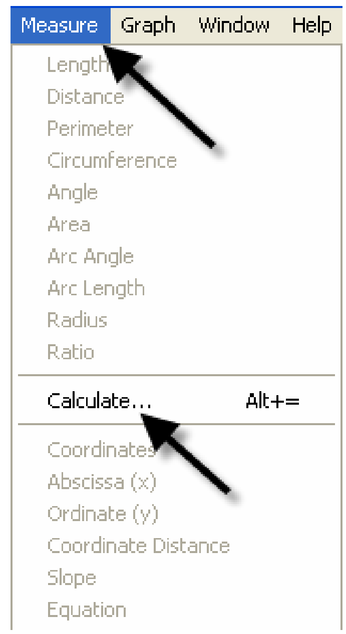
To use points to set up measures to use in the proportion, label points with the **Text** tool according to the sketch below.



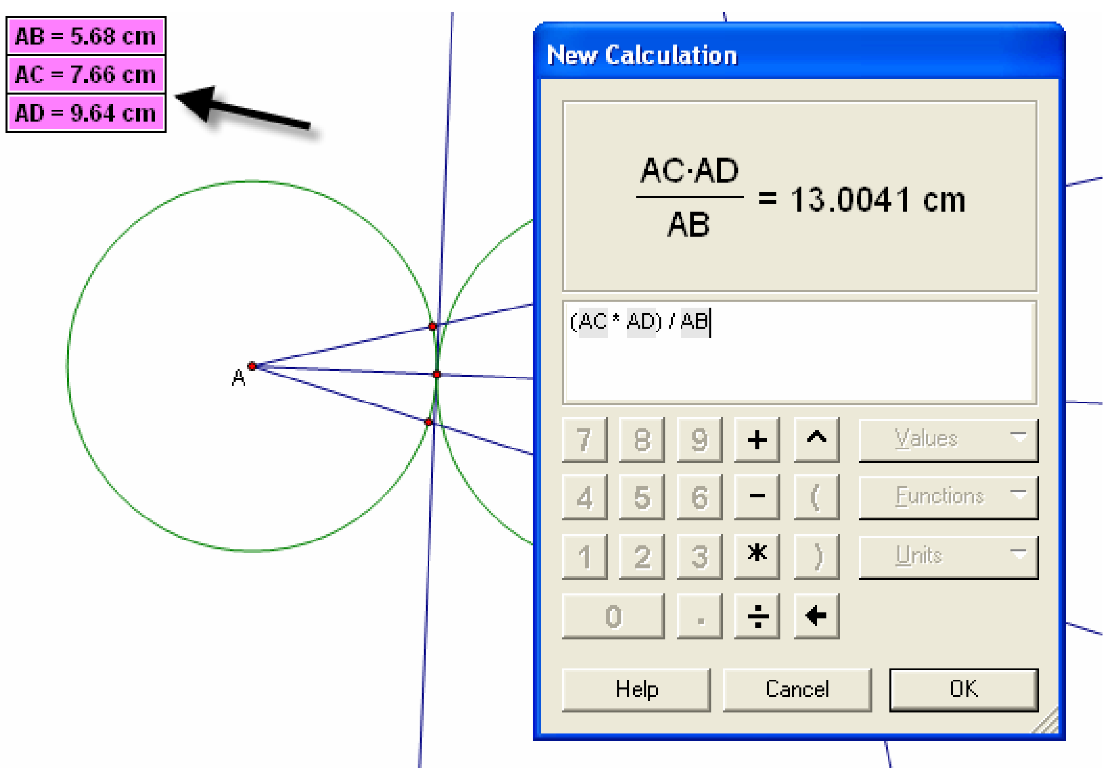

To set up the proportion, measure the following distances AB, AC and AD by selecting the endpoints of the segments and using **Measure** from the menu bar with the **Distance** option. The center of circle that will be tangent to the sides of the angle and tangent to the small circle at point D will have a proportional distance from A based on the following proportion: $\dfrac{AB}{AD} = \dfrac{AC}{Ax}$ which can be rewritten as $Ax = \dfrac{AC \cdot AD}{AB}$.



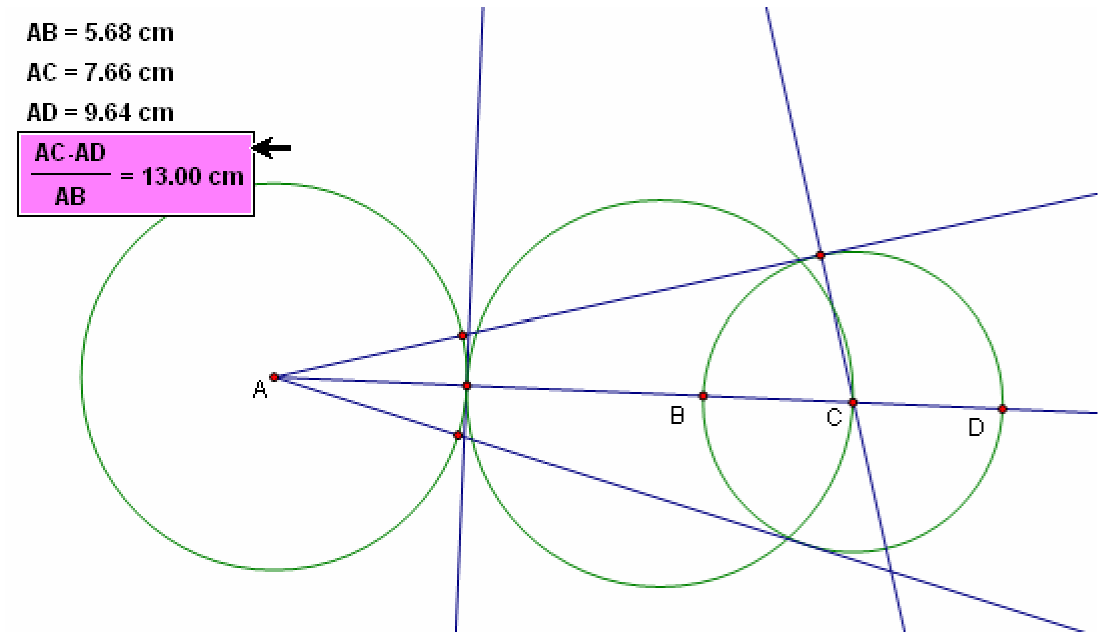
Use **Measure** from the menu bar with the **Calculate** option to compute the distance from A to the center of the new circle, $Ax = \dfrac{AC \cdot AD}{AB}$ .



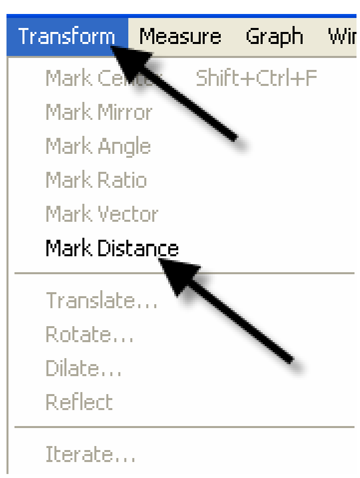

Click on the desired measure to enter the values in the calculator, then click **OK**.



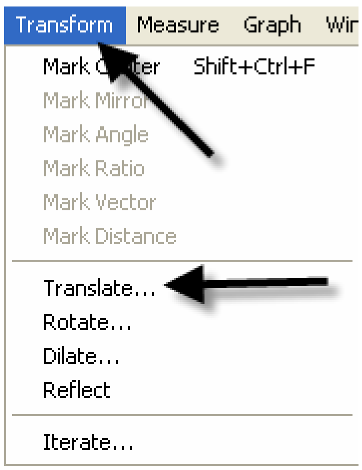
Select the solution for the distance of the new center from ***A***.

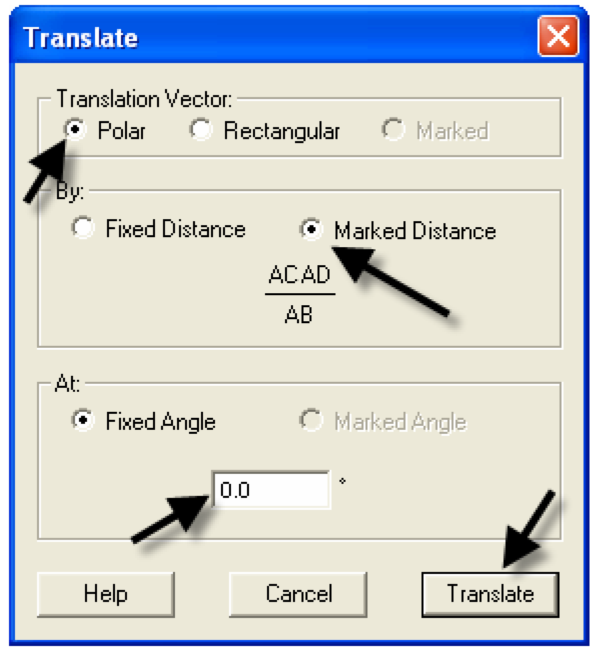



Use **Transform** from the menu bar with the **Mark Distance** option. The highlighted box with the solution in it will flash as it marked.

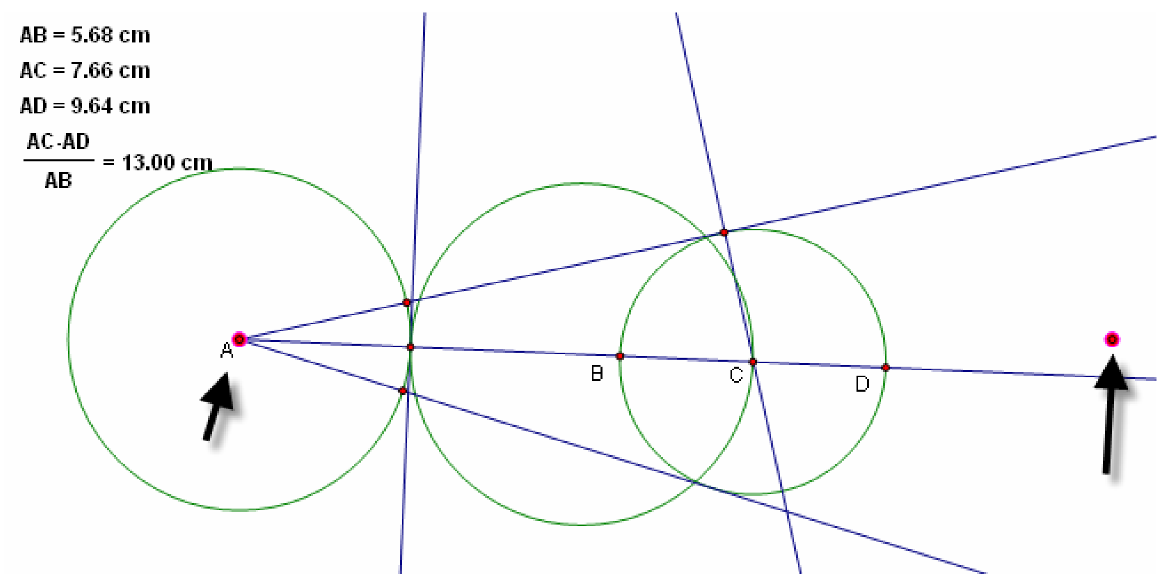


Select point ***A*** and use **Transform** from the menu bar with the **Translate** option. A pop-up box will appear that will allow you to select the following options.

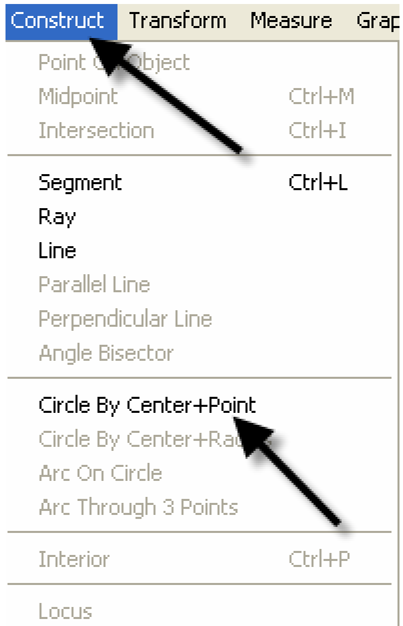


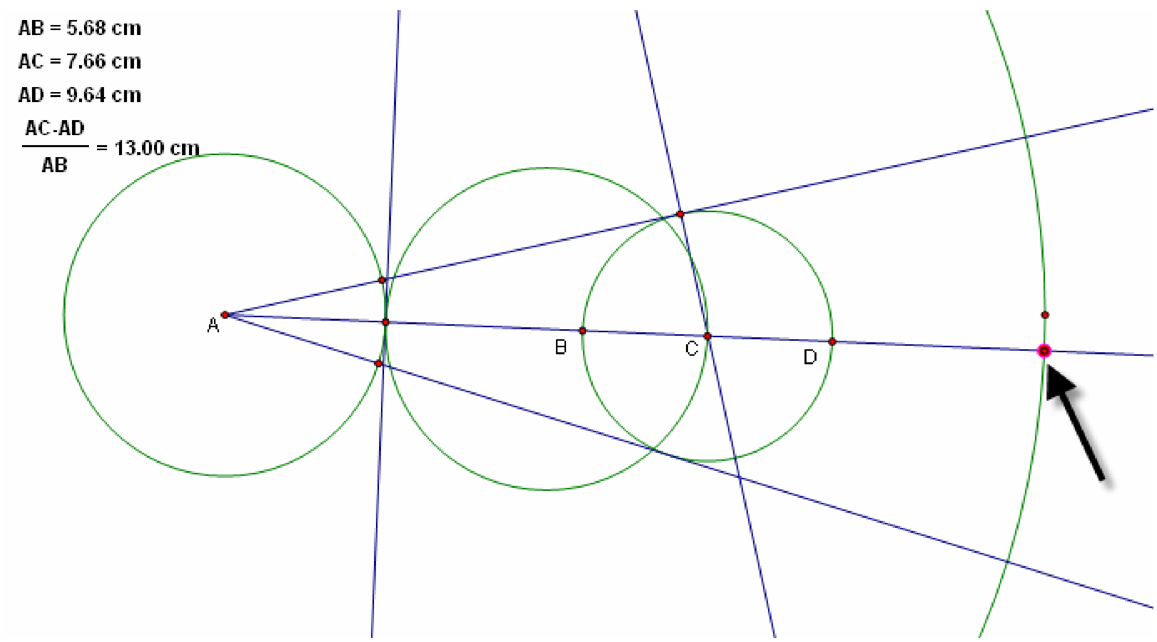

A new point will appear in the blank space of the sketch. In order, select point A followed by the newly translated point.



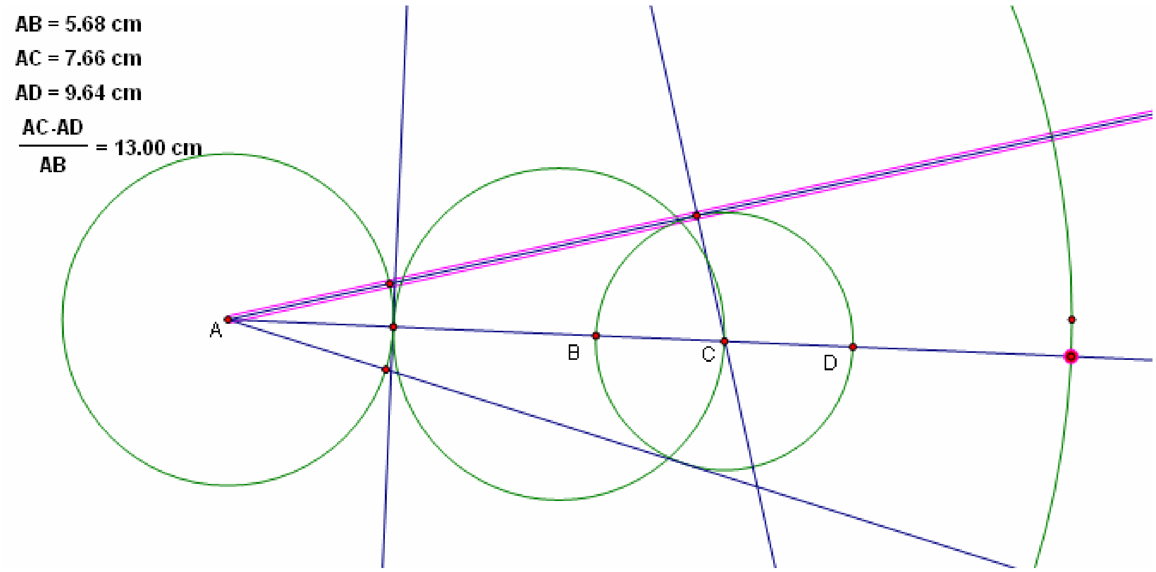
Use **Construct** from the menu bar with **Circle By Center+Point**.

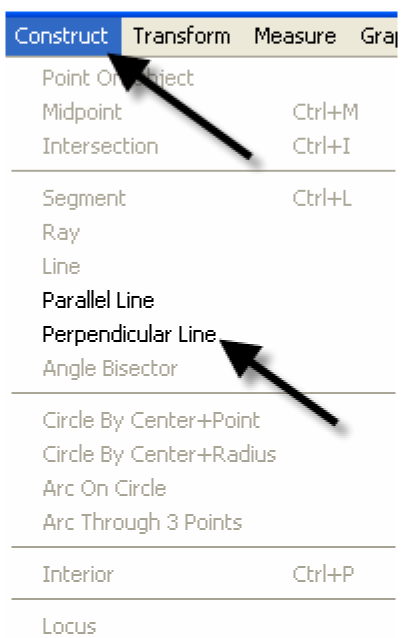



This will create a large circle whose point of intersection with the angle bisector will be the center of the new circle. Construct the intersection.

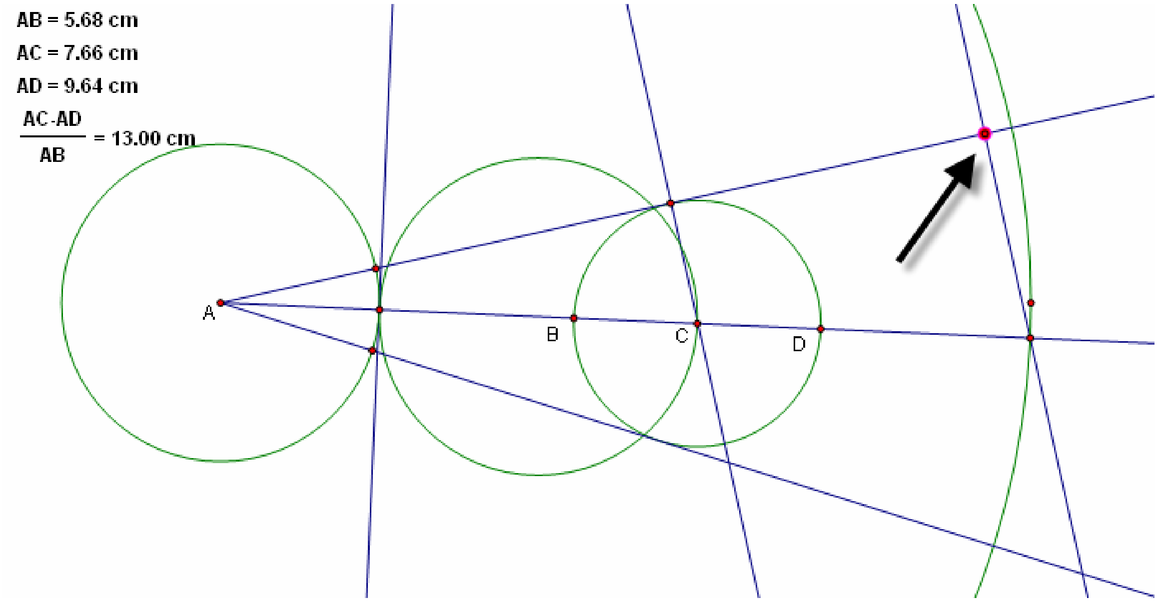


To find the radius of the circle, highlight the new point and the ray that makes the side of the angle. Then use **Construct** from the menu bar with the **Perpendicular Line** option.

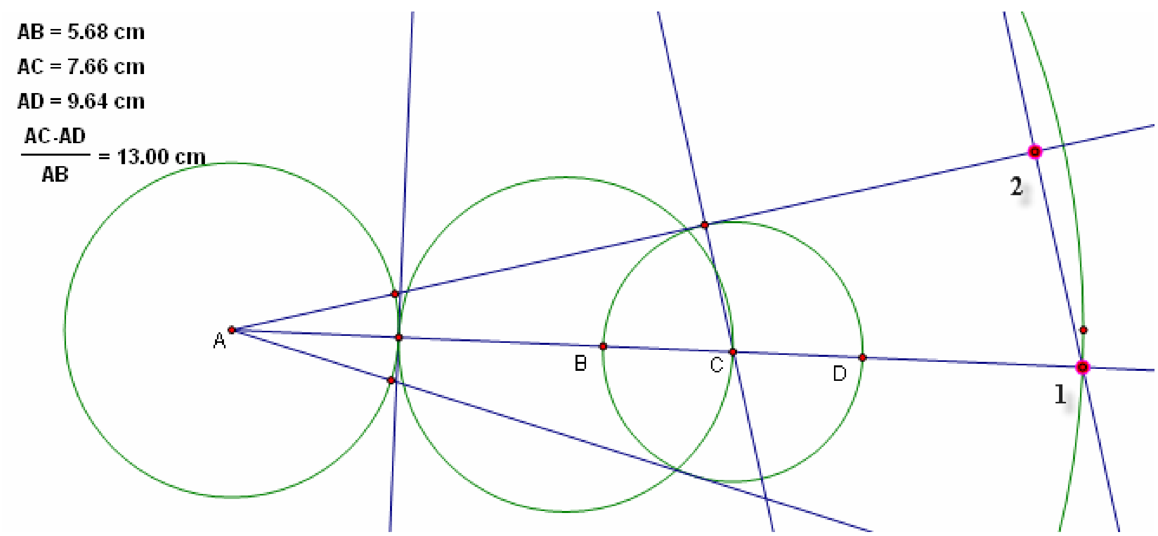


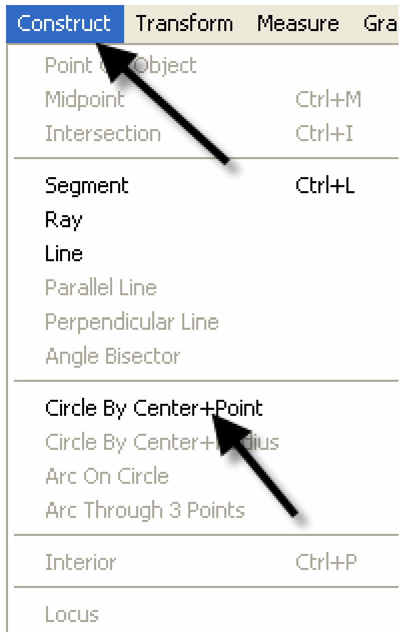



Construct a point of intersection where the new perpendicular line intersects with the side of the angle, then deselect it.

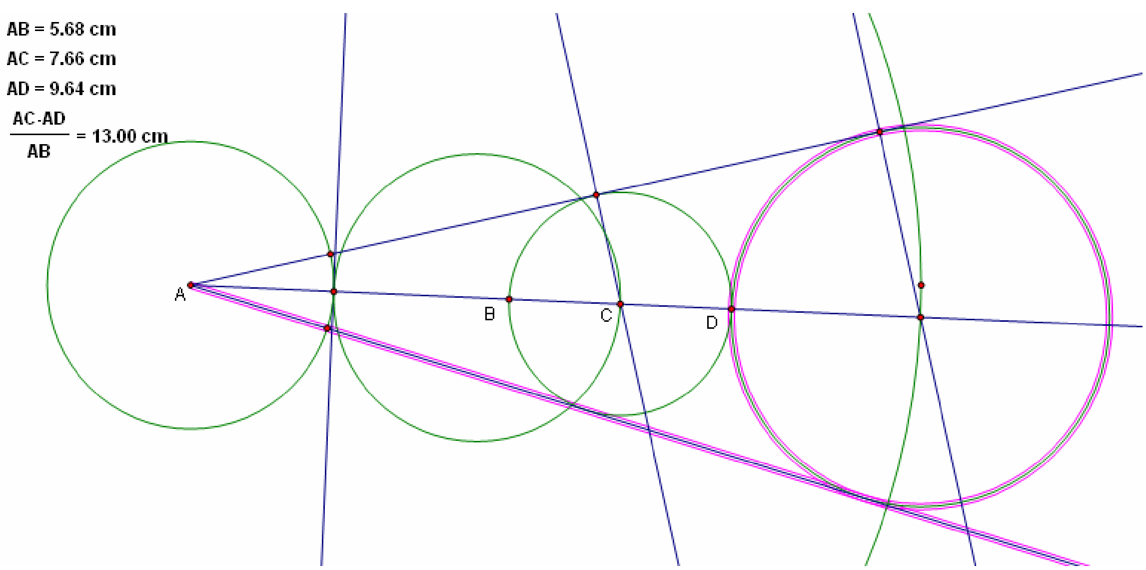


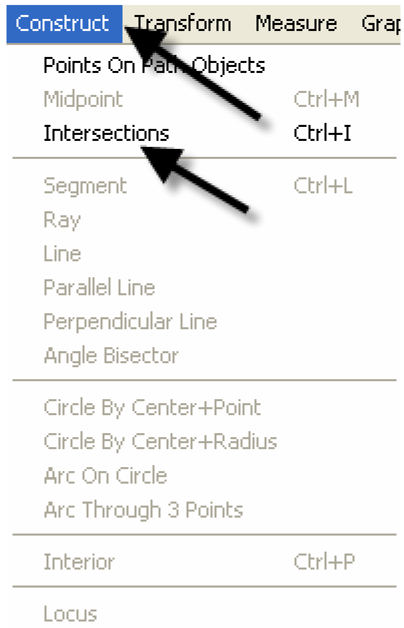

In order, select the new center point and the new point of intersection (see picture) and use **Construct** from the menu bar with the **Circle By Center+Point** option.



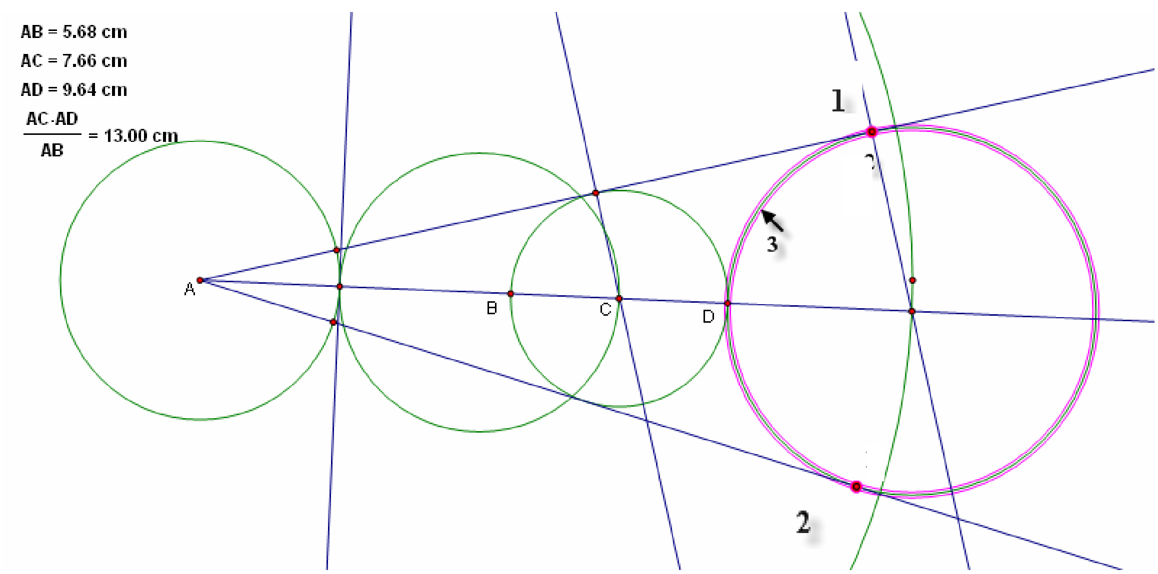



Select the new circle and the other ray that forms the angle. Use **Construct** from the menu bar with the **Intersection** option to create a point of tangency.

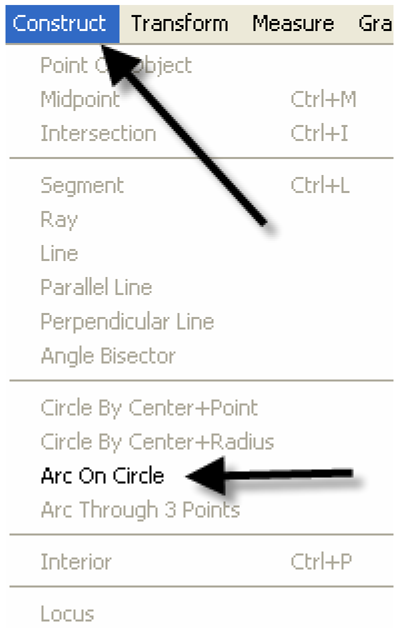


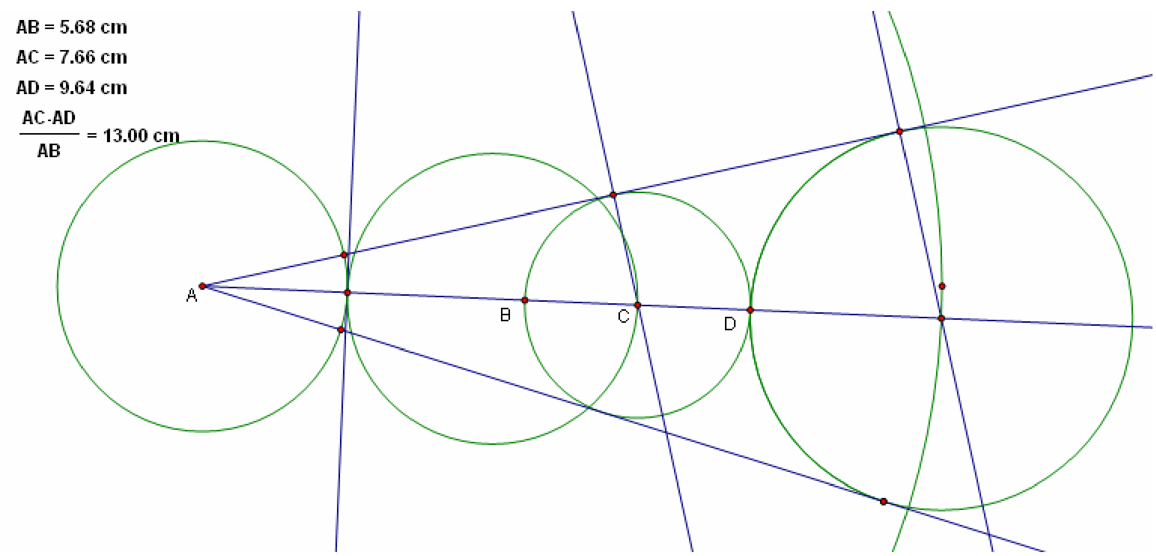



Construct the arc by selecting the points in a counter clockwise order, then selecting the circle.

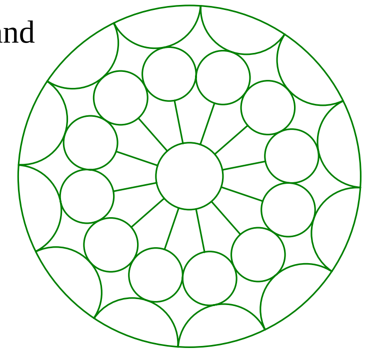


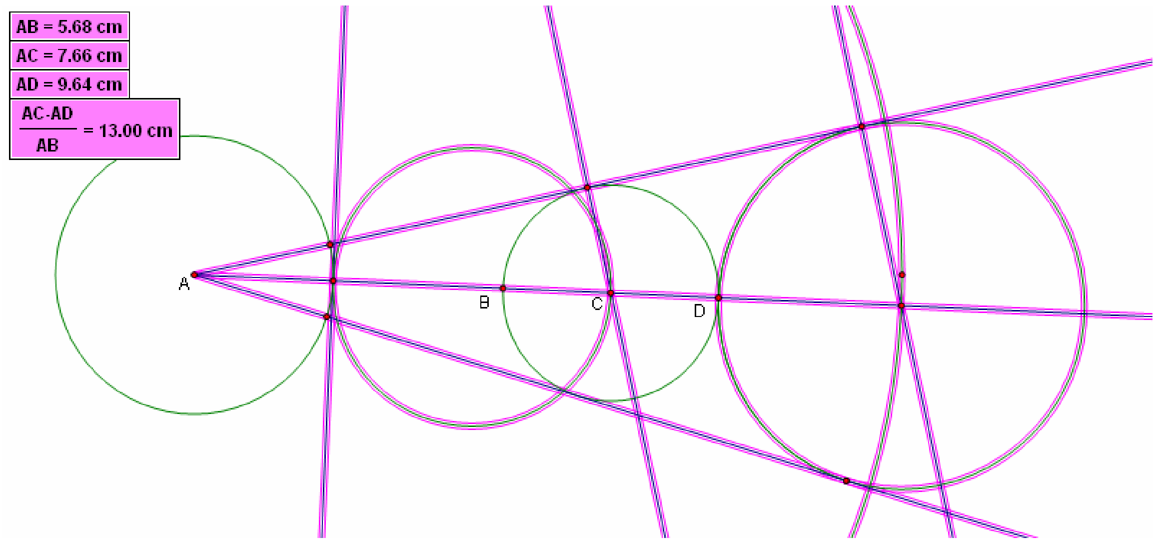
Next use **Construct** from the menu bar with the **Arc on a Circle** option.

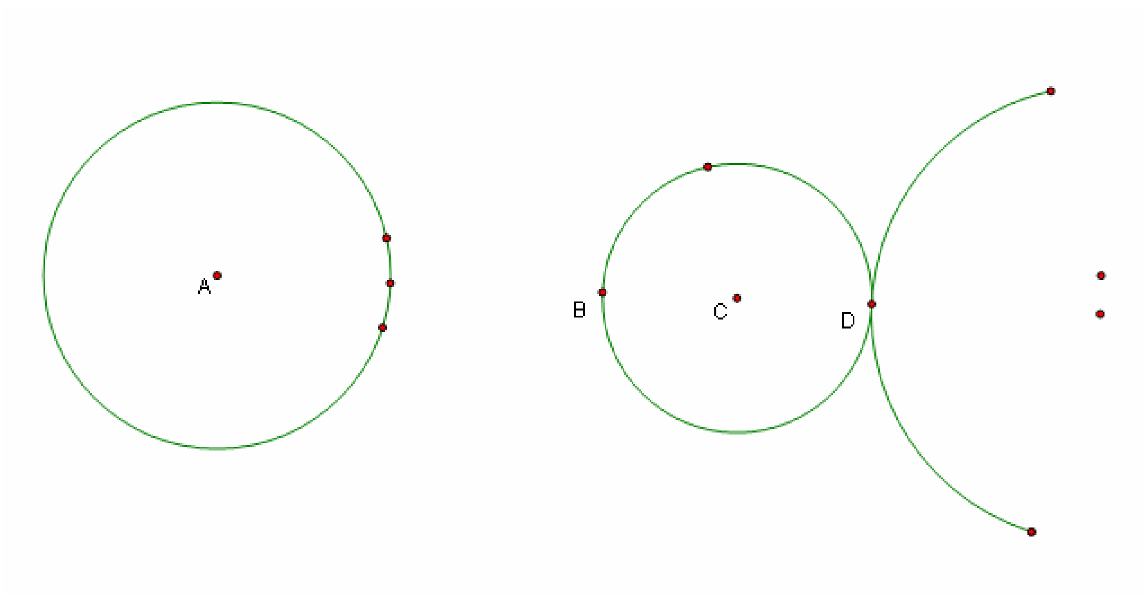


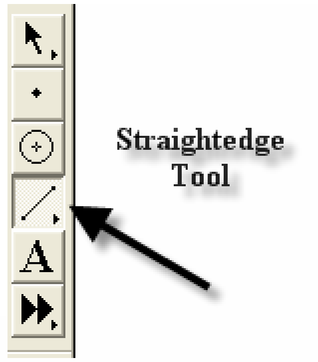
Hide all undesired parts of the construction by selecting them and using **Display** from the menu bar with the **Hide Path Objects** option.

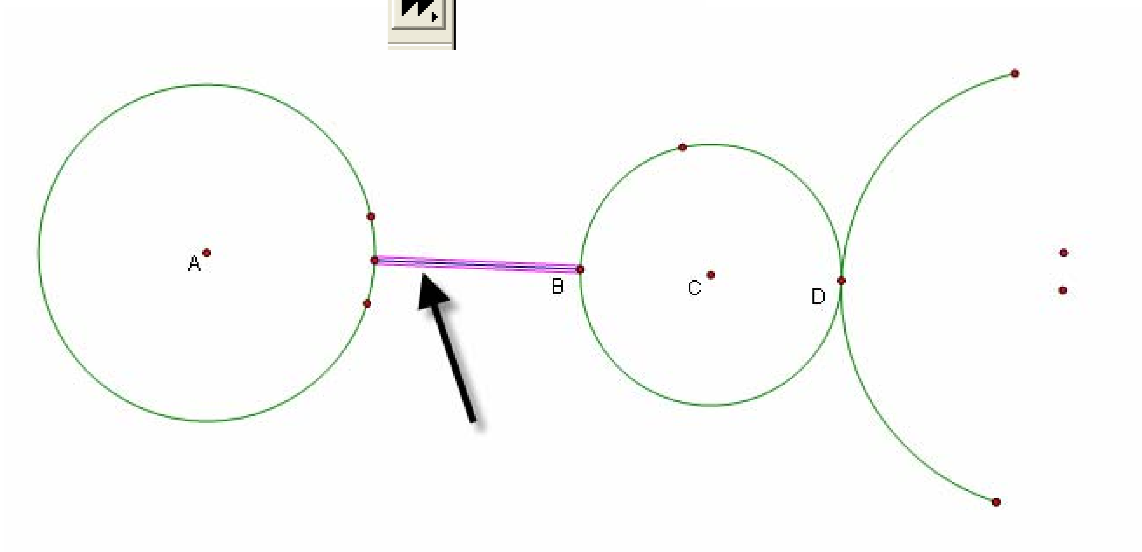




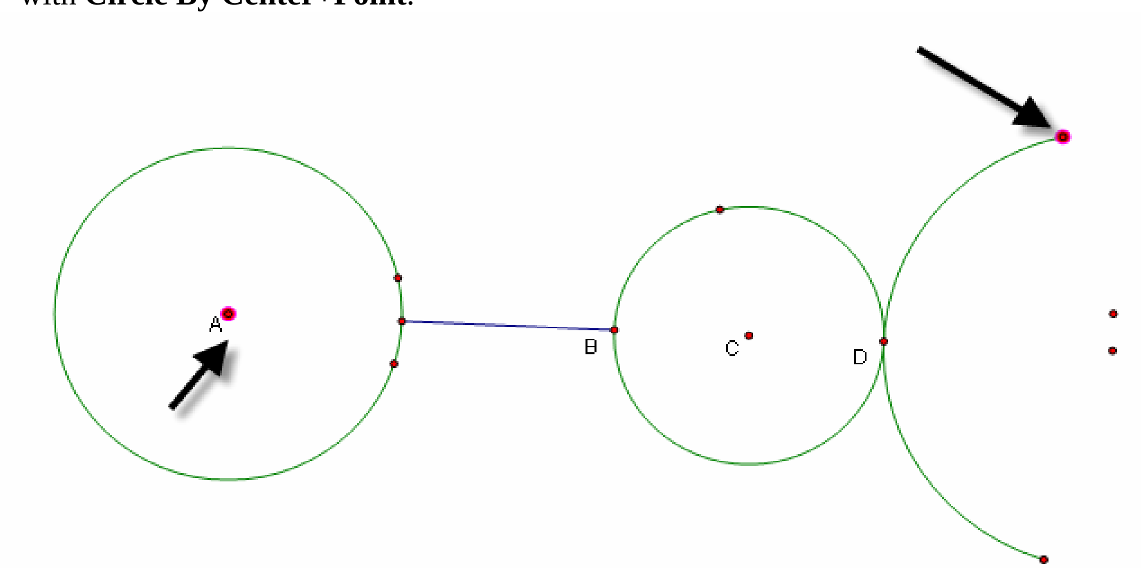



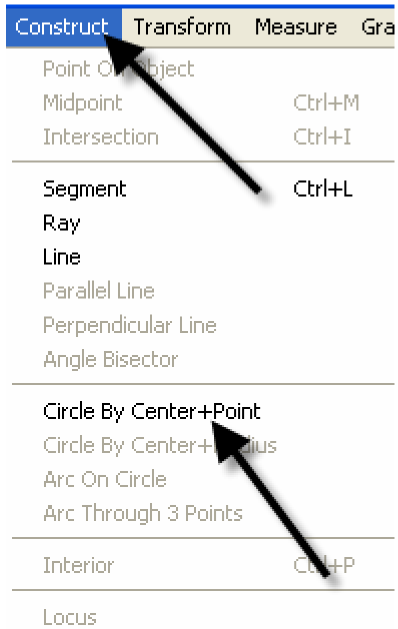
With the Straightedge Tool, construct a segment from the original circle to point ***B*** (See sketch.)



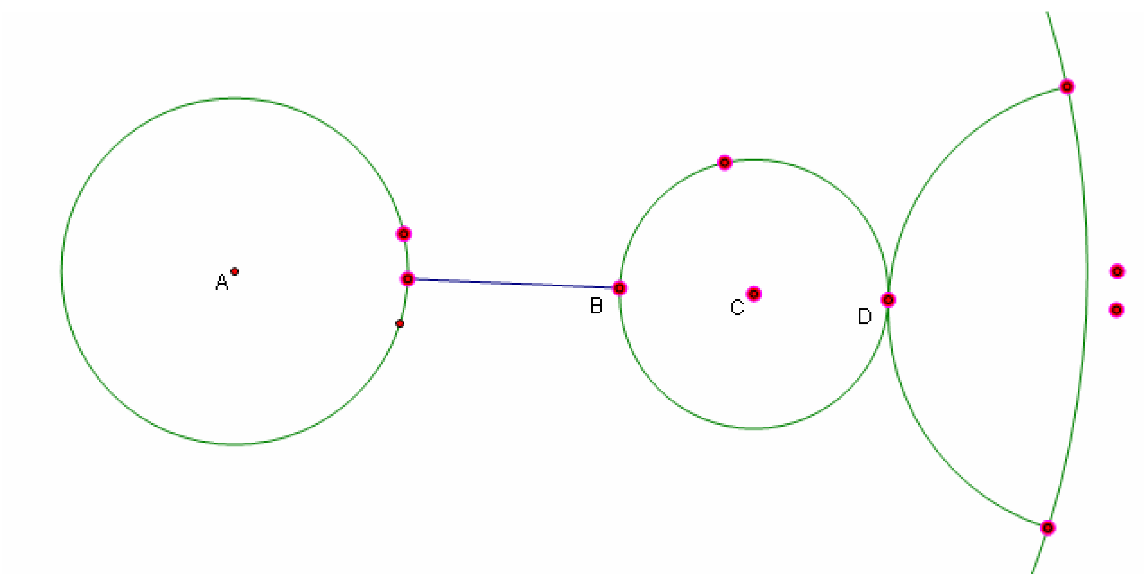



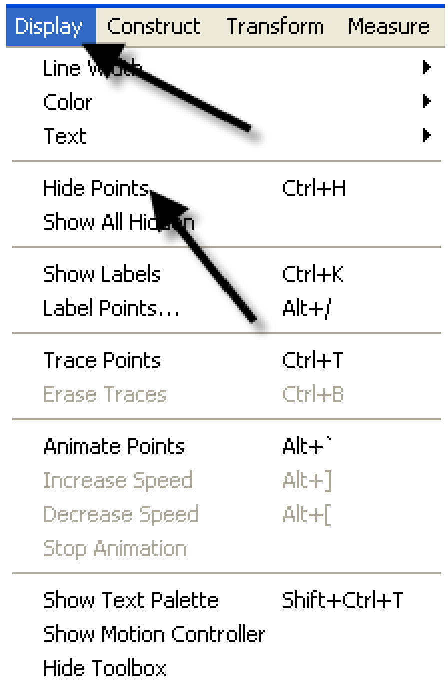

Select point ***A*** and one of the endpoints of the arc and use **Construct** from the menu bar with **Circle By Center+Point**.



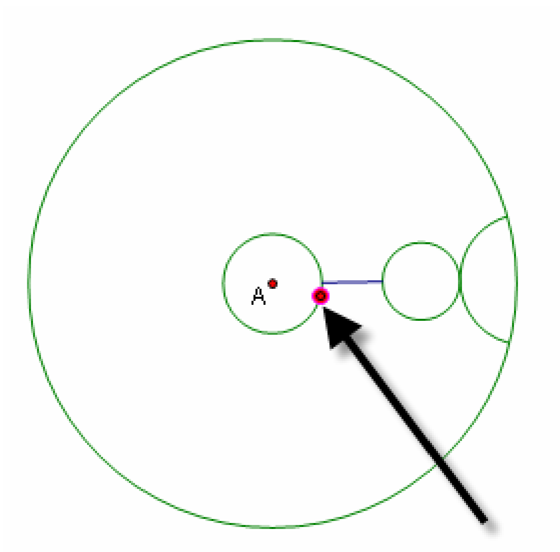



Hide the unnecessary points by first selecting them, then use Display from the menu bar with the **Hide Points** option.

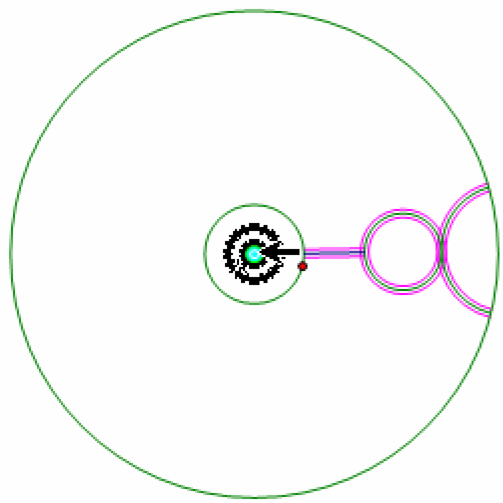


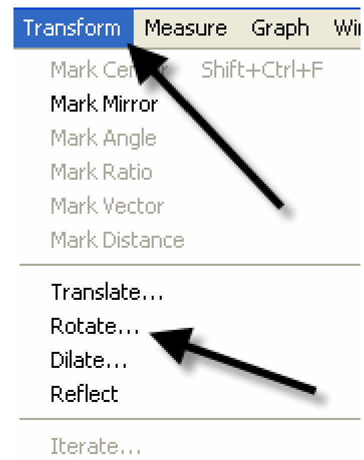



Shrink the construction by selecting the control point on circle ***A*** and moving it toward the center.

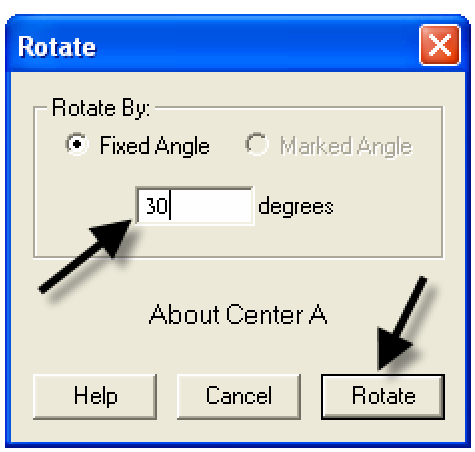


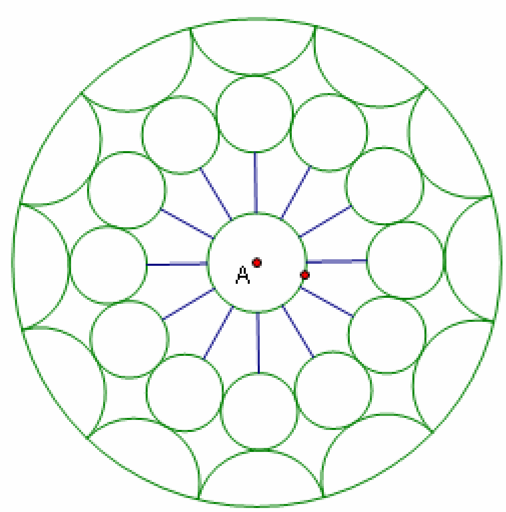
To rotate the construction around the circle, select the arc, small circle and segment. Double click on point ***A*** to mark the center of rotation. There will be concentric circles radiating out from point ***A*** as it is marked. Use **Transform** from the menu bar with the **Rotate** option.






A pop-up window will appear that will allow 30° to be entered in the window. Then select Rotate. Repeat the rotation until the construction is complete.



Hide point A if desired, but leave the control point for adjusting the size of the construction. If you want, you can select the entire construction and adjust the line thickness and color using **Display** from the menu bar with **Line Width/Thick** option, then **Display** with the **Color** option.